Math 1 – Problems instead of small tests

You can get $0, \frac{1}{2}, 1$ point from each small test.

- to get additional 0.5p you need to compute the first problem of the given topic
- to get additional 1p you need to compute all three subproblems for the given topic

Please give me solutions to these problems at the beginning of next exercise on a **signed, separate** paper.

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1. – LD/LI
   - Are given vectors LD?
     $\begin{pmatrix}4\\2\\1\end{pmatrix}, \begin{pmatrix}-2\\-1\\-4\end{pmatrix}$ and $\begin{pmatrix}5\\-1\\3\end{pmatrix}$.
   - Find vector $\vec{x}$ which satisfies: $2(\vec{x}+\vec{u}) = 3\vec{v} + \begin{pmatrix}0\\0\\2\end{pmatrix}$,
     where $\vec{u} = \begin{pmatrix}1\\-3\\0\end{pmatrix}$ and $\vec{v} = \begin{pmatrix}0\\2\\1\end{pmatrix}$.
   - Determine, if vector $\vec{x} = \begin{pmatrix}-1\\7\end{pmatrix}$ can be written as linear combination of vectors $\vec{a} = \begin{pmatrix}2\\2\end{pmatrix}$ and $\vec{b} = \begin{pmatrix}3\\-5\end{pmatrix}$?
     If yes, then calculate coefficients of such combination.
2. – Rank of matrix
   - Determine rank of matrix $A = \begin{pmatrix}1&2&1&1\\2&0&1&0\\1&1&0&1\\1&0&1&3\end{pmatrix}$.
   - Compute matrices $C = A \cdot B$ and $D = B \cdot A$ if possible, where $A = \begin{pmatrix}3&1&0\\5&2&-1\end{pmatrix}$ and $B = \begin{pmatrix}1&2\\-5&1\\0&-2\end{pmatrix}$.
   - Determine for which parameter $k \in \mathbb{R}$ is matrix $\begin{pmatrix}1&2&0\\k&k&1\\2&2&k-1\end{pmatrix}$
     regular. Consider computation of determinant.

3. – Inverse matrix

   - Compute inverse matrix to matrix $A = \begin{pmatrix} 1 & 2 & 3 \\ 0 & 3 & 1 \\ 0 & 0 & 2 \end{pmatrix}$.
   - For which parameter λ there is a solution?

$$\begin{aligned} 2x - y + z + u &= 1 \\ x + 2y - z + 4u &= 2 \\ x + 7y - 4z + 11u &= \lambda. \end{aligned}$$

   - Determine x_3 satisfying equations

$$\begin{aligned} x_1 - 2x_2 + 3x_3 - 4x_4 &= 4 \\ x_2 - x_3 + x_4 &= -3 \\ x_1 + 3x_2 - 3x_4 &= 1 \\ -7x_2 + 3x_3 + x_4 &= -3. \end{aligned}$$

4. – System of eqs.

   - Write the vectors $\vec{a} = \begin{pmatrix} 3 \\ 2 \\ 5 \end{pmatrix}$ and $\vec{b} = \begin{pmatrix} 5 \\ 6 \\ 7 \end{pmatrix}$ as a linear combination of following vectors $\begin{pmatrix} 1 \\ 3 \\ 2 \end{pmatrix}$, $\begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix}$ and $\begin{pmatrix} 5 \\ 1 \\ 8 \end{pmatrix}$. Is the expression unique?
   - Find a solution for x_1 depending on parameter $m \in \mathbb{R}$

$$\begin{aligned} -7x_2 - 5x_3 &= -1 \\ (2m-1)x_1 - x_2 &= 1 \\ 4mx_1 - 7x_2 - 5x_3 &= 0. \end{aligned}$$

   - Prove that sequence $a_n = \frac{n}{n+3}$ is increasing.

5. – Sequences and limits

   - Compute $\lim_{n\to+\infty} (\frac{4}{n} - \frac{3n}{n^2+1})$.
   - Compute $\lim_{n\to+\infty} \frac{6n^2-3n+2}{(3n-2)^2-9n^2}$.
   - Compute $\lim_{x\to 0} \frac{1}{x}$, $\lim_{x\to+\infty} \frac{1}{x}$ and $\lim_{x\to+\infty} \sin\left(\frac{1}{x}\right)$, $\lim_{x\to 0} \frac{\sin x}{x}$.

6. – Domain of function

   - Find domain of function $f(x) = \frac{1}{x-2}$ and its rank. Then plot its graph.
   - Find domain of function $f(x) = \ln\left(\frac{1}{x-2}\right)$ and its rank. Then plot its graph.
   - Compute derivative of functions:

     $$1)\ \cos x^2 \qquad 2)\ (\cos x)^2 \qquad 3)\ \cos(\cos x)$$

7. – Derivation

   - Compute derivative of functions:

     $$1)\ \tan(x+1) \qquad 2)\ (\arctan x)^2 \qquad 3)\ \ln(\cos x)$$

   - For which real numbers b, c has function $f(x) = x^2+bx+c$ tangent line $y = x$ in the point $x = 2$?
   - Write tangent line of function $f(x) = \frac{8}{4+x^2}$ in the point $x_0 = 2$ and approximate with it unknown value $f(1.9)$.

8. – Tangent

   - By using tangent line of function approximate value of $h(2) = \ln 5$, where $h(x) = \ln(x^2+1)$ with choice $x_0 = 3$.
   - Determine local extrema of function $f(x) = \frac{2x}{x^2+1}$.
   - A firm uses a single input, labor, to produce output q according to the production function $q = 10\sqrt{L}$. The commodity sells for \$250 per unit and the wage rate is \$50 per hour. Find:
     a) the profit-maximizing quantity of L,
     b) the profit-maximizing quantity of q,
     c) the maximum profit.

9. – Local extrema

   - Determine local extrema of function $g(x) = x^2 \ln x$
   - Find intervals where function $f(x) = x \ln x$ is convex.
   - Analyse behaviour of function $f(x) = x + \frac{1}{x}$.

10. – Integrals

    - Compute integral $\int x \ln x \ \mathrm{d}x$.
    - Compute integral $\int \frac{5x-4}{x^2-8x+12} \ \mathrm{d}x$.
    - Compute average production of company ABC during two years, if the production depends on season and it is given by $f(t) = 2 + \sin\left(\frac{t}{2\pi}\right)\cos\left(\frac{t}{2\pi}\right)$.