Aeroelastic problem with three degrees of freedom

Abstract

This report includes the formulation of the two dimensional aeroelastic problem with structure with three degrees of freedom.

Contents

1 Airfoil notation

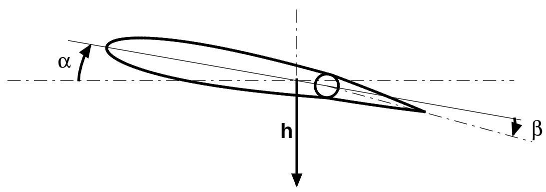

Figure 1: Airfoil pitching, plunging and rotation of the flap

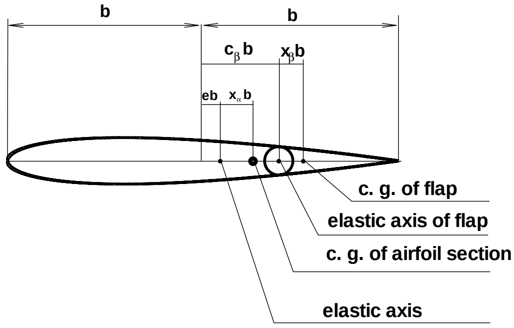

Figure 2: Typical airfoil section with three degrees of freedom.
By b the semichord of the airfoil is denoted, $e\,b$ denotes the location of the elastic axis of the wing after midchord, $x_\alpha\,b$ the location of the center of gravity after the elastic axis, $c_\beta\,b$ denotes the location of the flap hinge after the midchord and $x_\beta\,b$ the location of the center of gravity of the flap.

2 Mathematical description of the airfoil with three dof

A typical section airfoil (semichord b) in subsonic air flow is considered as shown in Figure 1. A trailing edge flap is hinged at the distance $c_\beta b$ after the midchord. By h, α and β the plunging of the elastic axis, pitching of the airfoil and rotation of the flap is denoted, respectively (see Figure 2). The fluid motion generates an aerodynamic lift $L = L(t)$, an aerodynamic moment $M = M(t)$ and an hinge moment $M_\beta = M_\beta(t)$. By k_h, k_α and k_β the spring constant of the wing bending, the wing torsional stiffness and the flap hinge moment are denoted, respectively. The mass matrix $\mathbb{M}$ of the structural system is defined with the aid of the entire airfoil mass m, the moment of inertia I_α of the airfoil around the elastic axis and the flap moment of inertia I_β of the flap around the hinge. The equations of the motion for a flexibly supported rigid airfoil with flap, cf. Dowell1995, read

$$\mathbb{M}\,\ddot{\boldsymbol{\eta}} + \mathbb{B}\dot{\boldsymbol{\eta}} + \mathbb{K}\,\boldsymbol{\eta} + \mathbf{f}_{\boldsymbol{NL}}(\boldsymbol{\eta}) = \mathbf{f}, \tag{1}$$

where

$$\mathbb{M} = \begin{pmatrix} m & S_\alpha & S_\beta \\ S_\alpha & I_\alpha & (c_\beta - e)bS_\beta + \mathbf{I}_\beta \\ S_\beta & (c_\beta - e)bS_\beta + \mathbf{I}_\beta & I_\beta \end{pmatrix},$$

$$\mathbb{K} = \begin{pmatrix} k_h & 0 & 0 \\ 0 & k_\alpha & 0 \\ 0 & 0 & k_\beta \end{pmatrix}, \qquad \mathbb{D} = \begin{pmatrix} d_h & 0 & 0 \\ 0 & d_\alpha & 0 \\ 0 & 0 & d_\beta \end{pmatrix},$$

where $\eta = (h, \alpha, \beta)^T$, $\mathbf{f} = (-L, M, M_\beta)^T$. By $\mathbf{f}_{NL}$ the nonlinear terms are denoted, e.g. the weakening/hardening effects of the flap hinge spring can be considered.

3 Methods description

3.1 Analytical/Euler's method

The Euler method of solution base on eigenvalues/eigenfrequencies is employed. The system (1) is transformed to the system of the first order by the substitution

$$\boldsymbol{x} = \begin{pmatrix} \boldsymbol{\eta} \\ \dot{\boldsymbol{\eta}} \end{pmatrix},$$

thus

$$\begin{pmatrix} \mathbb{E} & 0 \\ 0 & \mathbb{M} \end{pmatrix} \frac{d}{dt} \begin{pmatrix} \eta \\ \dot{\eta} \end{pmatrix} = \begin{pmatrix} 0 & \mathbb{E} \\ -\mathbb{K} & -\mathbb{D} \end{pmatrix} \begin{pmatrix} \eta \\ \dot{\eta} \end{pmatrix},$$

and

$$\frac{d}{dt} \begin{pmatrix} \eta \\ \dot{\eta} \end{pmatrix} = \begin{pmatrix} 0 & \mathbb{E} \\ -\mathbb{M}^{-1}\mathbb{K} & -\mathbb{K}^{-1}\mathbb{D} \end{pmatrix} \begin{pmatrix} \eta \\ \dot{\eta} \end{pmatrix}. \tag{2}$$

The frequency is then found as imaginary parts $f = \beta/(2\pi)$ of the eigenvalues $\lambda = \alpha + i\beta$ of the matrix $\mathbb{A}$

$$\mathbb{A} = \begin{pmatrix} 0 & \mathbb{E} \\ -\mathbb{M}^{-1}\mathbb{K} & -\mathbb{K}^{-1}\mathbb{D} \end{pmatrix}.$$

3.2 Numerical method/Fourier transformation

The system (2) was approximated by 4th order RK method. Here, the method is given for general system $\dot{\mathbf{y}} = f(t, \mathbf{y})$. The approximation at time t^n is denoted by $\mathbf{y}^n$. Then for $n = 0, 1, \ldots$ we have

$$\begin{aligned}
\mathbf{k}_1 &= \mathbf{f}(t_n, \mathbf{y}^n), \\
\mathbf{k}_2 &= \mathbf{f}(t_n + \frac{1}{2}\Delta t, \mathbf{y}^n + \frac{1}{2}\Delta t \mathbf{k}_1), \\
\mathbf{k}_3 &= \mathbf{f}(t_n + \frac{1}{2}\Delta t, \mathbf{y}^n + \frac{1}{2}\Delta t \mathbf{k}_2), \\
\mathbf{k}_4 &= \mathbf{f}(t_n + \Delta t, \mathbf{y}^n + \Delta t \mathbf{k}_3), \\
\mathbf{y}^{n+1} &= \mathbf{y}^n + \frac{\Delta t}{6}\left(\mathbf{k}_1 + 2\mathbf{k}_2 + 2\mathbf{k}_3 + \mathbf{k}_4\right).
\end{aligned} \tag{3}$$

In order to validate both the numerical method and the implementation the method was included into **Fluid-Structure Interaction Program**, where the fluid part of the solution was ommited, zero aerodynamical forces ($\mathbf{f} = 0$, $\mathbf{f}_{NON} = 0$) used and the solution of the system of ordinary differential equations obtained by (3).

4 Analysis of the eigenfrequencies

The problem

$$\mathbb{M} = \begin{pmatrix} m & S_\alpha & S_\beta \\ S_\alpha & I_\alpha & (c_\beta - e)bS_\beta + \mathbf{I}_\beta \\ S_\beta & (c_\beta - e)bS_\beta + \mathbf{I}_\beta & I_\beta \end{pmatrix} =$$

$$= \begin{pmatrix} 0.08662 & -0.7796 & S_\beta \\ -0.7796 & 4.87291 \cdot 10^{-4} & (c_\beta - e)bS_\beta + I_\beta \\ S_\beta & (c_\beta - e)bS_\beta + I_\beta & I_\beta \end{pmatrix},$$

$$\mathbb{K} = \begin{pmatrix} 105.109 & 0 & 0 \\ 0 & 3.695582 & 0 \\ 0 & 0 & k_\beta \end{pmatrix},$$

where $k_\beta \in [0.001, 0.05]$.

- **Model problem I.** (flap $\pm 0\%$)

$$S_\beta = 0 \text{ kg m}, \qquad I_\beta = 1 \cdot 10^{-6} \text{ kg m}^2, \qquad (c_\beta - e)b = 0.12 \text{ m}$$

- **Model problem II.** (flap $+5\%$, center of gravity of the flap closer to the elastic axis of the airfoil, distance 0.105 m.)

$$S_\beta = -9.09 \cdot 10^{-4} \text{ kg m}, \qquad I_\beta = 2.364 \cdot 10^{-6} \text{ kg m}^2, \qquad (c_\beta - e)b = 0.12 \text{ m}$$

- **Model problem III.** (flap -5%, center of gravity of the flap closer to the elastic axis of the airfoil, distance 0.135 m.)

$$S_\beta = 9.09 \cdot 10^{-4} \text{ kg m}, \qquad I_\beta = 2.364 \cdot 10^{-6} \text{ kg m}^2, \qquad (c_\beta - e)b = 0.12 \text{ m}$$

$l = 0.079$ **m ???**

4.1 Model problem I.

MATLAB script

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```
i=0;
F=zeros(4,3);

format long

m=    0.086622;
kh=   105.109;
ka=   3.695582;
Sa=  -7.796e-4;
Ia=   4.87291e-4; 3.981e-4;   %Ia=3.981e-4+vzdalenoast^2*m2;
dEO_FLAP=0.12;
dEO_TEZ= 0.12;

Sb=0; Ib=1e-6;
m2=0.00606;
for kb=0.0005:0.0005:0.05,
  M=[ m Sa Sb; Sa Ia Ib+Sb*dEO_FLAP; Sb Ib+Sb*dEO_FLAP Ib];
  K=diag([kh,ka,kb]);
  D=0.0*K;

  MM=[ zeros(3,3), eye(3,3); -inv(M)*K, -D];
  aa=eig(MM);  freq=imag(aa);
  pom=freq(1:2:end)/(2*pi);

  i=i+1; F(i,1:4)=[kb,pom'];

end

plot(F(:,1),F(:,2),'o',F(:,1),F(:,3),'o',F(:,1),F(:,4),'o')
print -deps freq0perc.eps
M
K
D
```

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MATLAB script output

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```
octave:1> analyza
M =

   0.0866220000000000  -0.0007796000000000   0.0000000000000000
  -0.0007796000000000   0.0004872910000000   0.0000010000000000
   0.0000000000000000   0.0000010000000000   0.0000010000000000

K =

  105.108999999999995    0.000000000000000    0.000000000000000
    0.000000000000000    3.695582000000000    0.000000000000000
    0.000000000000000    0.000000000000000    0.050000000000000

D =

  0  0  0
  0  0  0
  0  0  0

octave:2>
```

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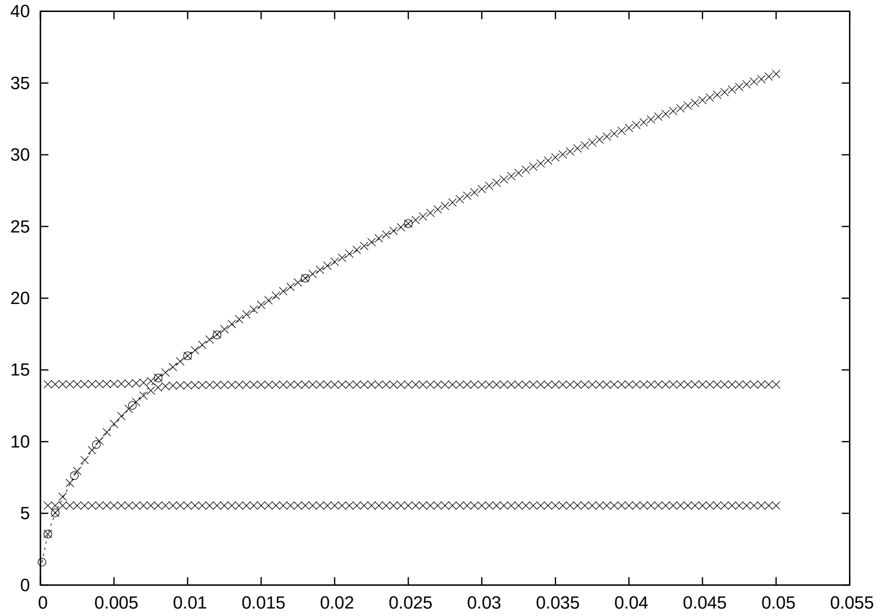
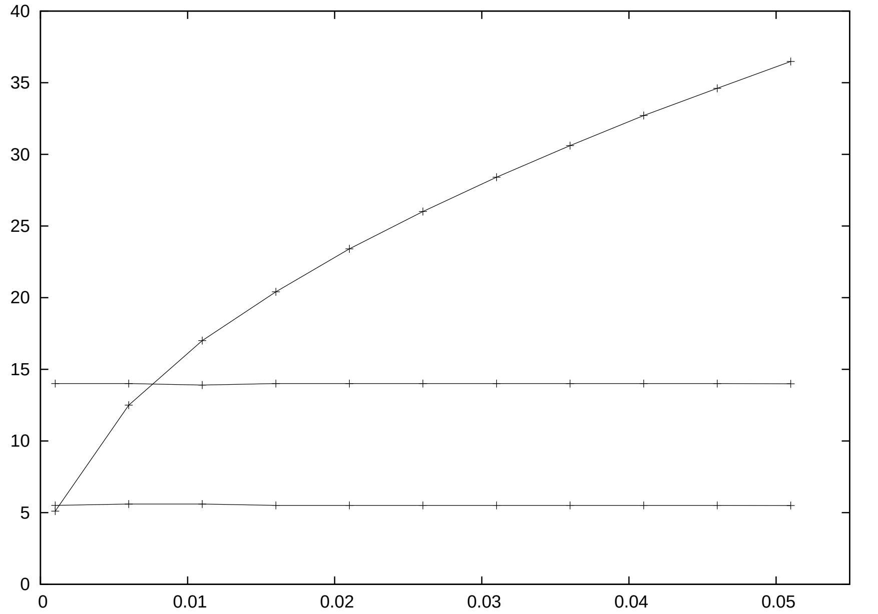
4.1.1 Analytical Frequencies



4.1.2 Numerical Frequencies



4.2 Model problem II.

MATLAB script

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```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
i=0;
F=zeros(4,3);

m=    0.08662;
kh= 105.109;
ka=   3.695582;
Sa=  -7.796e-4;
Ia=   4.87291e-4; 3.981e-4;   %Ia=3.981e-4+vzdalenoast^2*m2;
dEO_FLAP=0.12;
dEO_TEZ= 0.105;
Sb=-9.09e-5;
Ib=2.364e-6;
m2=0.00606;

for kb=0.0001:0.0001:0.05,
  M=[
        m       Sa               Sb;
        Sa      Ia               Ib+Sb*dEO_FLAP;
        Sb      Ib+Sb*dEO_FLAP  Ib];
  K=diag([kh,ka,kb]);
  D=0.0*K;

  MM=[ zeros(3,3), eye(3,3); -inv(M)*K, -D];
  aa=eig(MM);  freq=imag(aa);
  pom=abs(freq(2:2:end))/(2*pi);

  i=i+1; F(i,1:4)=[kb,pom'];

end

plot(F(:,1),F(:,2),'o',F(:,1),F(:,3),'+',F(:,1),F(:,4),'*')
print -deps freqpred.eps
```

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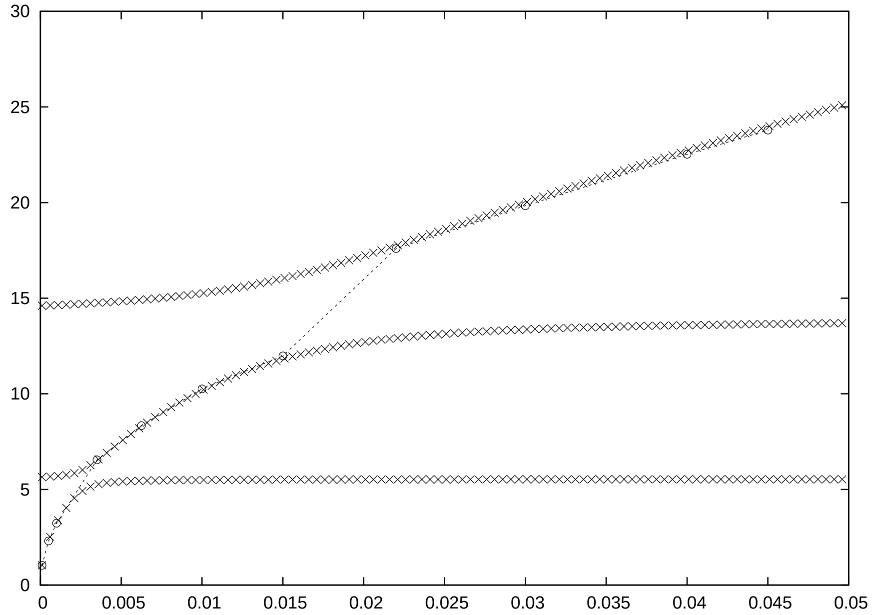
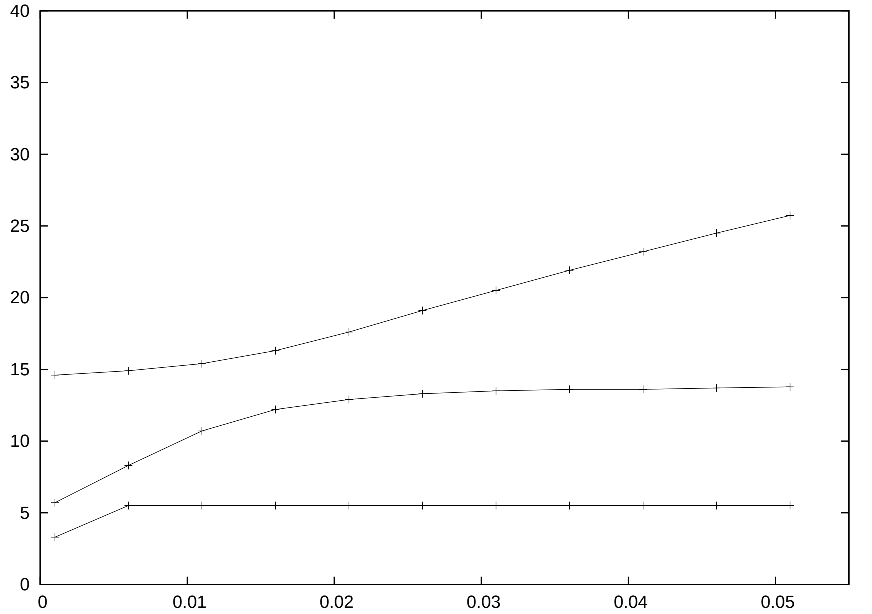
4.2.1 Analytical Frequencies



4.2.2 Numerical Frequencies



4.3 Model problem III.

MATLAB script

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```
i=0;
F=zeros(4,3);

m=    0.086622;
kh= 105.109;
ka=   3.695582;
Sa=  −7.796e−4;
Ia=   4.87291e−4; 3.981e−4;   %Ia=3.981e−4+vzdalenoast^2*m2;
dEO_FLAP=0.12;
dEO_TEZ= 0.135;
Sb=9.09e−5;
Ib=2.364e−6;
m2=0.00606;
for kb=0.0005:0.0005:0.05,
  M=[ m Sa Sb; Sa Ia Ib+Sb*dEO_FLAP; Sb Ib+Sb*dEO_FLAP Ib];
  K=diag([kh,ka,kb]);
  D=0.0*K;

  MM=[ zeros(3,3), eye(3,3); −inv(M)*K, −D];
  aa=eig(MM);  freq=imag(aa);
  pom=freq(1:2:end)/(2*pi);

  i=i+1; F(i,1:4)=[kb,pom’];

end

plot(F(:,1),F(:,2),’o’,F(:,1),F(:,3),’o’,F(:,1),F(:,4),’o’)
print −deps freqza.eps
```

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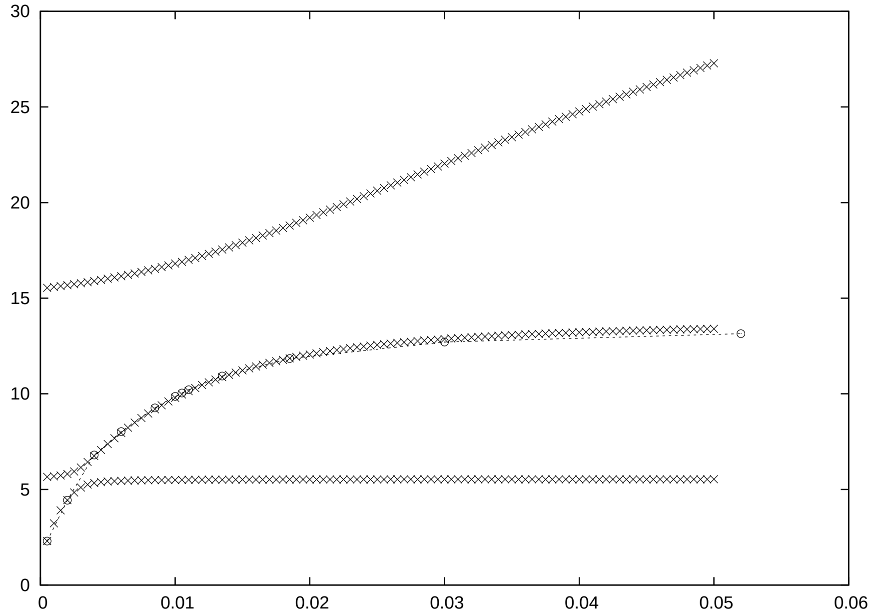
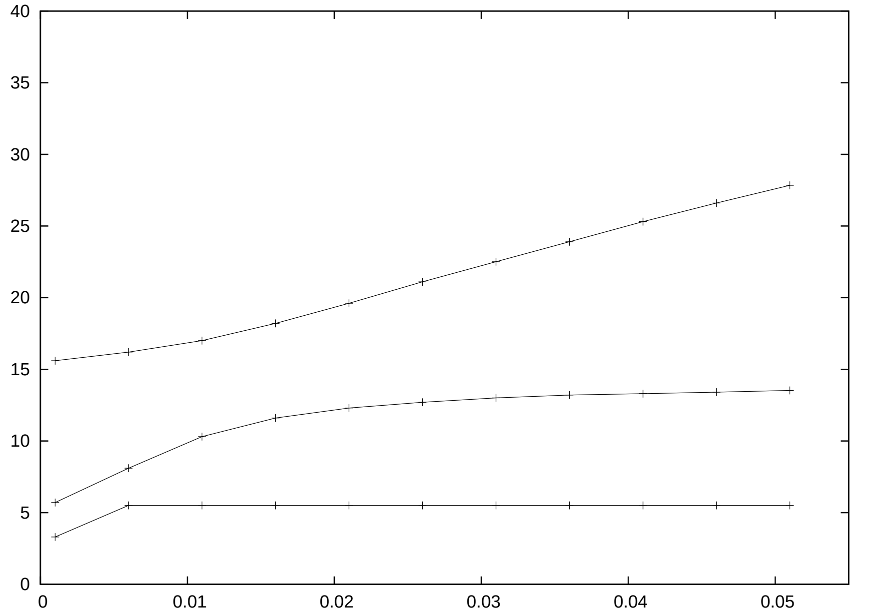
4.3.1 Analytical Frequencies



4.3.2 Numerical Frequencies



5 Influence of the proportional structural damping on frequencies

The problem

$$\mathbb{M} = \begin{pmatrix} m & S_\alpha & S_\beta \\ S_\alpha & I_\alpha & (c_\beta - e)bS_\beta + \mathbf{I}_\beta \\ S_\beta & (c_\beta - e)bS_\beta + \mathbf{I}_\beta & I_\beta \end{pmatrix} =$$

$$= \begin{pmatrix} 0.08662 & -0.7796 & S_\beta \\ -0.7796 & 4.87291 \cdot 10^{-4} & (c_\beta - e)bS_\beta + I_\beta \\ S_\beta & (c_\beta - e)bS_\beta + I_\beta & I_\beta \end{pmatrix},$$

$$\mathbb{K} = \begin{pmatrix} 105.109 & 0 & 0 \\ 0 & 3.695582 & 0 \\ 0 & 0 & k_\beta \end{pmatrix},$$

$$\mathbb{D} = 0.001\,\mathbb{K},$$

where $k_\beta \in [0.001, 0.05]$.

- **Model problem I.** (flap $\pm 0\%$)

$$S_\beta = 0 \text{ kg m}, \qquad I_\beta = 1 \cdot 10^{-6} \text{ kg m}^2, \qquad (c_\beta - e)b = 0.12 \text{ m}$$

- **Model problem II.** (flap $+5\%$, center of gravity of the flap closer to the elastic axis of the airfoil, distance 0.105 m.)

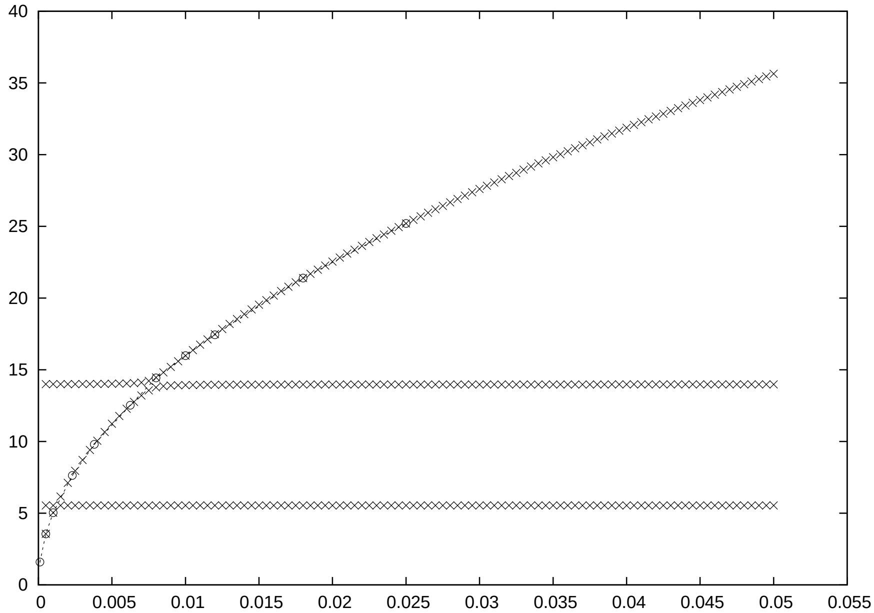
$$S_\beta = -9.09 \cdot 10^{-4} \text{ kg m}, \qquad I_\beta = 2.364 \cdot 10^{-6} \text{ kg m}^2, \qquad (c_\beta - e)b = 0.12 \text{ m}$$

- **Model problem III.** (flap -5%, center of gravity of the flap closer to the elastic axis of the airfoil, distance 0.135 m.)

$$S_\beta = 9.09 \cdot 10^{-4} \text{ kg m}, \qquad I_\beta = 2.364 \cdot 10^{-6} \text{ kg m}^2, \qquad (c_\beta - e)b = 0.12 \text{ m}$$

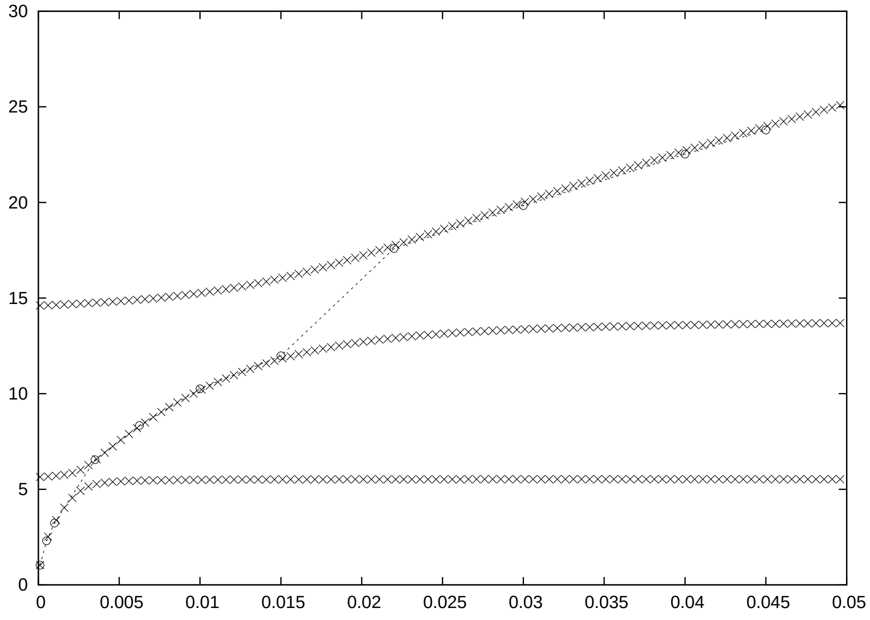
5.1 Model problem I.

5.1.1 Analytical Frequencies



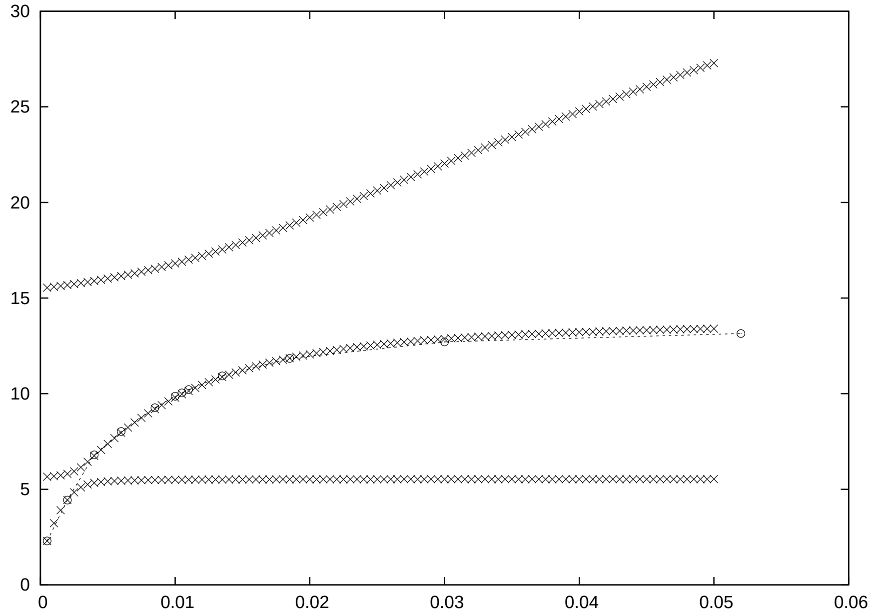
5.2 Model problem II.

5.2.1 Analytical Frequencies



5.3 Model problem III.

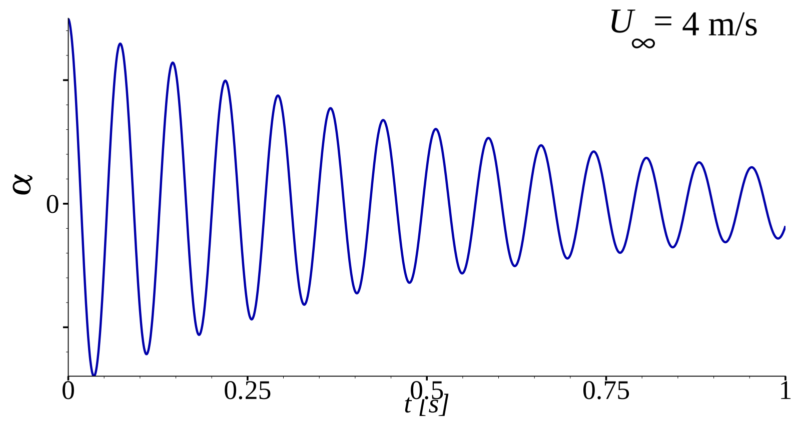
5.3.1 Analytical Frequencies

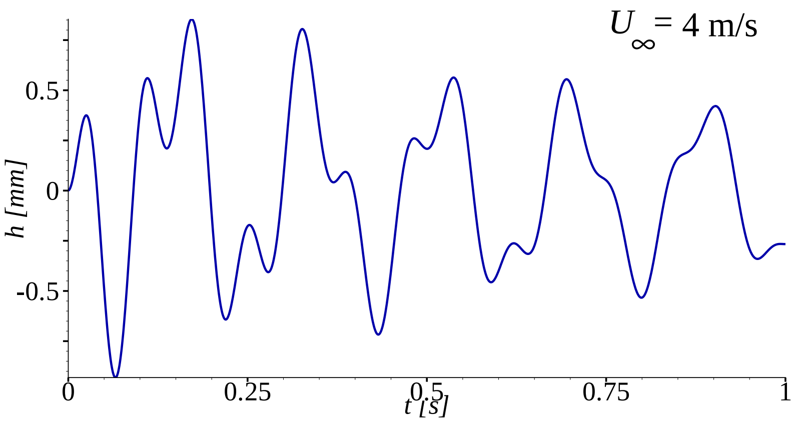


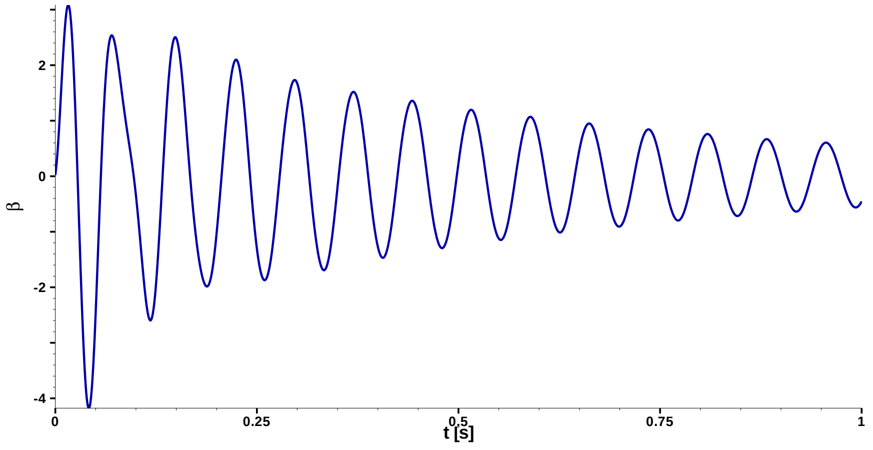
6 Aeroelastic Simulations

6.1 Model Problem I.

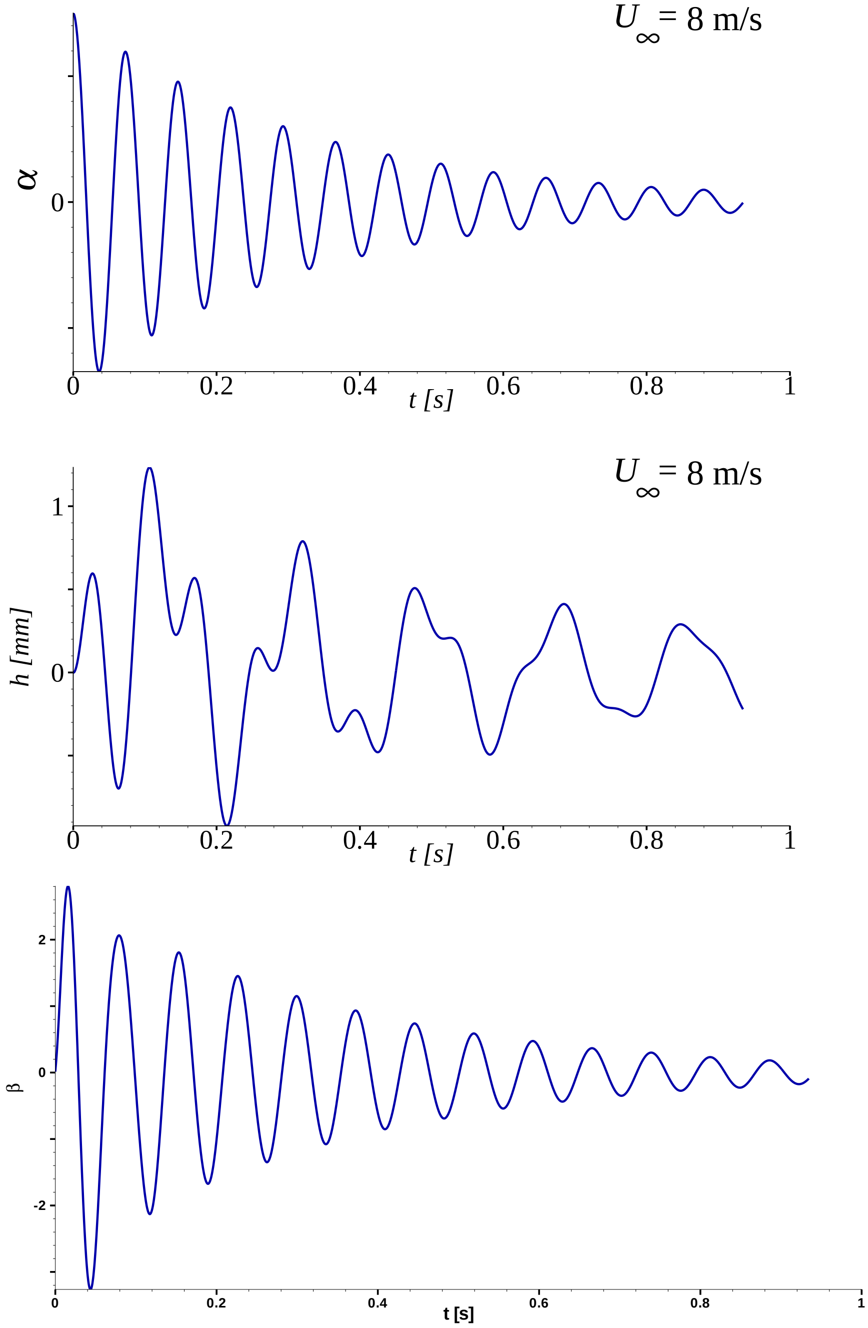
$$U = 4m/s$$



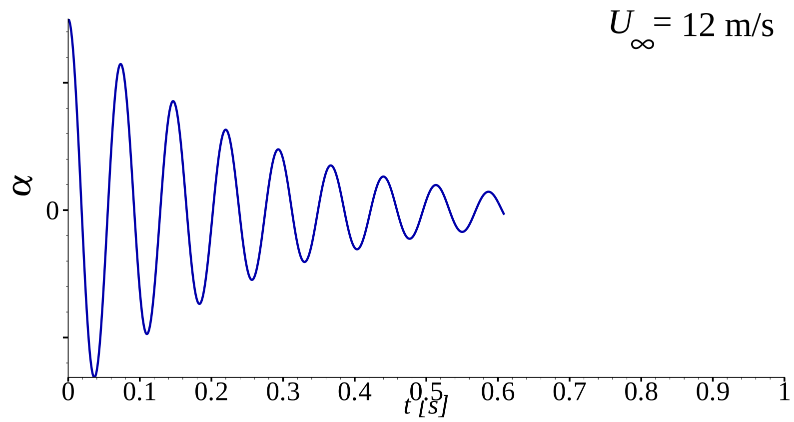


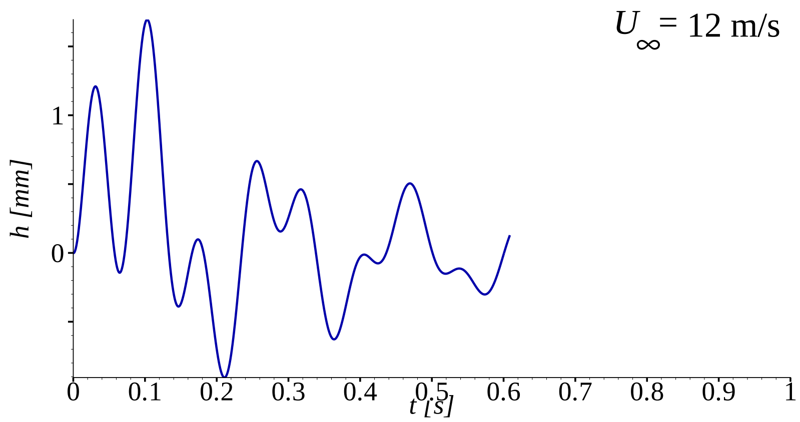


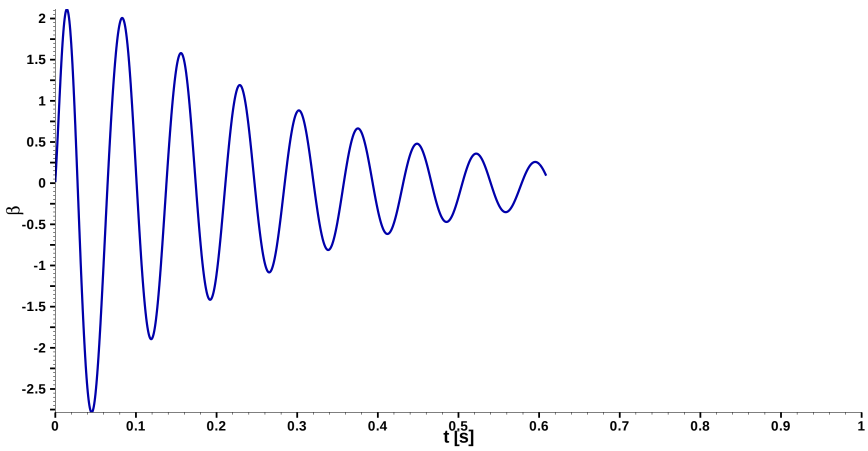
$U = 8m/s$


U∞ = 8 m/s
α
0
0
0.2
0.4
t [s]
0.6
0.8
1
U∞ = 8 m/s
h [mm]
1
0
0
0.2
0.4
t [s]
0.6
0.8
1
β
2
0
-2
0
0.2
0.4
t [s]
0.6
0.8
1

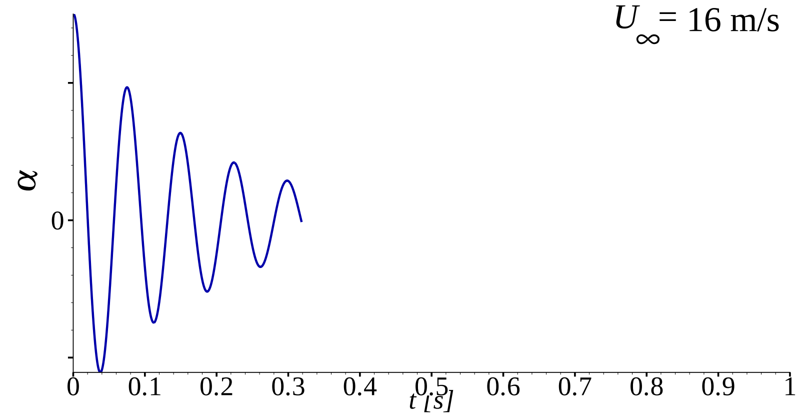
$U = 12m/s$

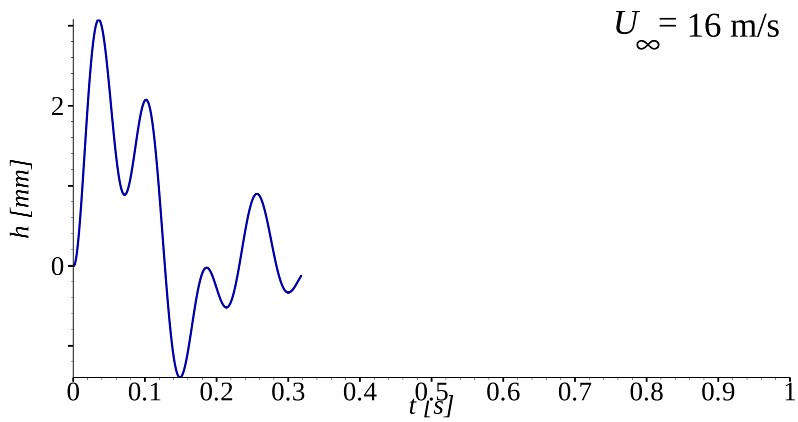

U_∞ = 12 m/s
α
0
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
t [s]

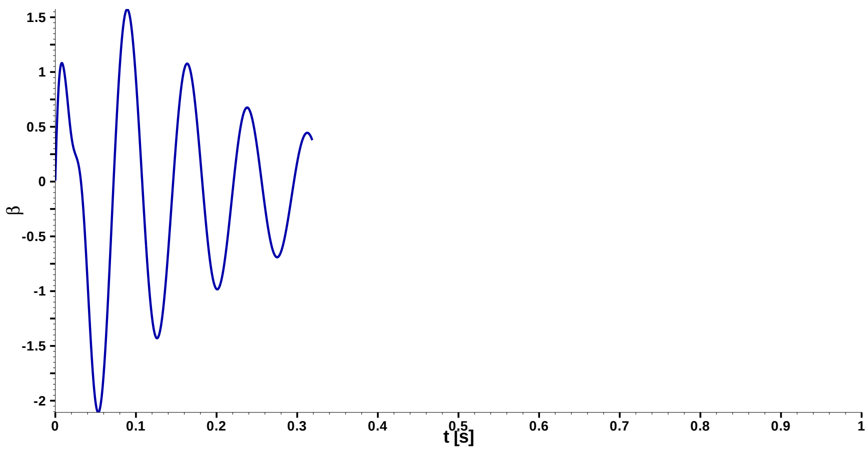

U_∞ = 12 m/s
h [mm]
1
0
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
t [s]


β
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-2.5
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
t [s]

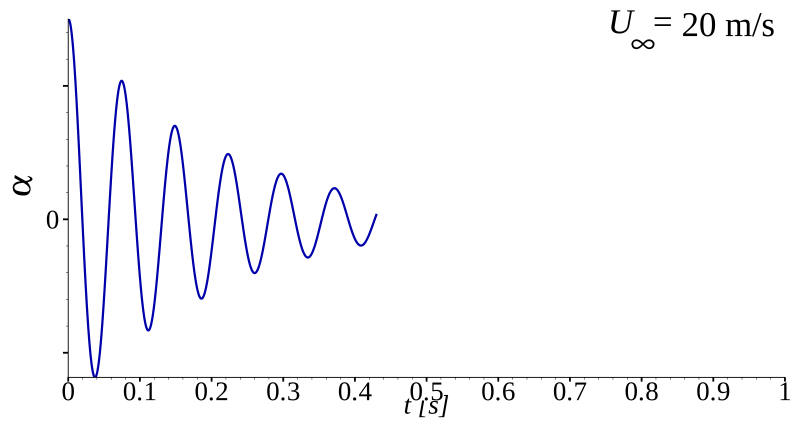
$U = 16m/s$

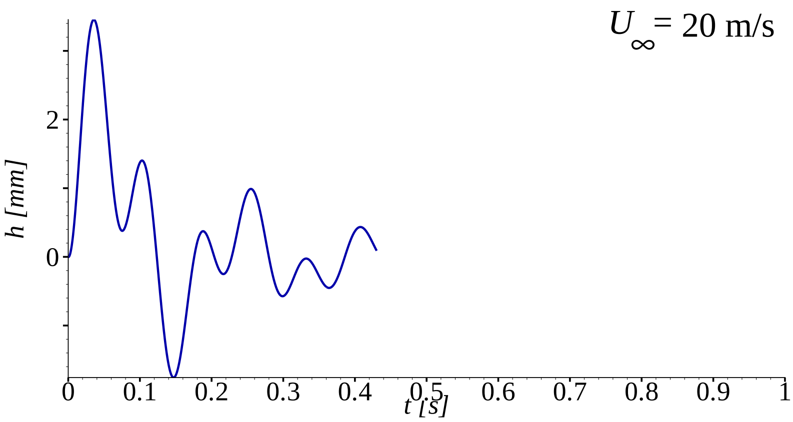

U_∞ = 16 m/s
α
0
0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1
t [s]

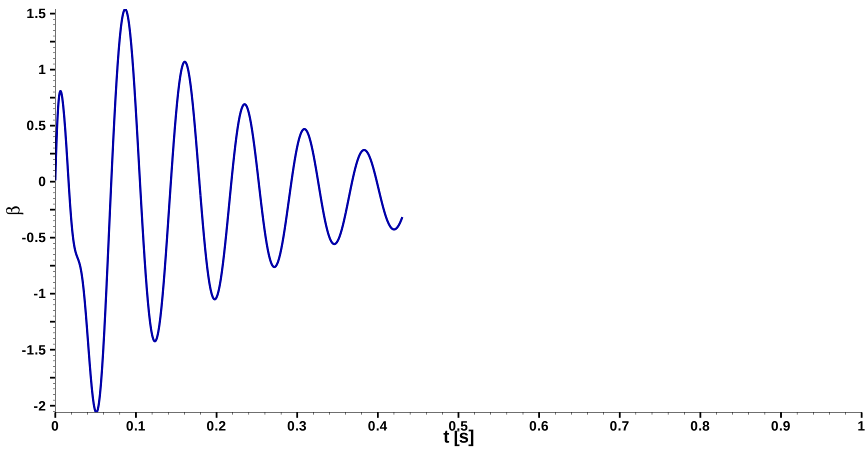

U_∞ = 16 m/s
h [mm]
2
0
0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1
t [s]


β
1.5
1
0.5
0
-0.5
-1
-1.5
-2
0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1
t [s]

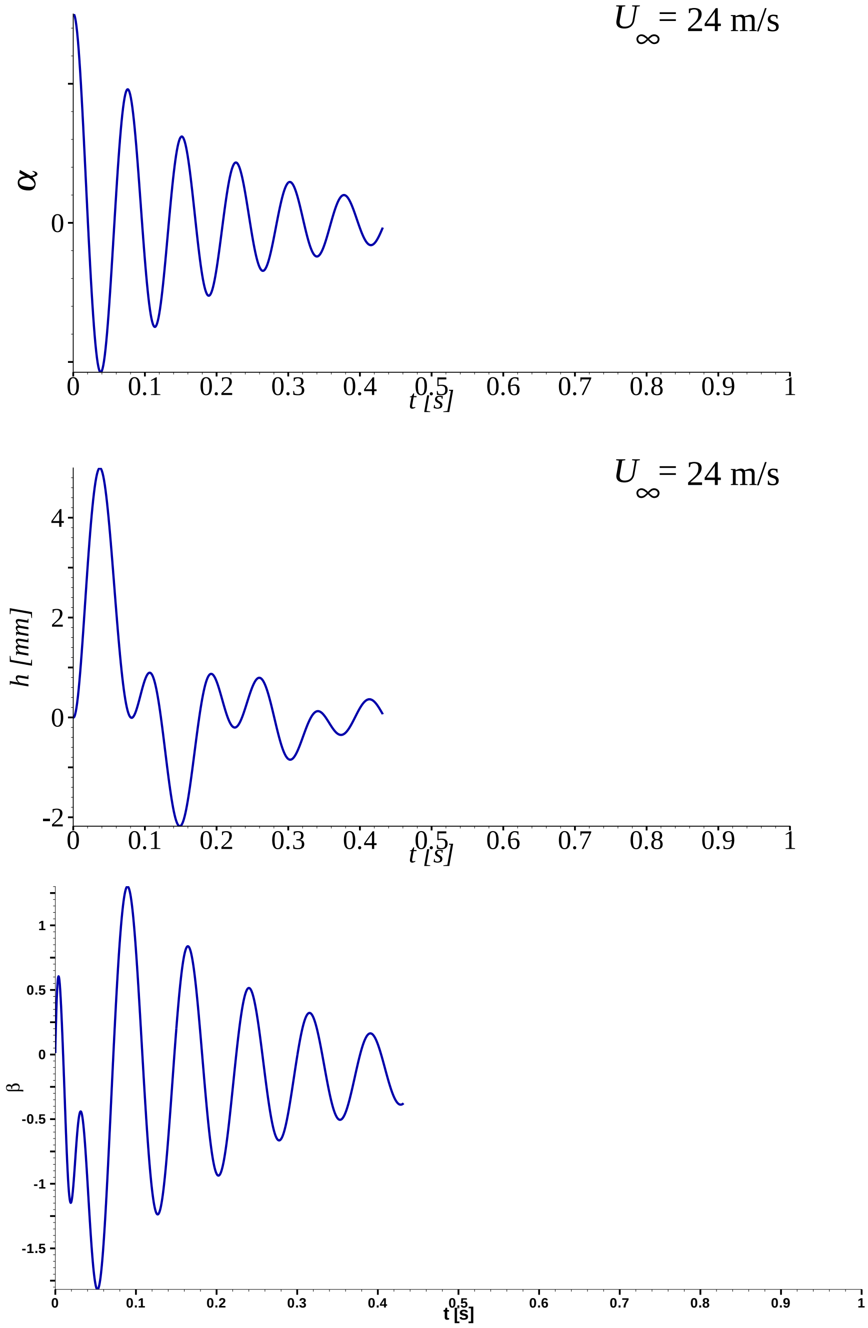
$U = 20m/s$


U_∞ = 20 m/s
α
0
0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1
t [s]

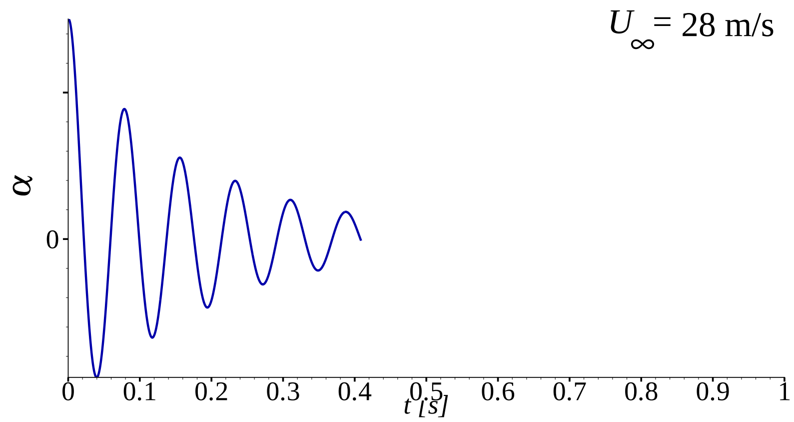

U_∞ = 20 m/s
h [mm]
2
0
0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1
t [s]

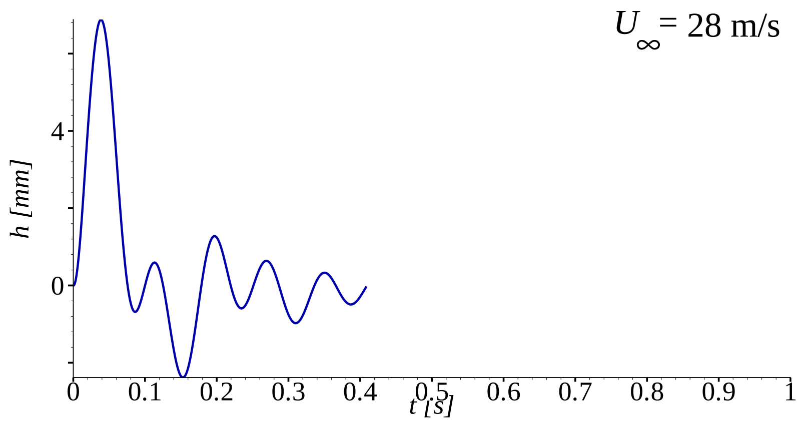

β
1.5
1
0.5
0
-0.5
-1
-1.5
-2
0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1
t [s]

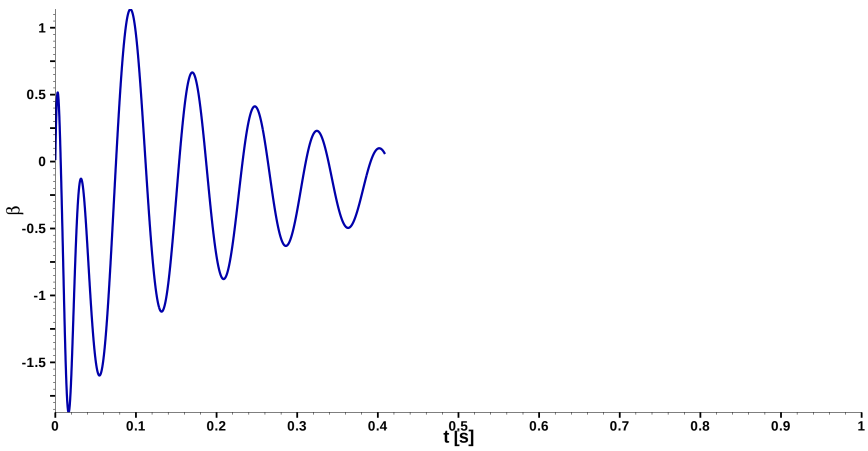
$U = 24m/s$


U_∞ = 24 m/s
α
0
0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1
t [s]
U_∞ = 24 m/s
h [mm]
4 2 0 -2
0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1
t [s]
β
1 0.5 0 -0.5 -1 -1.5
0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1
t [s]

$U = 28m/s$


U_∞ = 28 m/s
α
0
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
t [s]


U_∞ = 28 m/s
h [mm]
4
0
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
t [s]


β
1
0.5
0
-0.5
-1
-1.5
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
t [s]