(tangent (hyper-)plane)

1. Given $f(x,y) = 3y^2 - 2x^2 + x$ and a point $T = [2; -1; ?]$.
   a) Compute P.D. of the function in a point $[2; -1]$.
   b) Find an equation of the tangent plane (τ) to the graph of the function at the point T.

2. Find an equation of the plane (τ) tangent to the graph of $f(x,y) = x\sin(x+y)$ at a point $T = [-1; 1; ?]$. Find also an equation of a line (ν) normal to the graph of f at point T.

3. a) Find an equation of the plane tangent to the graph of $f(x,y) = \ln(x+y)$ at a point $[1; 0; ?]$.
   b) Use the result to approximate the functional value $f(A_1)$ in a point $A_1 = [1.1; 0.1]$.

4. Given $f(x,y) = 2x^2 - y^2$ and a plane $\sigma: \quad 8x - 6y - z + 12 = 0$.
   a) Find a plane (τ) tangent to the graph of f and parallel to the plane σ.
   b) Find a line (ν) normal to the graph of f and normal to the plane σ.

5. Find an equation of the hyper-plane (τ) tangent to the graph of $f(x,y,z) = \ln(x^2 - y + 3z)$ at a point $T = [2; 1; 1; ?]$.

6. Given $f(x,y,z) = \ln(z + \sqrt{9 - x^2 - y^2})$,
   a) Find Domain of definition of f and sketch it (at least in 2 cuts).
   b) Find an equation of the hyper-plane (τ) tangent to the graph of f at a point $T = [0; 0; 1; ?]$.

Gradient and directional derivative

7. Given $f(x,y) = \sqrt{1-x^2} - \sqrt{1-y^2}$,
   a) find Domain of definition of f and sketch it.
   b) Where is the function f differentiable? (Find the domain of differentiability.)
   c) Compute gradient of the function in a point $A = [1/2; 0]$

8. Given $f(x,y) = \frac{\sqrt{y-x^2}}{1-x^2}$ and a point $A = [0; 1]$,
   a) find the Domain of definition of f and sketch it.
   b) Where is the function f differentiable? (Find the domain of differentiability.)
   c) Determine the direction in which the graph of the function is increasing the most at point A.

9. Given $f(x,y,z) = \sin xz + x + y - \frac{z}{y}$ and a point $A = [2; 1; 0]$,
   Determine the direction of maximal decrease of the function f at the point A.

10. Given $f(x,y) = x^2 + 2xy - 3y^2$ and a point $A = [1; 1]$,
    a) compute the (directional) derivative of f at point A in direction given by vector $\vec{s} = (3; 4)$.
    b) Describe the behavior of the function in this direction.
    c) Compute the derivative of f at point A in the direction given by the vector $\vec{t} = \frac{1}{\sqrt{2}}(1; 1)$. What can you say about the function in this direction at the point A?

11. Given $f(x,y) = \cos xy + e^{xy}$ and a point $A = [1; 0]$,
    a) determine the direction $\vec{s}$ of maximal increase of the function f at a point A.
    b) Compute the (directional) derivative of f at point A in the direction given by a vector $\vec{s}$.
    c) Compute the derivative of f at point A in the direction given by a vector $\vec{t} = (1; 2)$. What can you say about the function in this direction?

12. Given $f(x,y) = \sqrt{9 - x - y^2}$ and a point $A = [1; -2]$,
    a) compute gradient of the function at point A.
    b) Find the direction vector $\vec{u}$ in which the function doesn't change its value.

13. Given $f(x,y,z) = x^2 - 2y^2 - 3z^3 - 17$ and a point $A = [1; 1; 1]$,
    compute the directional derivative of f at point A in the direction given by a vector $\vec{s} = (1; 1; 1)$. What can you say about the function in this direction?