Partial derivatives

1. Find a domain of definition of following functions (and sketch it), compute all partial derivatives:
   (a) $f(x, y) = \ln(9 - x^2 - 9y^2)$
   (b) $f(x, y) = x^y$
   (c) $f(x, y, z) = \sqrt{x} + \sqrt{y} + \sqrt{z}$
   (d) $f(x, y, z) = xz - 5x^2y^3z^4$

2. To given function $f(x, y, z, t) = x^2 y \cos(\frac{z}{t})$ find the $\frac{\partial f}{\partial t}$.

3. Compute all partial derivatives of $f(x, y, z) = x \sin(y - z)$ in a point $A = [1; 0; 0]$. What does these values mean?

4. Compute all partial derivatives of $f(x, y, z) = ze^{xyz}$ in a point $A = [0; 2; -1]$. What does these values mean?

5. a) Compute all partial derivatives of $f(x, y) = \ln(2x - y) + 3x^3 - xy$ in a point $A = [1; 1]$.
   b) Write a tangent line of the function in a cut $x \equiv 1$ in tangent point A.

6.* Compute first and second order partial derivatives of following functions:
   (a) $f(x, y) = x^2 + 5xy + \sin(xy) + xe^{y^2/2}$
   (b) $f(x, y) = y + x^2y + 4y^3x - \ln(y^2 + x)$

7. a) Compute all partial derivatives of $f(x, y) = \ln(2x - y) + 3x^3 - xy$ in a point $A = [1; 1]$.
   b) Write a tangent line of the function in a cut $x \equiv 1$ in tangent point A.

8. a) Compute all partial derivatives of $f(x, y, z) = ze^{xyz}$ in a point $A = [0; 2; -1]$.
   b) Write a tangent line of the function in a cut $y \equiv 2 \wedge z \equiv -1$ in tangent point A.

9. Verify that a function $u(x, y) = e^y(y^2 - x^2)$ is a solution of an equation
$$y\frac{\partial u}{\partial x} + x\frac{\partial u}{\partial y} = xu.$$

10. Verify that a function $u(x, t) = \sin(x - ta)$ (with parameter $a \in \mathbb{R}$) is a solution of an equation
$$\frac{\partial u}{\partial t} + a\frac{\partial u}{\partial x} = 0.$$

Differential and tangent (hyper-)plane

11. a) Write (total) differential of a function $f(x, y) = \frac{y}{x}$ in a point $A_0 = [2; 1]$.
    b) Approximate the increment of the function between points A_0 and $A_1 = [2.1; 1.2]$ (i.e. $\Delta f = f(A_1) - f(A_0) =?$)

12. By using the (total) differential, approximate the value of $f(0.97; 1.02; 0.99) = \frac{\sqrt[4]{0.97}}{1.02^3 \sqrt[3]{0.99}}$ (with 2 decimal places precision) hint: Use known value $f(1; 1; 1)$.

13. Given $f(x, y) = 3y^2 - 2x^2 + x$ and a point $T = [2; -1; ?]$.
    a) Compute P.D. of the function in a point $[2; -1]$.
    b) Find an equation of the tangent plane (τ) to the graph of the function at the point T.

14. Find an equation of the plane (τ) tangent to the graph of $f(x, y) = x \sin(x + y)$ at a point $T = [-1; 1; ?]$. Find also an equation of a line (ν) normal to the graph of f at point T.

15. a) Find an equation of the plane tangent to the graph of $f(x, y) = \ln(x + y)$ at a point $[1; 0; ?]$.
    b) Use the result to approximate the functional value $f(A_1)$ in a point $A_1 = [1.1; 0.1]$.

16. Given $f(x, y) = 2x^2 - y^2$ and a plane $\sigma: \quad 8x - 6y - z + 12 = 0$.
a) Find a plane (τ) tangent to the graph of f and parallel to the plane σ.
b) Find a line (ν) normal to the graph of f and normal to the plane σ.

17. Find an equation of the hyper-plane (τ) tangent to the graph of $f(x, y, z) = \ln(x^2 - y + 3z)$ at a point $T = [2; 1; 1; ?]$.

18. Given $f(x, y, z) = \ln(z + \sqrt{9 - x^2 - y^2})$,
a) Find Domain of definition of f and sketch it (at least in 2 cuts).
b) Find an equation of the hyper-plane (τ) tangent to the graph of f at a point $T = [0; 0; 1; ?]$.