Limits

1. (a) Find a candidate for a limit $\lim\limits_{[x,y]\to[0;0]} \frac{1}{\sqrt{x^2+y^2}}$
   (b)* Can you prove (in this special case) that the candidate is the only possibility?
   hint: Try the rotational symmetry

2. (a) Find a candidate for a limit $\lim\limits_{[x,y]\to[0;0]} \frac{\sin x \sin y}{xy}$
   (b)* Can you prove (in this special case) that the candidate is the only possibility?
   hint: Remember what is true for multiplication of the limits from M1

3. (a) Find a candidate for a limit $\lim\limits_{[x,y]\to\infty} (x^2+3y^2)e^{-x^2-y^2}$
   (b)* Can you prove that the candidate is the only possibility?

4. (a) Find a candidate for a limit $\lim\limits_{[x,y]\to[0;0]} \frac{x+y}{xy}$
   (b) Prove that the limit doesn't exist. hint: Try different lines

5. Prove that the $\lim\limits_{[x,y]\to[0;0]} \frac{xy^2}{x^2+y^4}$ doesn't exist. hint: Try different parabolas

6. Decide if the following function is continuous in point [0; 0]:
   $f(x,y) = \begin{cases} 2 & \text{for} \quad [x,y]=[0;0] \\ \frac{\sin(x^2+y^2)}{\sqrt{x^2+y^2+1}-1} & \text{elsewhere} \end{cases}$

Derivatives with parameters

7. Compute the derivative of the function $f(x) = \frac{1}{\tan\left(\frac{a}{x}\right)}$, where $a \in \mathbb{R}$ is a parameter.

8. a) Compute the derivative of the function $f(x) = \frac{1}{\sqrt{x^2+a^2+b^2}}$, where $a, b \in \mathbb{R}$ are parameters.
   b) Where is the function increasing?

9. a) Compute the derivative of the function $f(y) = a^2 + a(\sin y - y^4)$, where $a \in \mathbb{R}$ is a parameter.
   b) Decide if the function is decreasing or increasing in the neighborhood of point $y_0 = 0$.

10. Compute the derivative of the function $f(y) = ae^{ay^2} + b^5 y^{-4}$, where $a, b \in \mathbb{R}$ are parameters.

Partial derivatives

11. Find a domain of definition of following functions (and sketch it), compute all partial derivatives:
    (a) $f(x,y) = \sqrt{2x-y}$
    (b) $f(x,y) = x^2 + y^3 - 2y^2 - 4xy$
    (c) $f(x,y) = xe^y + x^2 - 2y^2 - 2$
    (d) $f(x,y) = \ln(x - y^2)$
    (e) $f(x,y) = 3\cos(4y)\sin(x) - \sin(2x)$
    (f) $f(x,y) = \sqrt{xy}$
    (g) $f(x,y) = \ln(9 - x^2 - 9y^2)$
    (h) $f(x,y) = x^y$
    (i) $f(x,y,z) = \sqrt{x} + \sqrt{y} + \sqrt{z}$
    (j) $f(x,y,z) = xz - 5x^2y^3z^4$

12. To given function $f(x,y,z,t) = x^2 y \cos(\frac{z}{t})$ find the $\frac{\partial f}{\partial t}$.

13. Compute all partial derivatives of $f(x, y, z) = x \sin(y - z)$ in a point $A = [1; 0; 0]$. What does these values mean?

14. Compute all partial derivatives of $f(x, y, z) = ze^{xyz}$ in a point $A = [0; 2; -1]$. What does these values mean?

15. a) Compute all partial derivatives of $f(x, y) = \ln(2x - y) + 3x^3 - xy$ in a point $A = [1; 1]$.
    b) Write a tangent line of the function in a cut $x \equiv 1$ in tangent point A.