(implicitly defined functions)

1. Verify that by the equation $xz^2 - x^2y + y^2z + 2x - y = 0$ is implicitly defined function $z = f(x, y)$ near the point $A = [0; 1; 1]$.
   Find a direction in which is the function $z = f(x, y)$ increasing the most at point $[0; 1]$.

Local extrema

2. Given $f(x, y) = x^2y + \cos y + y \sin x$,
   Find all partial derivatives of first and second order. Decide if the origin ($O = [0; 0]$) is the critical point of the function f (verify). Find the Hessian matrix in this point.

3. Given $f(x, y) = x^y$,
   Find all partial derivatives of first and second order. Decide if $P = [1; 1]$ is the critical point of the function f (verify).

4. Find the local extrema of the function $f(x, y) = \ln(1 - x^2 - y^2)$, i.e. find their position, type and value.

5. Find the local extrema of the function $f(x, y) = 2xy - 5x^2 - 2y^2 + 4x + 4y$, i.e. find their position, type and value.

6. Find the local extrema of the function $f(x, y) = x^3 + y^3 + 3x^2 - 3y^2 - 8$, i.e. find their position, type and value.

7. Determine if the function $f(x, y) = 4xy - x^4 - y^4 - 11$ has local extremes at points $A_0 = [0; 0]$ or $A_1 = [1; 1]$. If the answer is YES, find its type and value.

8. Has the function $f(x, y) = e^x \cos y$ local extrema?

9. Find all critical points of the function $f(x, y) = y \cos x$.