(implicitly defined functions)

1. (=8. fromt previous tut.) Given $F(x,y) = x^3 + 2x^2y + y^4 - 1$,
   a) Compute all first and second order partial derivatives of F.
   By the equation $F(x,y) = 0$ is implicitly defined function $y = f(x)$ near the point $A = [2;-1]$ with continuous first and second derivatives (you don't need to verify).
   b) Compute the first and the second derivative of $y = f(x)$ at point $x_0 = 2$.
   c) Approximate the function $y = f(x)$ by the second order Taylor's polynomial.

2. Prove that the equation $\ln(x+2y)+x-1 = 0$ imlicitly defines (in some neighborhood) a function $y = f(x)$ which satisfies $f(1) = 0$. Aproximate the value $f(1.1)$ by the second order Taylor's polynomial.

3. Given $F(x,y) = x^3 + y^3 - 2x^2 - xy + 1$,
   verify that by the equation $F(x,y) = 0$ is implicitly defined function $y = f(x)$ near the point $A = [1;0]$.
   Compute the first and the second derivative of $y = f(x)$ at point $x_0 = 1$ and describe the behavior of $y = f(x)$ near point A (is it increasing or decreasing, convex or concave?).

4. a) Find equation of an iso-curve for $F(x,y) = xye^{x-y}$ at point $P = [1;2]$.
   b) Find a tangent line to this iso-curve at point P.
   c) Near the point P approximate the iso-curve by second degree Taylor's polynomial.

5. Given $F(x,y) = \ln(xy+4) - 2\ln 2$ and a point $A = [0;2]$.
   Can the equation $F(x,y) = 0$ defined correctly the implicitly defined function $y = f(x)$ near the point A?
   If not, suggest how to compute tangent to the iso-curve $F(x,y) = 0$. (hint: switch the variables)

6. Given $F(x,y,z) = x^3 + y^3 + z^3 + xyz - 6$,
   a) verify that by the equation $F(x,y,z) = 0$ is implicitly defined function $z = f(x,y)$ near the point $A = [1;2;-1]$.
   b) Compute all the partial derivatives of $z = f(x,y)$ at point $T = [1;2]$.
   c) Find an equation of the tangent plain which is tangent to the graph of $z = f(x,y)$ at tangent point A.

7. Verify that by the equation $xz^2 - x^2y + y^2z + 2x - y = 0$ is implicitly defined function $z = f(x,y)$ near the point $A = [0;1;1]$.
   Find a direction in which is the function $z = f(x,y)$ increasing the most at point $[0;1]$.