

(tangent (hyper-)plane)

- Given $f(x, y) = 2x^2 - y^2$ and a plane $\sigma : 8x - 6y - z + 12 = 0$.
 - Find a plane (τ) tangent to the graph of f and parallel to the plane σ .
 - Find a line (ν) normal to the graph of f and normal to the plane σ .
- Find an equation of the hyper-plane (τ) tangent to the graph of $f(x, y, z) = \ln(x^2 - y + 3z)$ at a point $T = [2; 1; 1; ?]$.
- Given $f(x, y, z) = \ln(z + \sqrt{9 - x^2 - y^2})$,
 - find Domain of definition of f and sketch it (at least in 2 cuts).
 - Find an equation of the hyper-plane (τ) tangent to the graph of f at a point $T = [0; 0; 1; ?]$.

Gradient and directional derivative

- Given $f(x, y) = \sqrt{1 - x^2} - \sqrt{1 - y^2}$,
 - find Domain of definition of f and sketch it.
 - Where is the function f differentiable? (Find the domain of differentiability.)
 - Compute gradient of the function in a point $A = [1/2; 0]$
- Given $f(x, y) = \frac{\sqrt{y-x^2}}{1-x^2}$ and a point $A = [0; 1]$,
 - find the Domain of definition of f and sketch it.
 - Where is the function f differentiable? (Find the domain of differentiability.)
 - Determine the direction in which the graph of the function is increasing the most at point A .
- Given $f(x, y, z) = \sin xz + x + y - \frac{z}{y}$ and a point $A = [2; 1; 0]$,
Determine the direction of maximal decrease of the function f at the point A .
- Given $f(x, y) = x^2 + 2xy - 3y^2$ and a point $A = [1; 1]$,
 - compute the (directional) derivative of f at point A in direction given by vector $\vec{s} = (3; 4)$.
 - Describe the behavior of the function in this direction.
 - Compute the derivative of f at point A in the direction given by the vector $\vec{t} = \frac{1}{\sqrt{2}}(1; 1)$.
What can you say about the function in this direction at the point A ?
- Given $f(x, y) = \cos xy + e^{xy}$ and a point $A = [1; 0]$,
 - determine the direction \vec{s} of maximal increase of the function f at a point A .
 - Compute the (directional) derivative of f at point A in the direction given by a vector \vec{s} .
 - Compute the derivative of f at point A in the direction given by a vector $\vec{t} = (1; 2)$. What can you say about the function in this direction?
- Given $f(x, y) = \sqrt{9 - x^2 - y^2}$ and a point $A = [1; -2]$,
 - compute gradient of the function at point A .
 - Find the direction vector \vec{u} in which the function doesn't change its value.
- Given $f(x, y, z) = x^2 - 2y^2 - 3z^3 - 17$ and a point $A = [1; 1; 1]$,
compute the directional derivative of f at point A in the direction given by a vector $\vec{s} = (1; 1; 1)$.
What can you say about the function in this direction?