(Triple integrals - generalized cylindrical coordinates)

0. Given a body: $M = \{[x, y, z] \in \mathbb{R}^3 : \ 0 \le z \le 1 \ \wedge \ 0 \le y \le x \ \wedge \ \frac{x^2}{3} + y^2 \le 1\}$.
   (a) Transfer the following integral to generalized cylindrical coordinates:
   $$\iiint\limits_M 1 \,\mathrm{d}x\mathrm{d}y\mathrm{d}z.$$
   (b) Compute the integral.
   (c) Write one possible physical meaning of the integral.

Triple integrals: spheres and spherical coords.

1. Given a body: $M = \{[x, y, z] \in \mathbb{R}^3 : \ 1 \le z \le \sqrt{9 - x^2 - y^2}\}$.
   Sketch (in cuts) the body and compute its volume.
2. Given a body: $M = \{[x, y, z] \in \mathbb{R}^3 : \ 1 \le x^2 + y^2 + z^2 \le 9 \ \wedge \ z \ge 0\}$.
   (a) Transfer the following integral to spherical coordinates:
   $$\iiint\limits_M \sqrt{x^2 + y^2 + z^2} \,\mathrm{d}x\mathrm{d}y\mathrm{d}z.$$
   (b) Compute the integral.
3. Compute mass of a body $M = \{[x, y, z] \in \mathbb{R}^3 : \ x^2 + y^2 + z^2 \le 4 \ \wedge \ x \ge 0\}$
   for $\rho(x, y, z) = x^2 + y^2$.
4. Compute volume of the body $M = \{[x, y, z] \in \mathbb{R}^3 : \ \sqrt{x^2 + y^2} \le z \le \sqrt{1 - x^2 - y^2}\}$
5. Sketch (in cuts) a body $M = \{[x, y, z] \in \mathbb{R}^3 : \ x^2 + y^2 + z^2 \le 16 \ \wedge \ x^2 + y^2 \le 9\}$.
   Compute its volume.
6. Compute the center of mass of a half-ball with radius R = 1 which is homogenenous (ρ = const.)
   $\left[z_C = \frac{3}{8}\right]$