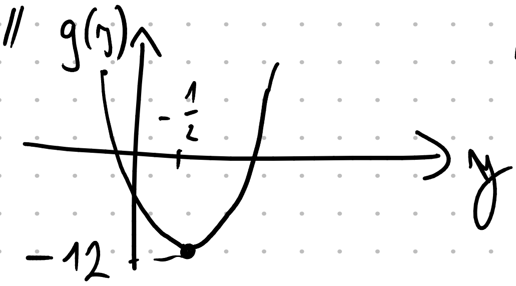
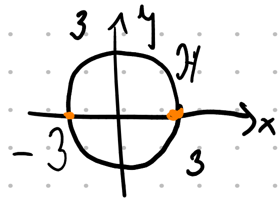


$A = \left[\frac{1}{2}, \frac{3}{2}\right]$ is constr. min. (glob.)

$g(A) = 9 - 36 + 15 = -12$

glob. max. $\nexists$ (parabola)

7.) $f(x,y) = x^2 - 2x + y^2$
$x^2 + y^2 = 9$ .... circle

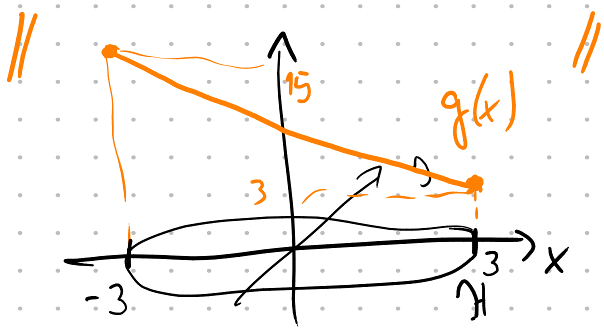


$g(x) = f(x,y)\big|_H = 9 - 2x \qquad x \in \langle -3, 3 \rangle$

$g'(x) = -2 \neq 0$

$x_1 = -3$ i.e. $[-3, 0]$ $\qquad g(-3) = 15$ glob. (constr.) max

$x_2 = 3$ i.e. $[3, 0]$ $\qquad g(3) = 3$ glob. (constr.) min.



$$\underbrace{f(B_2)}_{-3} < \underbrace{f(B_3)}_{-\frac{9}{4}} < \underbrace{f(P)}_{-2} < \underbrace{f(B_1) = f(B_4)}_{0}$$

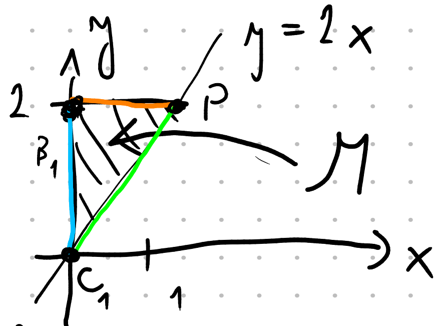
min max

glob. min -3 in $[0;3]$

glob. max. 0 in $[0;0]$ and $[3;0]$

2.) $f(x,y) = 2x^2 - 4x + y^2 - 4y + 2$

$x \geq 0 \wedge y \leq 2 \wedge y \geq 2x$



$[0,2]: 2 \geq 0$

M closed set $\wedge$ f cont $\Rightarrow \exists$ max. and min.

I) crit. p in M (inner)

$$\frac{\partial f}{\partial x} = 4x - 4 = 0 \quad \wedge \quad \frac{\partial f}{\partial y} = 2y - 4 = 0$$

$\boxed{P = [1;2]} \in M$

II) constrained crit. p. (∂M)

$\boxed{x=0}$ $\quad g_1(y) = f(0;y) = y^2 - 4y + 2$

$g_1' = 2y - 4 = 0 \Rightarrow y = 2$

$\boxed{B_1 = [0;2]}$

$\boxed{y=2}$ $\quad g_2(x) = f(x,2) = 2x^2 - 4x - 2$

$g_2' = 4x - 4 = 0 \Rightarrow x = 1$ ✓

$\boxed{y = 2x}$ $\quad g_3(t) = f(t, 2t) = 2t^2 - 4t + 4t^2 - 8t + 2$

$x = t \quad y = 2t$

$$g_3(t) = 6t^2 - 12t + 2$$

$$g_3' = 12t - 12 = 0 \quad \rightarrow t = 1$$

$$[1;2] \checkmark = P \text{ again}$$

III) corners $\quad \boxed{C_1 = [0;0]} \quad C_2 = B_1 \checkmark \quad C_3 = P \checkmark$

$\cancel{[1;2]} \qquad f(P) = -4$ glob. min. $\qquad f(P) < \dots < f(C_1)$

$[0;2] \qquad f(B_1) = -2$

$\cancel{[0;0]} \qquad f(C_1) = 2$ glob. max.

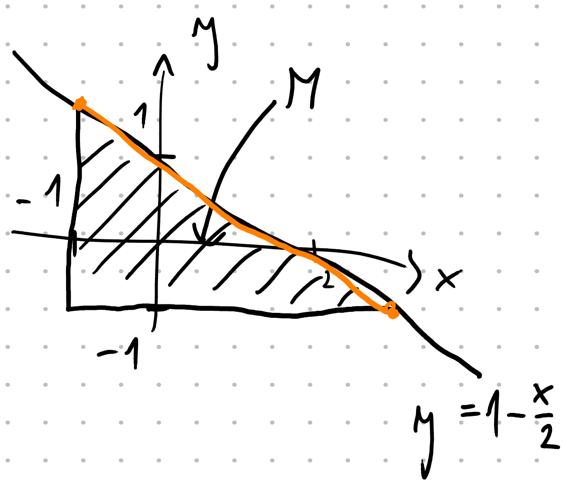
3.) $f(x,y) = x^2 - y^2$

$x \geq -1 \quad \wedge \quad y \geq -1 \quad \wedge$

$\wedge \; x + 2y \leq 2$

$2y = 2 - x$

$y = 1 - \frac{x}{2}$



M closed set $\wedge$ $f(x,y)$ cont. $\Rightarrow$

$\Rightarrow \exists$ glob. max and min.

I) inside M

$$\frac{\partial f}{\partial x} = 2x = 0 \quad \wedge \quad \frac{\partial f}{\partial y} = -2y = 0$$

$$P = [0; 0]$$

II) ∂M:

$y = -1$: $g_1(x) = f(x; -1) = x^2 - 1$ $\quad B_1 = [0; -1]$

$g_1' = 2x = 0 \Rightarrow x = 0$

$x = -1$: $g_2(y) = f(-1, y) = 1 - y^2$

$g_2' = -2y = 0$ $\quad B_2 = [-1; 0]$

ℓ: $y = 1 - \frac{x}{2}$: $g_3(x) = f(x, y)\big|_{\ell} = x^2 - \left(1 - \frac{x}{2}\right)^2 =$

$= x^2 - 1 + x - \frac{x^2}{4} = \frac{3x^2}{4} + x + 1$

$g_3' = \frac{6x}{4} + 1 = 0 \Rightarrow B_3 = \left[-\frac{2}{3}; \frac{4}{3}\right]$

$x = -\frac{2}{3}$ $\quad y = 1 + \frac{1}{3}$

III) corners

$C_1 = [-1; -1]$

$C_2 = \left[-1; \frac{3}{2}\right]$ $\quad y = 1 + \frac{1}{2}$

$C_3 = [4; -1]$ $\quad -1 = 1 - \frac{x}{2}$

$f(P) = 0$ $\quad f(B_3) = -\frac{4}{3}$ $\quad f(C_1) = 0$

$f(B_1) = -1$ $\quad f(C_2) = -\frac{5}{4}$

$f(B_2) = 1$ $\quad f(C_3) = 15$

$$-\frac{4}{3} < -\frac{5}{4} < -1 < \dots < 15$$

$f(B_3)$ glob. min at $[-\frac{2}{3}; \frac{4}{3}]$

$f(C_3)$ glob. max at $[4; -1]$

alternative proc. (Lagrange multipl.)

note: II for curve ℓ

(not mandatory)

$$g(x,y) = 1 - \frac{x}{2} - y = 0$$

$$L(x,y,\lambda) = \underbrace{x^2 - y^2}_{f} + \lambda \underbrace{\left(1 - \frac{x}{2} - y\right)}_{g}$$

$$\frac{\partial L}{\partial x} = 2x - \frac{\lambda}{2} = 0 \quad |\cdot 2$$

$$\frac{\partial L}{\partial y} = -2y - \lambda = 0$$

$$\frac{\partial L}{\partial \lambda} = 1 - \frac{x}{2} - y = 0 \quad |\cdot 2$$

$$\begin{aligned} 4x \qquad\quad &- \lambda = 0 \\ -2y &- \lambda = 0 \\ x + 2y \qquad &= 2 \end{aligned}$$

II + III:

$$\begin{aligned} 4x \quad &- \lambda = 0 \\ x \quad &- \lambda = 2 \end{aligned}$$

$$3x = -2$$

$$x = -\frac{2}{3}$$

$$-\frac{2}{3} + 2y = 2$$

$$y = \frac{4}{3}$$

crit. point

$$B_3 = [-\frac{2}{3}; \frac{4}{3}]$$