

Mathematics II - tutorials

Assessment requirements: Attendance is mandatory - 4 absences are allowed
3 parts:

1. Differential calculus (Ex. 1., 2.) → approximately the end of March
2. Double and Triple integrals (Ex. 3., 4.) → approximately the end of April
3. The rest (Ex. 5., 6.) → the end of the semester

consultations - after the Friday's tutorial (or after e-mail agreement)

Motivation

Find and sketch a Domain of definition of the following functions:

1. $f(x, y) = \sqrt{x^2 + y}$
2. $f(x, y, z) = \ln(z - x^2 - y^2)$

Quadratics and Geometry repetition

1. Identify the type of a Quadric, its center and other parameters (axes, half-axis ...):

- (a) $\frac{(x-4)^2}{4} + \frac{(y-1)^2}{4} + \frac{z^2}{8} = 1$
- (b) $(x - 2)^2 + (y + 1)^2 + (z + 10)^2 = 9$
- (c) $\frac{(x-1)^2}{4} - \frac{(y-2)^2}{3} + (z + 1)^2 = 1$
- (d) $-\frac{(x-1)^2}{4} + \frac{(y-2)^2}{3} - (z + 1)^2 = 1$

2. Write an equation of following Quadric:

- (a) Elliptic paraboloid ($C = [0; 0; 0]$, $a \parallel z$)
- (b) Hyperbolic paraboloid ($V = [0; 0; 0]$, $a \parallel z$)
- (c) Elliptic cylinder ($a \parallel z$)
- (d) Parabolic cylinder ($V = [0; 0; 0]$, $a \parallel z$)
- (e) Hyperbolic cylinder ($C = [0; 0; 0]$, $a \parallel z$)

3. Identify the type of a Quadric and sketch a given cut (+ write its equation)

- (a) $z = x^2 + 4y^2$, cut for $z = 1$
- (b) $x^2 + y^2 - z^2 = 0$, cut for $x = 0$
- (c) $x^2 - y^2 + z^2 = 0$, cut for $z = 1$
- (d) $x^2 + y^2 = z - 1$, cut for $y = 0$
- (e) $z^2 - x^2 - y^2 = 1$, cut for $z = 2$
- (f) $x^2 + 4y^2 + z^2 = 4$, cut for $z = 0$
- (g) $z^2 = y - x^2$, cut for $y = 0$ and $x = 0$

4. Write the canonical equation of a given Quadric and identify its type:

(a) $x^2 + y^2 + z^2 - 4x + 6y - 6z + 3 = 0$

(b) $-4x^2 + 9y^2 + 9z^2 + 18y - 90z + 270 = 0$

(c) $x^2 - 6x + y^2 - 16y + z^2 - 8z + 37 = 0$

(d) $2x^2 - 8x + 3y^2 + 18y - 6z + 35 = 0$

(e) $-3x^2 + 6x + 4y^2 - 16y + 12z^2 + 24z + 13 = 0$

5. Identify and sketch the domain Ω :

(a) $\Omega = \{[x, y] \in \mathbb{R}^2; x^2 + y^2 \leq 16 \quad \wedge \quad y \leq x^2 + 2\}$

(b) $\Omega = \{[x, y] \in \mathbb{R}^2; \frac{x^2}{36} + \frac{y^2}{25} \leq 1 \quad \wedge \quad \frac{y^2}{4} - x^2 > 1\}$

(c) $\Omega = \{[x, y, z] \in \mathbb{R}^3; z = x^2 + 4y^2 \quad \wedge \quad z = 16\}$

(d) $\Omega = \{[x, y, z] \in \mathbb{R}^3; x^2 + y^2 \leq 4 \quad \wedge \quad z = 7 + x^2 + y^2\}$

(e) $\Omega = \{[x, y, z] \in \mathbb{R}^3; x^2 + y^2 \leq z^2 \quad \wedge \quad z = 6 - x^2 - y^2\}$

(f) $\Omega = \{[x, y, z] \in \mathbb{R}^3; 2x^2 - y^2 - z^2 = 4 \quad \wedge \quad 0 \leq x \leq 2\}$

(g) $\Omega = \{[x, y, z] \in \mathbb{R}^3; z^2 \leq 10 - x^2 - y^2 \quad \wedge \quad x^2 + y^2 + z^2 = 10\}$