Repetition - $\iint_\sigma$

1. Given surface $\sigma = \{[x,y,z] \in \mathrm{R}^3 : z = \sqrt{4-x^2-y^2} \ \wedge \ x^2+y^2 \leq 1\}$.
   (a) Sketch the surface σ and its projection to xy plane. Write down the parametrization of σ.
   (b) Compute $\iint_\sigma z^3 \, \mathrm{d}p$.
2. Given surface $\sigma = \{[x,y,z] \in \langle 0;1\rangle \times \langle -1;1\rangle \times \mathrm{R} : z = x^2+y\}$.
   (a) Write down the parametrization of σ.
   (b) Compute the statical moment about the xy-plane when $\rho(x,y,z) = \frac{1}{\sqrt{4z-4y+2}}$.
3. Given surface $\sigma = \{[x,y,z] \in \mathrm{R}^3 : z = 4-x^2-y^2 \ \wedge \ z \geq 0\}$ oriented by a normal vector with positive z-coordinate.
   (a) Sketch the surface σ and its projection to xy plane. Find a suitable parametrization of σ including the corresponding range of parameters.
   (b) Compute $\iint_\sigma (2x,2y,2z) \cdot \vec{\mathrm{d}p}$.
4. Given surface $Q = \{[x,y,z] \in \mathrm{R}^3 : z = 1+x^2+y^2 \ \wedge \ z \leq 2\}$ oriented by a normal vector with positive z-coordinate.
   (a) Sketch the surface Q and its projection to xy plane. Find a suitable parametrization of σ including the corresponding range of parameters.
   (b) Compute flux of the vector field $\vec{f}(x,y,z) = (y,-x,1+z)$ through Q.
5. (a) Write down the divergence theorem (Gauss-Ostrogradsky) and check the assumptions for the flux of the field $\vec{f}(x,y,z) = (x^3,y^3,z)$ through surface of a body
   $\Omega = \{[x,y,z] \in \mathrm{R}^3 : x^2+y^2 \leq 1 \ \wedge \ 0 \leq z \leq 2-\sqrt{x^2+y^2}\}$.
   The surface is oriented with a normal heading inside.
   (b) Compute the given flux.

Results

1. (a) $||P_u \times P_v|| = \frac{4u}{\sqrt{4-u^2}}$ (b) 14π
2. (a) $||P_u \times P_v|| = \sqrt{4u^2+2}$ (b) $2/3$
3. (b) 48π
4. (b) $\frac{5}{2}\pi$
5. (a) like a hut (b) $-\frac{46}{15}\pi$