Repetition - $\int_C$

1. Compute the work done by a vector field $\vec{f}(x, y) = (xy^2, 2x^2y)$ along the curve which is positively oriented boundary of a triangle M = [0;0], N = [2;2], O = [2;4].

2. A curve is given as a line segment from $E = [1; 0; 2]$ to $F = [1; 2; 1]$.
   (a) Compute its mass when $\rho(x, y, z) = x^2 + y^2$.
   (b) For the given potential $\varphi(x, y, z) = x^2y + 2y^4z$ find the corresponding vector field and compute its work done along the curve.

3. Given a conservative vector field $\vec{f}(x, y) = (\sqrt{x} + y, \sqrt{y} + x)$.
   (a) Find the potential of the vector field (determine where it is possible).
   (b) Compute $\int_C \vec{f} \cdot \vec{ds}$ where $C = \{[x, y] \in \mathrm{R}^2 : y = x^2 \ \wedge \ 1 \le x \le 2\}$.

4. A curve is given as a segment of function $y = \tan x$ for $x \in \langle 0; \frac{\pi}{4} \rangle$.
   (a) Suggest its parametrization, compute the tangent vector and determine its length.
   (b) Compute line integral of a scalar function $f(x, y) = 4\cos^5 x \sin x$.
   (c) Compute line integral of a vector function $\vec{g}(x, y) = (x, \cos^3 x)$.

5. Given an incomplete potential $\varphi(x, y) = 2xy^{3/2} + K(y)$, containing an unknown function $K(y)$ depending just on one variable (y). The corresponding conservative/potential vector field is $\vec{f}(x, y) = (U(x, y); V(x, y))$, where $V(x, y) = 3x\sqrt{y} + y$.
   (a) From the definition of potential determine the component U of the given vector field $\vec{f}$.
   (b) Finish the computation of potential $\varphi(x, y)$ by finding the unknown function $K(y)$.
   (c) Find the domain where the vector field $\vec{f}(x, y)$ is conservative.
   (d) Compute the work done by the force $\vec{f}$ acting along the oriented line segment C from the point $A = [1; 0]$ to the point $B = [3; 4]$.

6. Compute the circulation of a vector field $\vec{f}(x, y) = (x+y, x-y)$ along a positively oriented circle $x^2 + y^2 = 4$.

Results

1. W = 12
2. (a) $\frac{7}{3}\sqrt{5}$ (b) $f(x, y, z) = (2xy,\ x^2 + 8y^3z,\ 2y^4)$, W = 34
3. (a) $\varphi(x, y) = \frac{2}{3}(\sqrt{x^3} + \sqrt{y^3}) + xy + C$, (for $x > 0,\ y > 0$) (b) $\frac{4\sqrt{2}}{3} + 11$
4. (a) $||\dot{P}(t)|| = \frac{\sqrt{\cos^4 t+1}}{\cos^2 t}$ (b) $\frac{5\sqrt{5}+16\sqrt{2}}{12}$ (c) $\frac{\pi^2}{32} + \frac{\sqrt{2}}{2}$
5. (a) $U(x, y) = 2\sqrt{y^3}$ (b) $K(y) = y^2/2 + C$ (c) $\{[x, y] \in \mathrm{R}^2;\ y > 0\}$ (d) 56
6. 0