Repetition - $\iint$ and $\iiint$

1. Given $\iint_D f(x,y)\,\mathrm{d}x\,\mathrm{d}y = \int\limits_0^1 \int\limits_1^{x+1} e^x\,\mathrm{d}y\,\mathrm{d}x$
   (a) Write down and sketch the domain D.
   (b) Reverse the order of integration.
   (c) Evaluate the given integral.
2. Given prismatic body $M = \{[x,y,z] \in \mathrm{R}^3 : 0 \le z \le 2-2x \ \wedge\ 0 \le y \le 2 \ \wedge\ 0 \le x \le ?\}$.
   (a) Sketch the projection of M to xy-plane and determine the upper limit for x.
   (b) Compute $\iiint_M z\,\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z$.
   (c) Give at least two examples of physical meaning of integral from (b).
3. Given a body $M = \{[x,y,z] \in \mathrm{R}^3 : 0 \le z \le x^2+y^2+2 \ \wedge\ 0 \le y \le 2 \ \wedge\ 0 \le x \le 1\}$.
   (a) Sketch the projection of M to xy-plane and the cut by plane $y=0$.
   (b) Compute $\iiint_M x\,\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z$.
   (c) Give at least two examples of physical meaning of integral from (b).
4. (a) Sketch bounded domain $D \in \mathrm{R}^2$ with boundary: $y=\sqrt{x}$ and $y=x$.
   (b) Compute $\iint_D x^2 y\,\mathrm{d}x\,\mathrm{d}y$.
5. (a) Sketch a body $\Omega \in \mathrm{R}^3$ bounded by surfaces: $16x^2+4y^2=64$, $z=0$ and $z=2$.
   (b) Compute the mass of the body if the density $\rho(x,y,z)=y^2z$.
6. (a) Sketch a body $\Omega = \{[x,y,z] \in \mathrm{R}^3 : x^2+y^2 \le 1 \ \wedge\ 0 \le z \le 1-x\}$.
   (b) Compute the volume of the body.

Results

1. (b) $\int\limits_1^2 \int\limits_{y-1}^1 e^x\,\mathrm{d}x\,\mathrm{d}y$ (c)=1
2. (a) $0 \le x \le 1$ (b) $\frac{4}{3}$ (c) mass, $\rho = z$, or static moment m_{xy}, $\rho = 1$
3. (b) 23/6 (c) mass, $\rho = x$, or static moment m_{yz}, $\rho = 1$
4. (b) 1/40
5. (b) $m = 32\pi$
6. (b) $V = \pi$