

(implicitly defined functions)

1. Given $F(x, y) = \ln(xy + 4) - 2 \ln 2$ and a point $A = [0; 2]$.
Can the equation $F(x, y) = 0$ defined correctly the implicitly defined function $y = f(x)$ near the point A ?
If not, suggest how to compute tangent to the iso-curve $F(x, y) = 0$. (hint: switch the variables)
2. Given $F(x, y, z) = x^3 + y^3 + z^3 + xyz - 6$,
 - a) verify that by the equation $F(x, y, z) = 0$ is implicitly defined function $z = f(x, y)$ near the point $A = [1; 2; -1]$.
 - b) Compute all the partial derivatives of $z = f(x, y)$ at point $T = [1; 2]$.
 - c) Find an equation of the tangent plain which is tangent to the graph of $z = f(x, y)$ at tangent point A .
3. Verify that by the equation $xz^2 - x^2y + y^2z + 2x - y = 0$ is implicitly defined function $z = f(x, y)$ near the point $A = [0; 1; 1]$.
Find a direction in which is the function $z = f(x, y)$ increasing the most at point $[0; 1]$.

Local extrema

4. Given $f(x, y) = x^2y + \cos y + y \sin x$,
Find all partial derivatives of first and second order. Decide if the origin ($O = [0; 0]$) is the critical point of the function f (verify). Find the Hessian matrix in this point.
5. Given $f(x, y) = x^y$,
Find all partial derivatives of first and second order. Decide if $P = [1; 1]$ is the critical point of the function f (verify).
6. Find the local extrema of the function $f(x, y) = \ln(1 - x^2 - y^2)$, i.e. find their position, type and value.
7. Find the local extrema of the function $f(x, y) = 2xy - 5x^2 - 2y^2 + 4x + 4y$, i.e. find their position, type and value.
8. Find the local extrema of the function $f(x, y) = x^3 + y^3 + 3x^2 - 3y^2 - 8$, i.e. find their position, type and value.
9. Determine if the function $f(x, y) = 4xy - x^4 - y^4 - 11$ has local extremes at points $A_0 = [0; 0]$ or $A_1 = [1; 1]$. If the answer is YES, find its type and value.
10. Has the function $f(x, y) = e^x \cos y$ local extrema?
11. Find all critical points of the function $f(x, y) = y \cos x$.