**(implicitly defined functions)**

1. Given $F(x,y) = \sin(x+y) - y^2 \cos x$,
verify that by the equation $F(x,y) = 0$ is implicitly defined function $y = f(x)$ in the neighborhood of the point $A = [\pi; 0]$.
Compute its derivative $\frac{\mathrm{d}f}{\mathrm{d}x}$ at point $x_0 = \pi$ and describe the behavior of $f(x)$ near point $A$ (is it increasing or decreasing, how fast?).

2. Given $F(x,y) = x^3 + y^3 - 2x^2 - xy + 1$,
verify that by the equation $F(x,y) = 0$ is implicitly defined function $y = f(x)$ near the point $A = [1; -0]$.
Compute the first and the second derivative of $y = f(x)$ at point $x_0 = 1$ and describe the behavior of $y = f(x)$ near point $A$ (is it increasing or decreasing, convex or concave?).

3. Given $F(x,y) = x^3 + 2x^2y + y^4$,
verify that by the equation $F(x,y) = 1$ is implicitly defined function $y = f(x)$ near the point $A = [2; -1]$.
Compute the first and the second derivative of $y = f(x)$ at point $x_0 = 2$. Approximate the function $y = f(x)$ by the second degree Taylor's polynomial.

4. a) Find equation of an iso-curve for $F(x,y) = xye^{x-y}$ at point $P = [1; 2]$.
b) Find a tangent line to this iso-curve at point $P$.
c) Near the point $P$ approximate the iso-curve by second degree Taylor's polynomial.

5. Given $F(x,y) = \ln(xy+4) - 2\ln 2$ and a point $A = [0; 2]$.
Can the equation $F(x,y) = 0$ defined correctly the implicitly defined function $y = f(x)$ near the point $A$?
If not, suggest how to compute tangent to the iso-curve $F(x,y) = 0$. (hint: switch the variables)

6. Given $F(x,y,z) = x^3 + y^3 + z^3 + xyz - 6$,
a) verify that by the equation $F(x,y,z) = 0$ is implicitly defined function $z = f(x,y)$ near the point $A = [1; 2; -1]$.
b) Compute all the partial derivatives of $z = f(x,y)$ at point $T = [1; 2]$.
c) Find an equation of the tangent plain which is tangent to the graph of $z = f(x,y)$ at tangent point $A$.

7. Verify that by the equation $xz^2 - x^2y + y^2z + 2x - y = 0$ is implicitly defined function $z = f(x,y)$ near the point $A = [0; 1; 1]$.
Find an equation of the tangent plain which is tangent to the graph of $z = f(x,y)$ at tangent point $A$.