(directional derivative)

1. Given $f(x,y) = \sqrt{9-x^2-y^2}$ and a point $A = [1;-2]$,
   a) compute gradient of the function at point A.
   b) Find the direction vector $\vec{u}$ in which the function doesn't change its value.

2. Given $f(x,y,z) = x^2 - 2y^2 - 3z^3 - 17$ and a point $A = [1;1;1]$,
   compute the directional derivative of f at point A in the direction given by a vector $\vec{s} = (1;1;1)$.
   What can you say about the function in this direction?

3. Given $f(x,y,z) = x^3y + \frac{x}{y^2} + 2z$ and a point $A = [-1;1;0]$,
   a) determine the direction $\vec{s}$ in which is the directional derivative at point A maximal.
   b) Compute the derivative in this direction ($\vec{s}$) at point A.

Chain rule (derivatives of composite functions)

4. Given $f(u,v) = u^2 \ln v$, compute $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ when you know that $u(x,y) = \frac{x}{y}$ and $v(x,y) = 3x - 2y$.

5. Given unknown function $z(x,y) = f(u,v) = f(u(x,y), v(x,y))$ and functions $u(x,y) = x^2 - y^2$, $v(x,y) = e^{xy}$. Compute $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ at point $A = [1;2]$ when you know (from physics) that $\frac{\partial f}{\partial u}(A) = 1$ and $\frac{\partial f}{\partial v}(A) = 0$.

6. Prove that the function $z(x,y) = f(\frac{y}{x})$ solve an equation
   $$x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = 0.$$
   hint: define function $u(x,y) = \frac{y}{x}$

7. From equation of state (for perfect gas)
   $$p(t) = C\frac{T(t)}{V(t)},$$
   where $C \in \mathbb{R}$ is a const., $T(t) = \ln t$ and $V(t) = \sqrt{t}$, compute the derivative $\frac{\mathrm{d}p}{\mathrm{d}t}$ in time $t_0 = 1$.

Implicitly defined functions

8. Given $F(x,y) = x^3 + y^3 - 6xy + 4$,
   verify that by the equation $F(x,y) = 0$ is implicitly defined function $y = f(x)$ near the point $A = [1;1]$. Compute its derivative $\frac{\mathrm{d}f}{\mathrm{d}x}$ at point $x_0 = 1$ and find an equation of tangent to the graph of $f(x)$.

9. Verify that by equation $x^3y + y^3x + x^2y - 3 = 0$ is implicitly defined function $y = f(x)$ near the point $A = [1;1]$.
   Compute its derivative $\frac{\mathrm{d}f}{\mathrm{d}x}$ at point $x_0 = 1$ and find an equation of normal to the graph of $f(x)$.

10. Given $F(x,y) = \sin(x+y) - y^2\cos x$,
    verify that by the equation $F(x,y) = 0$ is implicitly defined function $y = f(x)$ in the neighborhood of the point $A = [\pi;0]$.
    Compute its derivative $\frac{\mathrm{d}f}{\mathrm{d}x}$ at point $x_0 = \pi$ and describe the behavior of $f(x)$ near point A (is it increasing or decreasing, how fast?).

11. Given $F(x,y) = x^3 + 2x^2y + y^4$
    verify that by the equation $F(x,y) = 1$ is implicitly defined function $y = f(x)$ near the point $A = [2;-1]$.
    Compute the first and the second derivative of f at point $x_0 = 2$.