

1. Decide if the following function is continuous in point $[0; 0]$:

$$f(x, y) = \begin{cases} \frac{\sin(x^2+y^2)}{\sqrt{x^2+y^2+1}-1} & \text{for } [x, y] \in \mathcal{D}(f) \\ 2 & \text{for } [x, y] = [0; 0] \end{cases}$$

Derivatives with parameters

2. Compute the derivative of the function $f(x) = \frac{1}{\tan(\frac{a}{x})}$, where $a \in \mathbb{R}$ is a parameter.
3. a) Compute the derivative of the function $f(x) = \frac{1}{\sqrt{x^2+a^2+b^2}}$, where $a, b \in \mathbb{R}$ are parameters.
b) Where is the function increasing?
4. a) Compute the derivative of the function $f(y) = a^2 + a(\sin y - y^4)$, where $a \in \mathbb{R}$ is a parameter.
b) Decide if the function is decreasing or increasing in the neighborhood of point $y_0 = 0$.
5. Compute the derivative of the function $f(y) = ae^{ay^2} + b^5y^{-4}$, where $a, b \in \mathbb{R}$ are parameters.

Partial derivatives

6. Find a domain of definition of following functions (and sketch it), compute all partial derivatives:
- (a) $f(x, y) = \sqrt{2x - y}$
 - (b) $f(x, y) = x^2 + y^3 - 2y^2 - 4xy$
 - (c) $f(x, y) = xe^y + x^2 - 2y^2 - 2$
 - (d) $f(x, y) = \ln(x - y^2)$
 - (e) $f(x, y) = 3 \cos(4y) \sin(x) - \sin(2x)$
 - (f) $f(x, y) = \sqrt{xy}$
 - (g) $f(x, y) = \ln(9 - x^2 - 9y^2)$
 - (h) $f(x, y) = x^y$
 - (i) $f(x, y, z) = \sqrt{x} + \sqrt{y} + \sqrt{z}$
 - (j) $f(x, y, z) = xz - 5x^2y^3z^4$
7. To given function $f(x, y, z, t) = x^2y \cos(\frac{z}{t})$ find the $\frac{\partial f}{\partial t}$.
8. Compute all partial derivatives of $f(x, y, z) = x \sin(y - z)$ in a point $A = [1; 0; 0]$. What does these values mean?
9. Compute all partial derivatives of $f(x, y, z) = ze^{xyz}$ in a point $A = [0; 2; -1]$. What does these values mean?
10. a) Compute all partial derivatives of $f(x, y) = \ln(2x - y) + 3x^3 - xy$ in a point $A = [1; 1]$.
b) Write a tangent line of the function in a cut $x \equiv 1$ in tangent point A.