Domains of definition, iso-surface

1. Find a) a domain of definition of $f(x,y) = e^{\frac{1}{x-y}}$ and sketch it.
   b) an equation of iso-curve (a level-curve) $f(x,y) = e^2$, simplify it and sketch it.

2. Find a) a domain of definition of $f(x,y) = \frac{\sin xy}{\sqrt{xy}}$ and sketch it.
   b) an equation of iso-curves (a level-curves) $f(x,y) = 0$, simplify it and sketch it.

3. Find a) a domain of definition of $f(x,y) = \ln(x^2 + y^2 - 4)$ and sketch it.
   b) an equation of iso-curve (a level-curve $f(x,y) = K$) which contains a point $[2; e]$ simplify it and sketch it.

4. Find a) the domain of definition for $f(x,y,z) = \sqrt{y - x^2}\ln z$.
   b) Identify the iso-surface $f(x,y,z) = 0$

5. Find a) the domain of definition for $f(x,y,z) = \frac{x}{\sqrt{y^2 - z^2}}$.
   b) Identify the iso-surface which contains a point $[-1; 1; 0]$

6. Find a) the domain of definition for $f(x,y,z) = \frac{1}{\sqrt{x^2+y^2+z^2}}$.
   b) Identify the iso-surface $f(x,y,z) = \frac{1}{4}$

Limits

7. (a) Find a candidate for a limit $\lim\limits_{[x,y]\to[0;0]} \frac{1}{\sqrt{x^2+y^2}}$
   (b)* Can you prove (in this special case) that the candidate is the only possibility?
   hint: Try the rotational symmetry

8. (a) Find a candidate for a limit $\lim\limits_{[x,y]\to[0;0]} \frac{\sin x \sin y}{xy}$
   (b)* Can you prove (in this special case) that the candidate is the only possibility?
   hint: Remember what is true for multiplication of the limits from M1

9. (a) Find a candidate for a limit $\lim\limits_{[x,y]\to\infty} (x^2 + 3y^2)e^{-x^2-y^2}$
   (b)* Can you prove that the candidate is the only possibility?

10. (a) Find a candidate for a limit $\lim\limits_{[x,y]\to[0;0]} \frac{x+y}{xy}$
    (b) Prove that the limit doesn't exist. hint: Try different lines

11. Prove that the $\lim\limits_{[x,y]\to[0;0]} \frac{xy^2}{x^2+y^4}$ doesn't exist. hint: Try different parabolas

12. Decide if the following function is continuous in point [0; 0]:
    $$f(x,y) = \begin{cases} \frac{\sin(x^2+y^2)}{\sqrt{x^2+y^2+1}-1} & \text{for} \quad [x,y] \in \mathcal{D}(f) \\ 2 & \text{for} \quad [x,y] = [0;0] \end{cases}$$