Nabla, 3D potential, divergence theorem

1. Given $\vec{f}(x,y,z) = (xy^2, x^2+2z, 3yz)$.
   (a) div $\vec{f} = \nabla \cdot \vec{f} = ?$
   (b) rot $\vec{f} = \nabla \times \vec{f} = ?$
   (c) Is the field $\vec{f}$ conservative?

2. Given $\vec{f}(x,y,z) = (2xy^3z^4, 3x^2y^2z^4, 4x^2y^3z^3)$.
   (a) div $\vec{f} = \nabla \cdot \vec{f} = ?$
   (b) Where is the field $\vec{f}$ conservative?
   (c) Is the scalar function $\varphi(x,y,z) = x^2y^3z^4$ its potential?

3. Consider $\vec{f}(x,y,z) = (3x, 2y, 2y+z)$ and a surface Q = shell of a body bounded by planes: $x=0$, $y=0$, $z=0$, $y=2$ and $x+z=1$. The surface is oriented with outer normal.

   $$\oiint_Q \vec{f} \cdot \,\mathrm{d}\vec{p} = ?$$

   [6]

4. Compute the flux of a vector field $\vec{f}(x,y,z) = (x+\cos x, y+e^z, z+z\sin x)$ through the surface of a body $\Omega = \{[x,y,z] \in \mathbb{R}^3 :\ 0 \le z \le 4-x^2-y^2\}$. The flux is heading outside (outer normal).
   [$\Phi = 24\pi$]

5. For a surface $\sigma = \{[x,y,z] \in \mathbb{R}^3 :\ x^2+y^2+z^2 = 4 \ \wedge\ z \ge 0\}$, oriented by the normal which has positive third component, compute $\iint_\sigma (y,x,z^2) \cdot \mathrm{d}\vec{p}$.

   [8π]