(Surface integral I)

1. For the surface $\sigma = \{[x,y,z] \in \mathbb{R}^3 :\ 2x + 2y + z = 4 \ \wedge\ x, y, z \geq 0\}$ compute $\iint\limits_{\sigma} xy \, \mathrm{d}p$.

2. For the surface $\sigma = \{[x,y,z] \in \mathbb{R}^3 :\ x^2 + y^2 = 4 \ \wedge\ 0 \leq z \leq 1\}$ compute $\iint\limits_{\sigma} xy \, \mathrm{d}p$.

3. Compute a mass of a metal sheet in a shape of $\sigma = \{[x,y,z] \in \mathbb{R}^3 :\ z = \sqrt{4 - x^2 - y^2}\}$ if $\rho(x,y,z) = z$.

4. Compute an area of a surface $\sigma = \{[x,y,z] \in \mathbb{R}^3 :\ z = \sqrt{x^2 + y^2} \ \wedge\ x^2 + y^2 \leq 2x\}$.

Surface integral II

1. Surface $\sigma = \{[x,y,z] \in \mathbb{R}^3 :\ x, y \in \langle 0; a \rangle \ \wedge\ z = a; a > 0\}$ is oriented by normal $\vec{n}_\sigma$ that has positive third (i.e. z) component.

   (a) Find the parametrization and determine if it is oriented in agreement with the surface.

   (b) Compute the surface integral $\iint\limits_{\sigma} (x, y, z) \cdot \vec{\mathrm{d}p}$.

2. Consider a surface $Q = \{[x,y,z] \in \mathbb{R}^3 :\ x + 2y + z = 6 \ \wedge\ x, y, z \geq 0\}$ oriented by normal which makes an acute angle with $\hat{k} = (0,0,1)$ and a vector function $\vec{f}(x,y,z) = (z, y, 2x)$.

   (a) Find the parametrization and determine if it is oriented in agreement with the surface.

   (b) Compute the flux of a vector field $\vec{f}$ through the oriented surface Q.

3. Consider a surface $\sigma = \{[x,y,z] \in \mathbb{R}^3 :\ x^2 + y^2 = 16 \ \wedge\ x \leq 0 \ \wedge\ y \geq 0 \ \wedge\ 0 \leq z \leq 1\}$ oriented by normal $\vec{n}_\sigma = \hat{i} = (1,0,0)$ in a point [-4; 0; 0].

   (a) Find the parametrization and determine if it is oriented in agreement with the surface.

   (b) Compute the surface integral $\iint\limits_{\sigma} (y, z, x^2) \cdot \vec{\mathrm{d}p}$.

4. For the surface $\sigma = \{[x,y,z] \in \mathbb{R}^3 :\ z = x^2 + y^2 \ \wedge\ z \leq 9\}$ oriented with outer normal compute

$$\iint\limits_{\sigma} (x, 0, 2z) \, \vec{\mathrm{d}p}$$

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5. Compute the flux of a vector field $\vec{f}(x,y,z) = (0,0,z^2)$ through the surface $Q = \{[x,y,z] \in \mathbb{R}^3 :\ x^2 + y^2 + z^2 = 4 \ \wedge\ x \leq 0 \ \wedge\ y, z \geq 0\}$ oriented by normal which makes an acute angle with $\hat{k} = (0,0,1)$.