(Conservative fields, potential)

1. Determine a domain where $\vec{f}$ is conservative and find its potential (if possible).
   (a) $\vec{f}(x,y) = (x^3y^2 + x,\ y^2 + yx^4)$
   (b) $\vec{f}(x,y) = (\ln y - \frac{e^y}{x^2},\ \frac{e^y}{x} + \frac{x}{y})$
2. To the given potential $\varphi(x,y) = xe^{x+y}$
   (a) find the conservative $\vec{f}$ (corresponding to φ).
   (b) Compute $\int_C \vec{f} \cdot \mathrm{d}\vec{s}$. where $C:\ x^2 + y^2 = 1$ in a first quadrant, oriented counter-clockwise.
3. $\vec{f}(x,y) = (\frac{x}{\sqrt{x^2+y^2}}, \frac{y}{\sqrt{x^2+y^2}})$.
   (a) Where is $\vec{f}$ conservative?
   (b) Is $\varphi(x,y) = \sqrt{x^2+y^2}$ its potential?
   (c) Compute the work done along a curve $x^2 + y^2 = 4$ oriented counter-clockwise.

Surface integral I

1. Given a surface $\sigma = \{[x,y,z] \in \mathbb{R}^3 :\ z = 4 - x^2 - y^2\ \wedge\ z \geq 0\}$
   (a) Suggest its parametrization.
   (b) Verify that the suggested parametrization is correct.
   (c) Compute the length of the normal vector to the parametrization.
2. Parametrize a parallelogram ABCD and verify that the suggested parametrization is correct.
   A = [0;0;0] B = [0;2;3] C = [0;5;3] D = [0;3;0]
3. For the surface $\sigma = \{[x,y,z] \in \mathbb{R}^3 :\ 2x + 2y + z = 4\ \wedge\ x,y,z \geq 0\}$ compute $\iint_\sigma xy\,\mathrm{d}p$.