(Circulation)

0. Given curve is a boundary of domain $\{[x, y] \in \mathbb{R}^2 : \ y \geq x^2 \ \wedge \ x \geq 0 \ \wedge \ y \leq 1\}$ oriented negatively. Compute the circulation of a vector field $\vec{f}(x, y) = (-y/2, x/2)$ over the curve.

Conservative fields, potential

1. $\vec{f}(x, y) = (xy, x + y)$.
   (a) Where is $\vec{f}$ conservative?
   (b) Find its potential $\varphi(x, y)$.
   (c) Compute the work done (line integral) along line segment between $A = [1; 3]$ and $B = [1; 2]$.
2. $\vec{f}(x, y) = (x^2, y^2)$.
   (a) Where is $\vec{f}$ conservative?
   (b) Find its potential $\varphi(x, y)$.
   (c) Compute the work done (line integral) along line segment between $A = [0; 0]$ and $B = [1; 2]$.
3. $\vec{f}(x, y) = -(x - y)^{-2}\vec{i} + (x - y)^{-2}\vec{j}$.
   (a) Where is $\vec{f}$ conservative?
   (b) Choose correct potential:
   (i) $\varphi(x, y) = \frac{1}{y-x} + 2$ (ii) $\varphi(x, y) = \frac{-1}{y-x} + 1$
   (iii) $\varphi(x, y) = \frac{1}{x-y} + 1$ (iv) $\varphi(x, y) = \frac{1}{x-y} - \pi$
   (c) Compute the work done (line integral) along a curve $x^2 + y^2/2 = 2$ in a second quadrant, oriented clockwise.
4. To the given potential $\varphi(x, y) = x^2y + c$, $(c \in \mathbb{R})$
   (a) determine the constant c if $\varphi = 9$ in $A = [\sqrt{2}; \sqrt{2}]$.
   (b) find the conservative $\vec{f}$ (corresponding to φ).
   (c) Compute $\oint_C \vec{f} \cdot \vec{ds}$ where $C : \ x^2 + y^2 = 4$ oriented counter-clockwise.
5. Determine a domain where $\vec{f}$ is conservative and find its potential (if possible).
   (a) $\vec{f}(x, y) = (x^3y^2 + x, \ y^2 + yx^4)$
   (b) $\vec{f}(x, y) = (\ln y - \frac{e^y}{x^2}, \ \frac{e^y}{x} + \frac{x}{y})$
6. To the given potential $\varphi(x, y) = xe^{x+y}$
   (a) find the conservative $\vec{f}$ (corresponding to φ).
   (b) Compute $\int_C \vec{f} \cdot \vec{ds}$. where $C : \ x^2 + y^2 = 1$ in a first quadrant, oriented counter-clockwise.