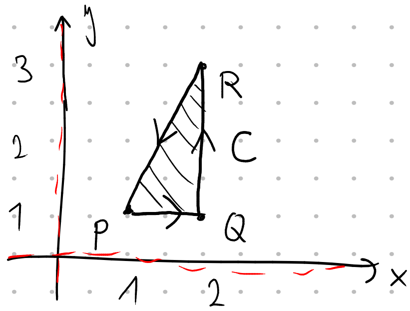
5.)



$\vec{f}(x,y) = \left(\frac{1}{y}; -\frac{1}{x}\right) = (U, V)$

C positively oriented

(Int C lies on the left)

Green? $\quad \mathcal{D}(\vec{f}) = \{[x,y] \in E^2;\ x \neq 0 \wedge y \neq 0\}$

i) $\frac{\partial U}{\partial x} = 0 \quad \frac{\partial U}{\partial y} = -\frac{1}{y^2}$

$\frac{\partial V}{\partial x} = \frac{1}{x^2} \quad \frac{\partial V}{\partial y} = 0$

$\Big\}$ cont. in $G = \mathcal{D}(\vec{f})$

$\uparrow$ $x \neq 0$, $y \neq 0$ OK

ii) C positively oriented curve; $\quad \text{Int}\, C \subset G \quad \checkmark$

$$\oint_C \vec{f} \cdot d\vec{s} \overset{\text{Green}}{=} \iint_{\text{Int}\,C} \left(\frac{\partial V}{\partial x} - \frac{\partial U}{\partial y}\right) dx\,dy = \iint_{\text{Int}\,C} \left(\frac{1}{x^2} + \frac{1}{y^2}\right) dx\,dy = I$$

line p: $\quad y = kx + q$

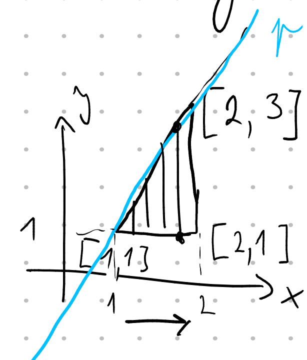
$P \in p$: $\quad 1 = 1k + q$

$\ominus \quad R \in p$: $\quad 3 = 2k + q$

$-2 = -k \quad \Rightarrow \quad k = 2$

$q = -1$

$y = 2x - 1$



EDI$_x$:

$$\boxed{\begin{array}{l} 1 \leq x \leq 2 \\ 1 \leq y \leq 2x - 1 \end{array}}$$

$\int y^{-2}\, dy = -y^{-1}$

$$I = \int_1^2 \left(\int_1^{2x-1} \left(\frac{1}{x^2} + \frac{1}{y^2}\right) dy \right) dx = \int_1^2 \left[\frac{1}{x^2} y - \frac{1}{y}\right]_1^{2x-1} dx =$$

$$= \int_1^2 \left(\frac{1}{x^2}(2x-1) - \frac{1}{x^2} - \frac{1}{2x-1} + 1\right) dx =$$

(5)

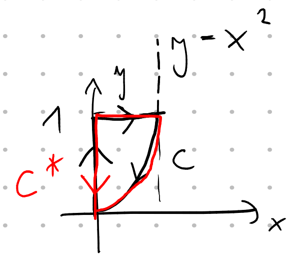
$$= \int_1^2 \left(\frac{2}{x} - \frac{2}{x^2} - \frac{1}{2x-1} + 1 \right) dx =$$

$$= \left[2 \ln|x| + \frac{2}{x} - \frac{1}{2} \ln|2x-1| + x \right]_1^2 =$$

$$= 2 \ln 2 + \cancel{1} - \frac{1}{2} \ln 3 + \cancel{2} - \cancel{(0 + 2 + 1)} =$$

$$= \ln\left(\frac{4}{\sqrt{3}}\right) = \ln\left(\frac{4\sqrt{3}}{3}\right)$$

6.) $\oint \underbrace{\left(-\frac{y}{2}; \frac{x}{2}\right)}_{\vec{f}(x,y)} \cdot d\vec{s} = *$



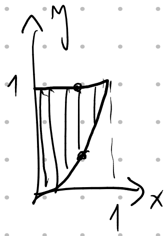
Green? $U = -\frac{y}{2}$ $V = \frac{x}{2}$

i) $\frac{\partial U}{\partial x} = 0$ $\frac{\partial U}{\partial y} = -\frac{1}{2}$ $\frac{\partial V}{\partial x} = \frac{1}{2}$ $\frac{\partial V}{\partial y} = 0$ ✓ cont. in $E_2 = G$ $(= D(\vec{f}))$

ii) C^* positively oriented curve; Int $C^* \subset G$ ✓

opposite orientation (than C)

$$* \overset{\text{Green}}{=} - \iint_{\text{Int } C} \left(\frac{\partial V}{\partial x} - \frac{\partial U}{\partial y} \right) dx\,dy = - \iint_{\text{Int } C} \left(\frac{1}{2} + \frac{1}{2} \right) dx\,dy =$$



$$\boxed{\begin{array}{l} 0 \le x \le 1 \\ x^2 \le y \le 1 \end{array}}$$

$$= - \int_0^1 \left(\int_{x^2}^1 1\, dy \right) dx = - \int_0^1 [y]_{x^2}^1 dx =$$

$$= - \int_0^1 (1 - x^2)\, dx = \left[\frac{x^3}{3} - x \right]_0^1 = \frac{1}{3} - 1 = -\frac{2}{3}$$