Domains of definition

1. Identify and sketch the domain Ω:

   (a) $\Omega = \{[x,y] \in \mathbb{R}^2;\ x^2+y^2 \le 16 \quad \wedge \quad y \le x^2+2\}$
   (b) $\Omega = \{[x,y] \in \mathbb{R}^2;\ \frac{x^2}{36}+\frac{y^2}{25} \le 1 \quad \wedge \quad \frac{y^2}{4}-x^2 > 1\}$
   (c) $\Omega = \{[x,y,z] \in \mathbb{R}^3;\ z = x^2+4y^2 \quad \wedge \quad z = 16\}$
   (d) $\Omega = \{[x,y,z] \in \mathbb{R}^3;\ x^2+y^2 \le 4 \quad \wedge \quad z = 7+x^2+y^2\}$
   (e) $\Omega = \{[x,y,z] \in \mathbb{R}^3;\ x^2+y^2 \le z^2 \quad \wedge \quad z = 6-x^2-y^2\}$
   (f) $\Omega = \{[x,y,z] \in \mathbb{R}^3;\ 2x^2-y^2-z^2 = 4 \quad \wedge \quad 0 \le x \le 2\}$
   (g) $\Omega = \{[x,y,z] \in \mathbb{R}^3;\ z^2 \le 10-x^2-y^2 \quad \wedge \quad 3z = x^2+y^2\}$

2. Find and sketch a domain of definition of following functions:

   (a) $f(x,y) = \frac{\sqrt{x-y^2}}{\ln(1-x^2-y^2)}$
   (b) $f(x,y) = \sqrt{\ln\left(\frac{16}{x^2+y^2}\right)}$
   (c) $f(x,y) = 3-7\ln(x+\ln y)$
   (d) $f(x,y) = \sqrt{2x+y-4}+\sqrt{16-x^2-y^2}$
   (e) $z(x,y) = \frac{\ln(x^2y)}{\sqrt{y-x}}$
   (f) $z(x,y) = \arcsin\left(\frac{y-1}{x}\right)$

3. Find and sketch a domain of definition. Find an equation of iso-curve (level-curve) $f(\vec{x}) = K$, simplify it and sketch it.

   (a) $f(x,y) = \frac{1}{x^2-2y}$, $K = 1$ and $K = 2$
   (b) $f(x,y) = e^{\frac{1}{x-y}}$, $K = e^2$
   (c) $f(x,y) = \frac{\sin xy}{\sqrt{xy}}$, $K = 0$
   (d) $f(x,y) = \ln(x^2+y^2-4)$, $K = 2$

4. Find a) the domain of definition for $f(x,y,z) = \sqrt{y-x^2}\ln z$.
   b) Identify the iso-surface $f(x,y,z) = 0$

5. Find a) the domain of definition for $f(x,y,z) = \frac{x}{\sqrt{y^2-z^2}}$.
   b) Identify the iso-surface $f(x,y,z) = -1$

6. Find a) the domain of definition for $f(x,y,z) = \frac{1}{\sqrt{x^2+y^2+z^2}}$.
   b) Identify the iso-surface $f(x,y,z) = \frac{1}{4}$