Double integrals: applications, polar coordinates

1. Given 2D body: $D = \{[x, y] \in \mathbb{R}^2 : \ 0 \le x \le 1 \ \wedge \ 0 \le y \le 2x + 1\}$.
   Its (2D) density $\rho(x, y) = x$.
   (a) Compute its mass.
   (b) Compute the static moment according to y-axis (m_y =?).
   (c) Determine the x-coordinate of center of mass (x_C =?).
   (HW:) Determine the y-coordinate of center of mass (y_C =?). $[y_C = 17/14]$

2. Given 2D body bounded by curves: $y = \frac{1}{x} - 1; \quad y = x; \quad y = 0$.
   $\rho(x, y) = (y + 1)^2$. J_y =?

3. Given $D = \{[x, y] \in \mathbb{R}^2 : \ x^2 + y^2 \le 4 \ \wedge \ y \ge 0\}$.
   (a) Transfer the following integral to polar coordinates:
   $$\iint_D xy \, \mathrm{d}x\mathrm{d}y.$$
   (b) Compute the integral.
   (c) Write one possible physical meaning of the integral, $\rho(x, y)$ =?.
   (HW:) Determine the center of mass (C =?) when $\rho(x, y) = y$. $[y_C = 3\pi/8]$.

4. Given $f(x, y) = \frac{1}{\sqrt{9 - x^2 - y^2}}$
   and $D = \{[x, y] \in \mathbb{R}^2; \ x \ge 0 \ \wedge \ x^2 + y^2 \le 8\}$.
   $$\iint_D f(x, y) \, \mathrm{d}x\mathrm{d}y = ?$$

5. Given $D = \{[x, y] \in \mathbb{R}^2 : \ \frac{x^2}{9} + \frac{y^2}{4} \le 1 \ \wedge \ x \ge 0 \ \wedge \ y \ge 0\}$.
   (a) Transfer the following integral to generalized polar coordinates:
   $$\iint_D xy^2 \, \mathrm{d}x\mathrm{d}y.$$
   (b) Compute the integral.
   (c) Write all possible physical meanings of the integral, $\rho(x, y)$ =?