(constrained extremes)

1. Given $f(x, y) = 2x^2 + y^2 - xy + 3x + y + 1$,
   a) Find local extremes of the function f, i.e. find their position, type and value.
   b) Find (glob.) extremes of the function f constrained on the line $x = 2 - y$.

2. Find global extremes of $f(x, y) = x^2 - 2x + y^2$ on a set $\mathcal{H} = \{[x, y] \in \mathbb{R}^2;\ x^2 + y^2 = 9\}$.

Global (absolute) extremes

3. Find global extremes of $f(x, y) = x^2 + xy - 3x - y$
   on a set $\mathcal{M} = \{[x, y] \in \mathbb{R}^2;\ x + y \leq 3 \ \wedge\ x \geq 0 \ \wedge\ y \geq 0\}$.

4. Find global extremes of $f(x, y) = 2x^2 - 4x + y^2 - 4y + 2$
   on a set $\mathcal{M} = \{[x, y] \in \mathbb{R}^2;\ x \geq 0 \ \wedge\ 2 \geq y \geq 2x\}$.

5. Find global extremes of $f(x, y) = x^2 - y^2$
   on a set $\mathcal{M} = \{[x, y] \in \mathbb{R}^2;\ x \geq -1 \ \wedge\ y \geq -1 \ \wedge\ x + 2y \leq 2\}$.
   .
   [glob. min. in $P_4 = [-2/3; 4/3]$, $f(P_4) = -4/3$, glob. max. in $P_5 = [-1; -1]$, $f(P_5) = 15$]

possible repetition of implicitly defined function

1. $F(x, y) = x^2 + \frac{1}{2}y^2 + xy - 9\ln(x)$
   (a) Find iso-curve $\iota: \quad F(x, y) = 1$.
   (b) Write an equation of a line tangent to the iso-curve ι in a tangent point $A = [1; 0]$.
   hint: Use implicitly defined function ($F(x, y) = 1$) and compute its derivative.
   (c) Approximate the iso-curve ι in a point A with the 2^{nd} order Taylor polynomial.

   [b) $y = 7(x - 1)$ c) $y(x) \approx 7(x - 1) - 37(x - 1)^2$]

2. $F(x, y, z) = z^3 + 3x^2z - 2xy = 0$
   (a) $\exists ?\ z = f(x, y)$ near the point $A = [-1; -2; 1]$ defined implicitly?
   (b) $\nabla f = ?$ at point $A_0 = [-1; -2]$.
   (c) Compute a tangent plane (τ) to $z = f(x, y)$ at point A.

   [b) $\nabla f(A_0) = (1/3; -1/3)$ c) $z - 1 = 1/3(x + 1) - 1/3(y + 2)$]