7.) $\ln 1{,}2 \approx ?$ 18.7.

n=4 $\Rightarrow |R_{4+1}| \le \varepsilon$ (2)

$f = \ln x \qquad x_0 = 1 \qquad x_1 = 1{,}2 \qquad \varepsilon = 10^{-3}$

$f(1) = 0 \qquad f'(1) = \frac{1}{x}\Big|_1 = 1 \qquad f'' = -\frac{1}{x^2}\Big|_1 = -1$

$f'''(1) = +2\frac{1}{x^3}\Big|_1 = 2 \qquad f^{IV}(1) = -6\frac{1}{x^4}\Big|_1 = -6$

$f^{V}(x) = +24\frac{1}{x^5}$

$T_n(x) = 0 + (x-1) - \frac{(x-1)^2}{2} + \frac{\cancel{2}\,1}{\cancel{6}\,3}(x-1)^3 - \frac{\cancel{6}}{\cancel{6}\,4}(x-1)^4 + \dots$

decr.

$|R_{n+1}(x)| = \frac{\cancel{24}\,4}{24 \cdot 5}\left(\frac{1}{\xi^5}\right)(x-1)^5$ is degree ~~5~~ 4 enough?

$\xi \in (1; 1{,}2) \qquad \xi = 1$ (worst)

$|R_5(1{,}2)| \le \frac{1}{5} 1 (0{,}2)^5 = \frac{1}{5}\left(\frac{2}{10}\right)^5 = \frac{1}{5}\frac{32}{10^5} < 10^{-3}$ ✓

n=4 is enough

$\ln 1{,}2 \approx 0 + 0{,}2 - \frac{0{,}2^2}{2} + \frac{0{,}2^3}{3} - \cancel{\frac{0{,}2^4}{4}} =$

$= 0{,}2 - 0{,}02 + \frac{8}{3}10^{-3}$

$|R_{3+1}(1{,}2)| < \frac{1}{4} 1 \cdot (0{,}2)^4 = \frac{\cancel{16}\,4}{\cancel{4}} \cdot 10^{-4} < 10^{-3}$:)

also n=3 is enough :-)

8.) $\sqrt[3]{1.5} \doteq ?$

$\varepsilon = 10^{-2}$

$f(x) = \sqrt[3]{1+x}$

$f(0) = 1 \qquad (x_0 = 0 \; , \; x_1 = 0.5)$

$f'(0) = \frac{1}{3} \frac{1}{\sqrt[3]{1+x}^2}\Big|_{x=0} = \frac{1}{3} \qquad \left(\frac{1}{3}(1+x)^{-\frac{2}{3}}\right)$

$f''(0) = -\frac{2}{9} \frac{1}{\sqrt[3]{1+x}^5}\Big|_{x=0} = -\frac{2}{9} \qquad \left(-\frac{2}{9}(1+x)^{-\frac{5}{3}}\right)$

is $n=2$ enough? : $T_2(x) = 1 + \frac{1}{3}x - \frac{1}{9}x^2 \approx f(x)$

b) $f'''(\xi) = +\frac{10}{3^3}(1+x)^{-\frac{8}{3}}\Big|_{x=\xi} = \frac{10}{27} \frac{1}{\sqrt[3]{1+\xi}^8}$

$R_{2+1}(x) = \frac{1}{2 \cdot 3} \frac{10}{27} \frac{1}{\sqrt[3]{1+\xi}^8} (x-0)^3 \qquad \xi \in (0, \frac{1}{2})$

c) $\sqrt[3]{1.5} = f(0.5) \approx 1 + \frac{1}{3} \cdot \frac{1}{2} - \frac{1}{9} \cdot \frac{1}{4} = \frac{36+6-1}{36} = \frac{41}{36}$

error = $|f(0.5) - T_2(0.5)| \leq |R_{2+1}(0.5)| = \left|\frac{5}{81} \frac{1}{\sqrt[3]{1+\xi}^8} \left(\frac{1}{2}\right)^3\right| < \varepsilon$?

decreasing

$(1+\frac{1}{2}$ .... smaller number$)$

worst case $\xi = 0$

$|R_{2+1}(0.5)| \leq \frac{5}{81} \frac{1}{8} = \frac{5}{648} < \frac{1}{100}$ (ε) ✓