Determinants and applications

Inverse matrix and its determinant

1. Find the inverse matrix (A^{-1}) and compute its determinant, $A = \begin{pmatrix} 1 & 2 \\ 2 & 2 \end{pmatrix}$

2. Compute the determinant of an inverse matrix A^{-1}:

   (a) $A = \begin{pmatrix} 3 & -5 & 0 \\ 0 & 2 & 3 \\ 1 & 2 & 1 \end{pmatrix}$

   (b) $A = \begin{pmatrix} 0 & 1 & 2 \\ -1 & 0 & 1 \\ 1 & 1 & 1 \end{pmatrix}$

Linear Independence of vectors with parameters

3. Find the parameter $p \in \mathbb{R}$ for which the vectors are linear independent, $\vec{u} = (3+p; 7; 1)$, $\vec{v} = (-2; 2p; 4)$ and $\vec{w} = (1; 0; 1)$

4. Find the parameter $k \in \mathbb{R}$ for which the vectors are linear independent, $\vec{u} = (k; 1; 0)$, $\vec{v} = (0; k-1; 3)$ and $\vec{w} = (0; 2; k)$

Cramer's rule

5.
$$\begin{aligned} 3x_1 + 2x_2 &= 0 \\ 4x_1 - 5x_2 &= 40 \end{aligned}$$

6. Find the solution for z:
$$\begin{aligned} 2x + 3y - 3z &= -1 \\ 4x - 4y - z &= 3 \\ 8x - 9z &= 0 \end{aligned}$$

7. Find all solutions depending on parameter $m \in \mathbb{R}$
$$\begin{aligned} 4x_1 + 2x_2 - 2x_3 &= 0 \\ 2x_1 + x_2 + 3x_3 &= 0 \\ mx_1 + x_2 + mx_3 &= 0 \end{aligned}$$

8. Find a solution for x_1 depending on parameter $m \in \mathbb{R}$
$$\begin{aligned} -7x_2 - 5x_3 &= -1 \\ (2m-1)x_1 - x_2 &= 1 \\ 4mx_1 - 7x_2 - 5x_3 &= 0 \end{aligned}$$

9. Find a parameter $p \in \mathbb{R}$ for which the system has non-trivial (not only zero) solution:
$$\begin{aligned} px + 4y + 7z &= 0 \\ 3x - 4y + 5z &= 0 \\ x + py + 4z &= 0 \end{aligned}$$