Higher derivatives

Compute $\frac{\mathrm{d}y}{\mathrm{d}x}$ and $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$ of following functions:

1. $y(x) = e^{-x^2}$
                2. $y(x) = x^2 \ln x$

3. $y(x) = \frac{1+x}{1-x}$

Tangent to the function

4. To the given function $f(x) = 4x - x^2$, find the slope of a tangent to the graph in points (a) $x_0 = 0$, (b) $x_0 = 4$. Determine if the function is increasing or decreasing near these points and how fast it is (inclination of the tangent).

5. Write the equation of the tangent line to the graph of $f(x) = \frac{1}{3}x^3$ in a point $x_0 = -1$. Use this result to calculate an approximate value of $f(-\frac{2}{3})$.

6. Write the equation of the tangent line to the graph of $f(x) = \sqrt{2x+3} - x$ in a tangent point $T = [3; ?]$. Use this result to calculate an approximate value of $f(3.2)$.

7. Write the equations of the tangent and normal lines to the graph of $f(x) = e^{-x} \cos 2x$ in point $x_0 = 0$.

8. Find a tangent point, such that the tangent line of a function $f(x) = x^2 + 4x$ (in the point) is parallel to the x-axes.

HW Approximate the value of $\sqrt{101}$ with 2 decimal places precision.
      hint: use tangent line to $f(x) = \sqrt{x}$

Intervals of monotonicity and local extrema

Determine the intervals of monotonicity, find local extrema (and determine their types) of following functions:

9. $f(x) = 3x - x^3$

10. $f(x) = x^2 - \ln(x^2)$

11. $f(x) = x^2 e^x$

12. $f(x) = x^2 \ln x$