Eigenvalues and eigenvectors

1. Find all eigenvalues of matrix A, choose two of them and find the corresponding eigenvectors.
$A = \begin{pmatrix} 2 & -3 & 1 \\ 1 & -2 & 1 \\ 1 & -3 & 2 \end{pmatrix}$

2. You have a 3×3 matrix, which of following statements can be true:
   (a) $\lambda_1 = 2$, $\lambda_2 = 3$
   (b) $\lambda_1 = 3$, $\lambda_2 = 2 + i$, $\lambda_3 = -2 - i$
   (c) $\lambda_1 = \lambda_2 = \lambda_3 = 1$
   (d) $\lambda_1 = 0$, $\lambda_2 = i$, $\lambda_3 = -i$
   (e) $\lambda_1 = 2$, $\lambda_2 = 1$, $\lambda_3 = 2 + i$
   (f) given eigenvectors (on tutorial)

3. Find all eigenvalues of matrix A, choose two of them and find the corresponding eigenvectors.
$A = \begin{pmatrix} -2 & -2 & -9 \\ -1 & 1 & -3 \\ 1 & 1 & 4 \end{pmatrix}$

4. (a) Check if $\lambda_1 = -1$ is an eigenvalue of the following matrix.
$A = \begin{pmatrix} 2 & 1 & 2 \\ 5 & 2 & 6 \\ 1 & 0 & -1 \end{pmatrix}$
(b) Choose one of the eigenvalues and find a corresponding eigenvector.
(c) Find the eigenvalues to A^2, choose one of them and find the corresponding eigenvector.

Repetition

5. (a) Are vectors $(1;2)$, $(2;3)$ and $(0;4)$ linear independent, why?
   (b) Find the parameters $a \in \mathbb{R}$ for which the following vectors form a base:
   $\vec{u} = (a; 0; a+3)$, $\vec{v} = (1;0;0)$ and $\vec{w} = (0; a-1; a)$.
   (c) If possible, express vectors $\vec{b} = (1;0;-3)$ and $\vec{c} = (2;-5;3)$ as a linear combination of vectors $\vec{u}$, $\vec{v}$ and $\vec{w}$ for $a = 1$.

6. (a) Determine how many solutions will have the following system depending on parameter $a \in \mathbb{R}$:
$$\begin{aligned} -x - 2y + z &= 3 \\ az + x + 3y &= 2a \\ 5y + 3x &= a \end{aligned}$$
(b) Find the solution for the parameter value $a = -4$.

7. Check if exists an inverse to A. If it exists, compute its determinant: $A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \\ 1 & 4 & 5 \end{pmatrix}$.

8. compute $A \cdot B$.
$A = \begin{pmatrix} 2 & 1 & 3 \\ 0 & 5 & 2 \end{pmatrix}$, $B = \begin{pmatrix} 3 & 1 \\ 0 & 3 \\ 5 & 0 \end{pmatrix}$.

9. $\begin{vmatrix} 1 & 0 & -1 & -1 \\ 0 & -1 & -1 & 1 \\ 0 & 2 & 0 & 0 \\ -1 & -1 & 1 & 0 \end{vmatrix} = ?$