

Determinants and applications

Compute determinants of the following matrices and decide if there are singular or not. Try to write down the rank of the matrices.

$$\begin{aligned} \bullet A_1 &= \begin{pmatrix} -1 & -3 \\ -2 & 5 \end{pmatrix} & \bullet A_2 &= \begin{pmatrix} 2 & 5 & 0 \\ -1 & 7 & 1 \\ 4 & 1 & -4 \end{pmatrix} & \bullet A_3 &= \begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \\ 1 & 4 & 5 \end{pmatrix} \\ \bullet A_4 &= \begin{pmatrix} 1 & 1 & -1 \\ 2 & 1 & 0 \\ 1 & -1 & 1 \end{pmatrix} & \bullet A_5 &= \begin{pmatrix} 1 & i & 1+i \\ -i & 1 & 0 \\ 1-i & 0 & 1 \end{pmatrix} \end{aligned}$$

Compute determinants of the following matrices and decide if there are singular or not:

$$\bullet A_6 = \begin{pmatrix} 0 & 5 & -2 & 3 \\ 1 & 2 & 0 & 0 \\ 5 & 2 & 3 & 2 \\ 2 & -1 & 2 & 3 \end{pmatrix} \quad \bullet A_7 = \begin{pmatrix} 1 & 0 & -1 & -1 \\ 0 & -1 & -1 & 1 \\ a & b & 0 & 0 \\ -1 & -1 & 1 & 0 \end{pmatrix} \quad \bullet A_8 = \begin{pmatrix} a & a & a \\ -a & a & x \\ -a & -a & x \end{pmatrix}$$

Inverse matrix and its determinant

1. Find the inverse matrix (A^{-1}) and compute its determinant, $A = \begin{pmatrix} 1 & 2 \\ 2 & 2 \end{pmatrix}$
2. Compute the determinant of an inverse matrix A^{-1} :

$$(a) A = \begin{pmatrix} 3 & -5 & 0 \\ 0 & 2 & 3 \\ 1 & 2 & 1 \end{pmatrix}$$

$$(b) A = \begin{pmatrix} 0 & 1 & 2 \\ -1 & 0 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$

Linear Independence of vectors with parameters

3. Find the parameter $p \in \mathbb{R}$ for which the vectors are linear independent, $\vec{u} = (3 + p; 7; 1)$, $\vec{v} = (-2; 2p; 4)$ and $\vec{w} = (1; 0; 1)$
4. Find the parameter $k \in \mathbb{R}$ for which the vectors are linear independent, $\vec{u} = (k; 1; 0)$, $\vec{v} = (0; k - 1; 3)$ and $\vec{w} = (0; 2; k)$