

Systems of Linear Algebraic Equations

In the following examples

(a) Determine the number of solution (of the SLAE)

(b) Find all the possible solutions.

1.

$$\begin{aligned} 2x - 3y + z &= 0 \\ x + 2y - z &= 3 \\ 2x + y + z &= 12 \end{aligned}$$

2.

$$\begin{aligned} -x + 3y + 2z &= 0 \\ -5x + y - 2z &= 1 \\ 2x + y + 2z &= 0 \end{aligned}$$

3.

$$\begin{aligned} x + 2y + 3z &= 4 \\ 2x + y - z &= 3 \\ 3x + 3y + 2z &= 7 \end{aligned}$$

4.

$$\begin{aligned} x + 3y + 2z &= 0 \\ 2x - y + 3z &= 0 \\ 3x - 5y + 4z &= 0 \\ x + 17y + 4z &= 0 \end{aligned}$$

5.

$$\begin{aligned} x_1 - 2x_2 + 3x_3 - 4x_4 &= 4 \\ x_2 - x_3 + x_4 &= -3 \\ x_1 + 3x_2 - 3x_4 &= 1 \\ -7x_2 + 3x_3 + x_4 &= -3 \end{aligned}$$

6.

$$\begin{aligned} x_1 - 2x_2 + x_3 + x_4 &= 1 \\ x_1 - 2x_2 + x_3 - x_4 &= -1 \\ x_1 - 2x_2 + x_3 + 5x_4 &= 5 \end{aligned}$$

7. For parameter $k \in \mathbb{R}$ solve:

$$\begin{aligned} x + 2y + 3z &= 5 \\ 3x + y + 2z &= k \\ 2x - y - z &= 0 \end{aligned}$$

Determinants

Compute determinants of the following matrices and decide if there are singular or not. Try to write down the rank of the matrices.

$$\bullet A_1 = \begin{pmatrix} -1 & -3 \\ -2 & 5 \end{pmatrix}$$

$$\bullet A_2 = \begin{pmatrix} 2 & 5 & 0 \\ -1 & 7 & 1 \\ 4 & 1 & -4 \end{pmatrix}$$

$$\bullet A_3 = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \\ 1 & 4 & 5 \end{pmatrix}$$

$$\bullet A_4 = \begin{pmatrix} 1 & 1 & -1 \\ 2 & 1 & 0 \\ 1 & -1 & 1 \end{pmatrix}$$

$$\bullet A_5 = \begin{pmatrix} 1 & i & 1+i \\ -i & 1 & 0 \\ 1-i & 0 & 1 \end{pmatrix}$$

Compute determinants of the following matrices and decide if there are singular or not:

$$\bullet A_6 = \begin{pmatrix} 0 & 5 & -2 & 3 \\ 1 & 2 & 0 & 0 \\ 5 & 2 & 3 & 2 \\ 2 & -1 & 2 & 3 \end{pmatrix}$$

$$\bullet A_7 = \begin{pmatrix} 1 & 0 & -1 & -1 \\ 0 & -1 & -1 & 1 \\ a & b & 0 & 0 \\ -1 & -1 & 1 & 0 \end{pmatrix}$$

$$\bullet A_8 = \begin{pmatrix} a & a & a \\ -a & a & x \\ -a & -a & x \end{pmatrix}$$