

## Inverse matrix, matrix equations

Find the inverse matrix ( $A^{-1}$ ) to following matrices:

1.  $A = \begin{pmatrix} 1 & 2 & 1 & 1 \\ 2 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 3 \end{pmatrix}$

2.  $A = \begin{pmatrix} 1 & -5 & 8 \\ 1 & -2 & 1 \\ 2 & -1 & -5 \end{pmatrix}$

Find the matrix  $X$  which solves the following matrix equation:

3.  $AX = (A - B)^2$   
where  $A = \begin{pmatrix} 1 & 2 \\ 0 & 2 \end{pmatrix}$  and  $B = \begin{pmatrix} 3 & 2 \\ 1 & -1 \end{pmatrix}$ .

4.  $XA = B$   
where  $A = \begin{pmatrix} 1 & 1 & -1 \\ 2 & 1 & 0 \\ 1 & -1 & 1 \end{pmatrix}$  and  $B = \begin{pmatrix} 1 & -1 & 3 \\ 4 & 3 & 2 \\ 1 & -2 & 5 \end{pmatrix}$ .

5.  $AXB = C$   
where  $A = \begin{pmatrix} 3 & 1 \\ 5 & 2 \end{pmatrix}$ ,  $B = \begin{pmatrix} 0 & 1 \\ -2 & 3 \end{pmatrix}$  and  $C = \begin{pmatrix} 2 & 2 \\ 2 & -1 \end{pmatrix}$ .

6. Find inverse matrix for a given matrix  $A = \begin{pmatrix} p_1 & 0 \\ 0 & p_2 \end{pmatrix}$  where  $p_1$  and  $p_2$  are Real parameters.  
Determine when the inverse exist.

## Systems of Linear Algebraic Equations

In the following examples (a) Determine the number of solution (of the SLAE) and (b) Find all the possible solutions.

1.

$$\begin{aligned} 2x - 3y + z &= 0 \\ x + 2y - z &= 3 \\ 2x + y + z &= 12 \end{aligned}$$

2.

$$\begin{aligned} -x + 3y + 2z &= 0 \\ -5x + y - 2z &= 1 \\ 2x + y + 2z &= 0 \end{aligned}$$

3.

$$\begin{aligned} x + 2y + 3z &= 4 \\ 2x + y - z &= 3 \\ 3x + 3y + 2z &= 7 \end{aligned}$$

4.

$$\begin{aligned} x + 3y + 2z &= 0 \\ 2x - y + 3z &= 0 \\ 3x - 5y + 4z &= 0 \\ x + 17y + 4z &= 0 \end{aligned}$$

5.

$$\begin{aligned} x_1 - 2x_2 + 3x_3 - 4x_4 &= 4 \\ x_2 - x_3 + x_4 &= -3 \\ x_1 + 3x_2 - 3x_4 &= 1 \\ -7x_2 + 3x_3 + x_4 &= -3 \end{aligned}$$

6.

$$\begin{aligned} x_1 - 2x_2 + x_3 + x_4 &= 1 \\ x_1 - 2x_2 + x_3 - x_4 &= -1 \\ x_1 - 2x_2 + x_3 + 5x_4 &= 5 \end{aligned}$$

7. For parameter  $k \in \mathbb{R}$  solve:

$$\begin{aligned} x + 2y + 3z &= 5 \\ 3x + y + 2z &= k \\ 2x - y - z &= 0 \end{aligned}$$