(Linear Independence, basis, dimension.)

Write the vectors $\vec{a}$ and $\vec{b}$ as a linear combination of vectors $\vec{u}$, $\vec{v}$ and $\vec{w}$. Is the expression unique?

1. $\vec{u} = (1; 3; 2)$, $\vec{v} = (2; -1; 3)$, $\vec{w} = (5; 1; 8)$
   $\vec{a} = (3; 2; 5)$, $\vec{b} = (5; 6; 7)$
2. $\vec{u} = (3; 4; 5)$, $\vec{v} = (-6; 7; 0)$, $\vec{w} = (8; -9; 1)$
   $\vec{a} = (23; -19; 6)$, $\vec{b} = (-20; 23; -1)$
3. $\vec{u} = (4; 0; 7; 2)$, $\vec{v} = (3; 1; 0; 5)$, $\vec{w} = (5; 3; 1; 0)$
   $\vec{a} = (3; 1; 8; -8)$, $\vec{b} = (0; 0; 0; 1)$

Matrices - multiplication

Compute matrix $C = A \cdot B$ and $D = B \cdot A$ if possible:

4. $$A = \begin{pmatrix} 2 & -1 & 3 \\ 0 & 5 & 2 \end{pmatrix}, \qquad B = \begin{pmatrix} 3 & 1 & 2 & 4 \\ 0 & 3 & 1 & 0 \\ 5 & 2 & 0 & 1 \end{pmatrix}$$
5. $$A = \begin{pmatrix} 2 & 1 \\ 1 & 3 \end{pmatrix} \qquad B = A^2$$
6. $$A = \begin{pmatrix} 2 & -1 \\ -1 & 1 \end{pmatrix} \qquad B = \begin{pmatrix} 3 & 0 \\ 0 & 2 \end{pmatrix}$$
7. $$A = \begin{pmatrix} \cos\varphi & -\sin\varphi \\ \sin\varphi & \cos\varphi \end{pmatrix} \qquad B = A^T$$

Compute unknowns x and $y \in \mathbb{R}$ in following equality:

8. $$\begin{pmatrix} x+y & -3 \\ -2 & x-y \end{pmatrix} = \begin{pmatrix} 6 & -2 \\ -3 & 2 \end{pmatrix}^T$$