Applications of definite integral

Mean Value

1. Compute a mean value of function $f(x) = x\cos x$ on the interval $x \in \langle 0; \frac{\pi}{2}\rangle$.
2. Compute a mean value of function $f(x) = \sin^2 x$ on the interval $x \in \langle 0; \pi\rangle$.

Surface

3. Compute the area between the graph of the function $y(x) = x\sqrt{1-x^2}$ and x-axis for $x \in \langle 0; 1\rangle$.
4. Sketch the region bounded by given curves and evaluate its area: $y = 3 - 2x - x^2$ and $y = 0$.
5. Sketch the region bounded by given curves and evaluate its area: $y = x^2$ and $y = \sqrt{x}$.

Volume of rotational bodies

6. Evaluate the volume of the circular body that arises by rotation of a curve $y = \sin x$ about the x–axis for $x \in \langle 0; \frac{\pi}{2}\rangle$.
7. Evaluate the volume of the circular body that arises by rotation of a region bounded between $y = \sqrt{8x}$ and $y = x^2$
   (a) about x-axis.
   (b) about y-axis.

Improper Riemann integral

1. $\int_0^4 \frac{1}{\sqrt{x}}\,\mathrm{d}x$
2. $\int_1^e \frac{1}{x\ln x}\,\mathrm{d}x$
3. $\int_1^\infty \frac{1}{\sqrt[3]{x}}\,\mathrm{d}x$
4. $\int_{16}^\infty \frac{1}{\sqrt[4]{x^5}}\,\mathrm{d}x$
5. $\int_{\pi/2}^\infty \sin x\,\mathrm{d}x$