(L'Hospital's rule for limits)

Let $L = \lim \frac{f(x)}{g(x)} = "\frac{\infty}{\infty}" \vee "\frac{0}{0}"$
than $L = \lim \frac{f'(x)}{g'(x)}$ if the limit exists

1. $\lim\limits_{x\to 0^+} x \ln x$
2. $\lim\limits_{x\to\infty} x \sin\left(\frac{\pi}{x}\right)$
3. $\lim\limits_{x\to 0^+} \sqrt{x}\ln(\sqrt{x})$
4. $\lim\limits_{x\to\pi} \ln(x-\pi)^2$

Higher derivatives

Compute $\frac{\mathrm{d}y}{\mathrm{d}x}$ and $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$ of following functions:

1. $y(x) = e^{-x^2}$
2. $y(x) = x^2 \ln x$
3. $y(x) = \frac{1+x}{1-x}$

Differential and approximate computations

4. Given $f(x) = \frac{1}{2}x^2 - 2x + 2$, compute its differential in a point $x_0 = 0$ and approximate the functional values (a) $f(0.5)$, (b) $f(0.1)$ and (c*) $f(a)$ for parameter $a \in \mathbb{R}$, assume $a << 1$. Compare them to the real functional values.
5. Given $f(x) = \ln x$, compute its differential in a point $x_0 = 1$ and approximate the functional values (a) $f(2)$, (b) $f(1.1)$.
6. Approximate the value of $\sqrt{101}$ with 2 decimal places precision.
   hint: use the differential
7. Approximate the value of $e^{0.05}$ with 2 decimal places precision.

Tangent to the function

8. To the given function $f(x) = 4x - x^2$, find the slope of a tangent to the graph in points (a) $x_0 = 0$, (b) $x_0 = 4$. Determine if the function is increasing or decreasing near these points and how fast it is (inclination of the tangent).
9. Write the equation of the tangent line to the graph of $f(x) = \frac{1}{3}x^3$ in a point $x_0 = -1$. Use this result to calculate an approximate value of $f(-\frac{2}{3})$.
10. Write the equation of the tangent line to the graph of $f(x) = \sqrt{2x+3} - x$ in a tangent point $T = [3; ?]$. Use this result to calculate an approximate value of $f(3.2)$. Writhe the normal line to the graph of a function.
11. Write the equations of the tangent and normal lines to the graph of $f(x) = e^{-x}\cos 2x$ in point $x_0 = 0$.
12. Find a tangent point, such that the tangent line of a function $f(x) = x^2 + 4x$ (in the point) is parallel to the x-axes.