Functions and limits

1. For a function $f(x) = \frac{1}{x^2}$ find a limit $L = \lim_{x\to 0} f(x)$ and a number $\delta > 0$ such that $\forall x \in P_\delta(0)$ is $f(x) > 100$.
2. For a function $f(x) = 3x + 2$ find a limit $L = \lim_{x\to 1} f(x)$ and a number $\delta > 0$ such that $\forall x \in P_\delta(1)$ is $f(x) \in U_\epsilon(L)$. $\epsilon = 0.1$

Compute the following limits

3. $\lim_{x\to\infty} \sin\frac{1}{x}$
4. $\lim_{x\to\infty} (\frac{x^3}{x^2+1} - x)$
5. $\lim_{x\to\infty} (\sqrt{x^2-1} - x)$
6. $\lim_{x\to 2} \frac{x^2-4x+1}{2x+1}$
7. $\lim_{x\to 1} \frac{x^2-2x+1}{2x^2-x-1}$
8. $\lim_{x\to 0} \frac{3x^2+x^3}{x^2+x^5}$
9. $\lim_{x\to 0} \arctan(x^{-2})$
10. $\lim_{x\to 0} \frac{\tan x - \sin x}{\sin^3 x}$

Limits of composed function

11. $\lim_{x\to\infty} \arcsin\frac{x}{x+1}$
12. $\lim_{x\to\infty} \arctan\frac{x}{x^3+3x^2}$
13. $\lim_{x\to 1} \cos^3\left(\pi\frac{x^2-1}{x-1}\right)$

One sided limits

Find both one sided limits and decide if the original limit exists:

14. $\lim_{x\to 2} \frac{x^2+\sin(x^2)}{x-2}$
    i.e. find $\lim_{x\to 2^+} \frac{x^2+\sin(x^2)}{x-2}$ and $\lim_{x\to 2^-} \frac{x^2+\sin(x^2)}{x-2}$ and compare the results.
15. $\lim_{x\to 2} \frac{x^2+\sin(x^2)}{(x-2)^2}$
16. $\lim_{x\to\frac{\pi}{2}} \tan x$
17. $\lim_{x\to 0} \frac{\ln 3x}{3x}$

Find (a) Domain of definition ($\mathcal{D}(f)$) and Range of the given function, (b) compute limits in boundary points of $\mathcal{D}(f)$.

18. $f(x) = \ln(x - \sqrt{x+1})$
19. $f(x) = \arccos\frac{1-2x}{4}$
20. $f(x) = \ln(x+3) + \sqrt{5-2x}$

Continuity

21. Where is the function $f(x) = \frac{1}{9-x^2}$ continuous?

22. Is the following function continuous in a point $x_0 = 0$?
$$f(x) = \begin{cases} \frac{(x+2)^2-4}{x} & \text{for} \quad x \in \mathbb{R} - \{0\} \\ 4 & \text{for} \quad x = 0 \end{cases}$$

23. Find parameter $a \in \mathbb{R}$ such that the following function is continuous:
$$f(x) = \begin{cases} ax & \text{for} \quad x > 1 \\ 4 - ax & \text{for} \quad x \leq 1 \end{cases}$$

24. Find parameter $a \in \mathbb{R}$ such that the following function is continuous:
$$f(x) = \begin{cases} e^{ax} & \text{for} \quad x < 0 \\ a - x & \text{for} \quad x \geq 0 \end{cases}$$