Sequences and limits

Find the first few members of the sequence, (a) sketch the graph, (b) Is the sequence strictly monotonic? proof it! (c) Is it bounded? Compute the limit.

1. $a_n = 2 + 3^n$

2. $a_n = \frac{n}{n+1}$

3. $a_n = -\frac{n^2}{n+1}$

4. $a_n = \frac{(-1)^n}{n^2+1}$

Find (a) the limit a of following sequences and (b) compute n_0 such that for $n > n_0$ is a_n in the ϵ-neighborhood of a.

5. $a_n = \frac{1}{n^2}$, $\epsilon = 0.05$

6. $a_n = 1 + 2^{-n}$, $\epsilon = 0.1$

Find the limit of following sequences:

7. $a_n = \frac{n^2 - n + 3}{n^3 + 2n + 2}$

8. $a_n = \frac{2n^2 - 3n + 5}{3n^2 + 1}$

9. $a_n = \frac{n + \sqrt{10n}}{\sqrt{5n^2 + 3n + 1}}$

10. $a_n = \frac{\sqrt{n+1} + \sqrt{n}}{\sqrt{n+3}}$

11. $a_n = \frac{(2n+1)! + (2n+2)!}{(2n+3)!}$

12. $a_n = \sqrt{n+2} - \sqrt{n+5}$

13. $a_n = n(\sqrt{n^2 - 2n} - \sqrt{n^2 - 3})$

14. $a_n = n - \sqrt{n(n-1)}$

More complicated limits

15. $\lim\limits_{n \to \infty} \frac{n + \cos(n!)}{2n + 1}$

16. $\lim\limits_{n \to \infty} \left(\frac{n+1}{n-1}\right)^n$

17. $\lim\limits_{n \to \infty} \frac{\arctan(n^2)}{n+1}$