Computer graphics
Lesson 1

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Information

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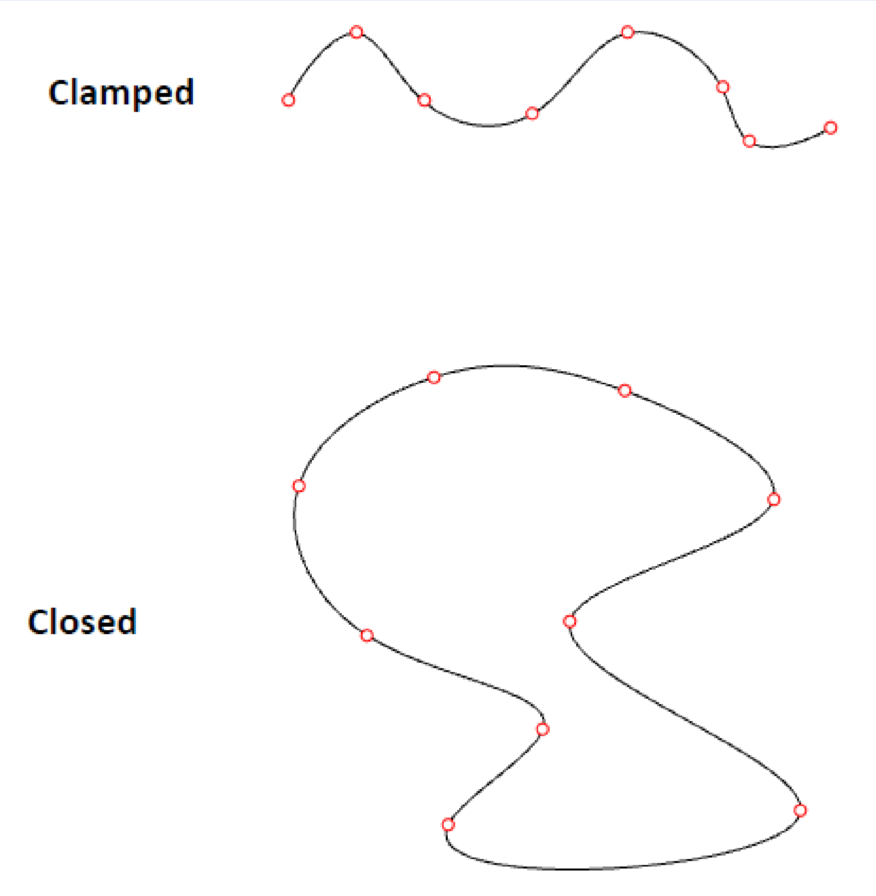
- *Lectures* – even week
- *Tutorials* – odd week (presence required)

- ***Moodle*** - B222-E012037 - Computer Graphics
- *Textbook* – Linkeová, I.: Curves and Surfaces for Computer Graphics

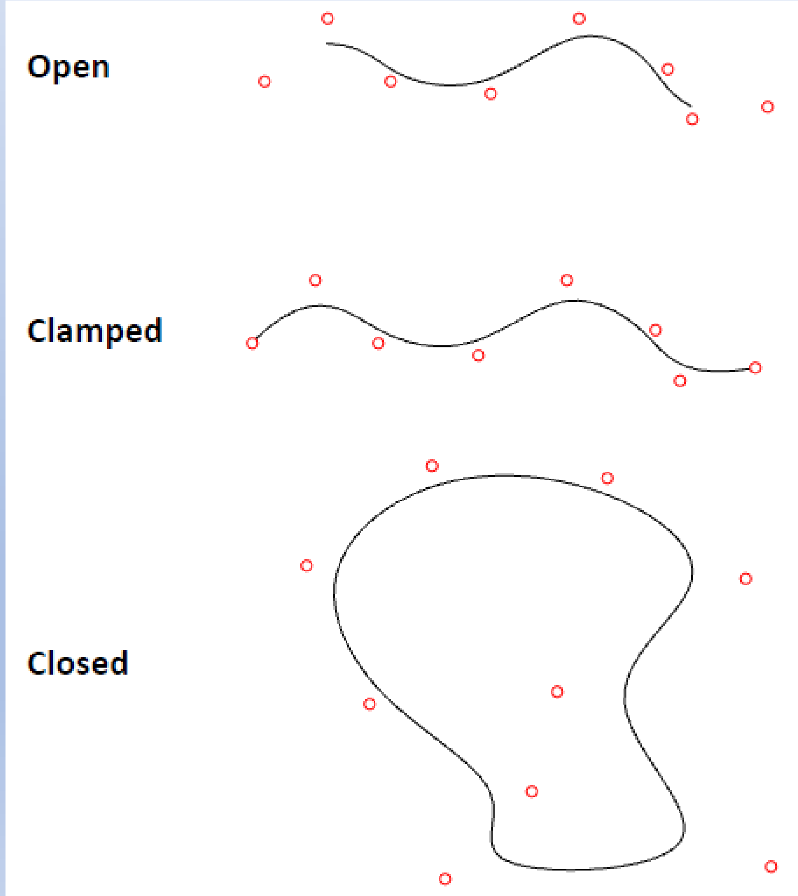
- ***Drawing aids*** – set-square/ruler, pencil, colour pencils, rubber, A4 squared paper (5 mm squared grid)
- *Completion of subject* – award of graded assessment + conditions (details in Moodle, 1 absence possible)

Curve types

1. Interpolation curves:
    - definitions points, definition polygon



2. Approximation curves:
    - Control points, control polygon



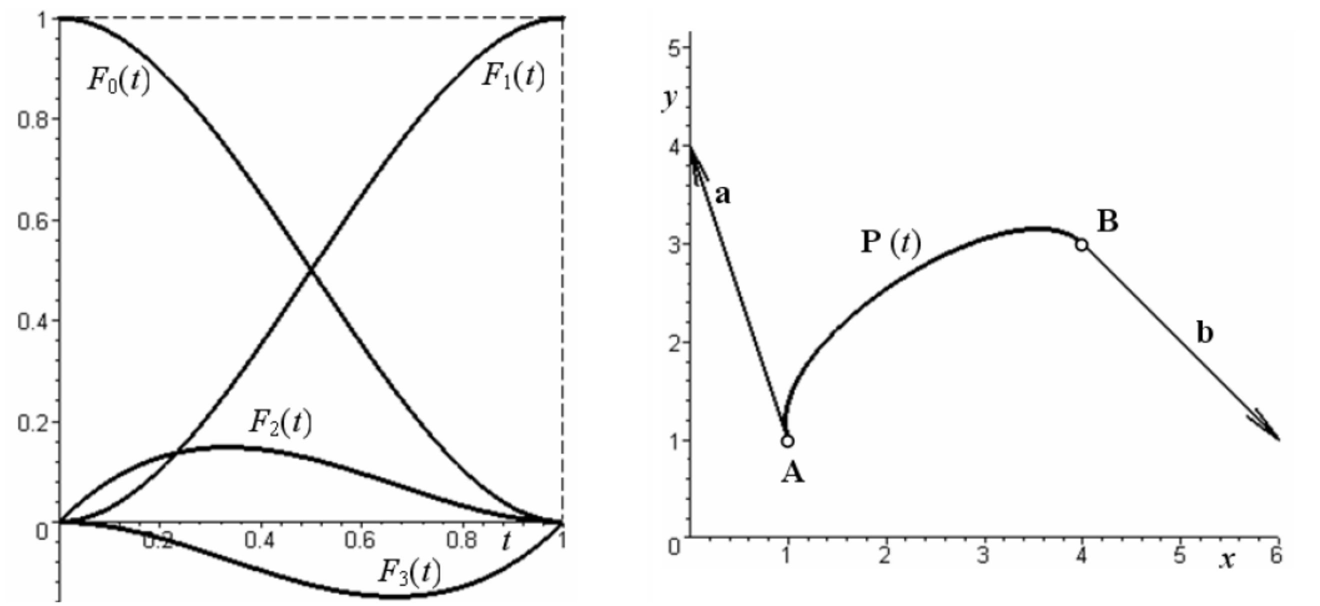
Ferguson cubic curve

- interpolation curve
- vector equation:

$$\mathbf{P}(t) = F_0(t)\mathbf{A} + F_1(t)\mathbf{B} + F_2(t)\mathbf{a} + F_3(t)\mathbf{b},\ t \in [0,1],$$

basis functions are Hermit polynomials

$$\begin{aligned} F_0(t) &= 2t^3 - 3t^2 + 1, \\ F_1(t) &= -2t^3 + 3t^2, \\ F_2(t) &= t^3 - 2t^2 + t, \\ F_3(t) &= t^3 - t^2,\ t \in [0,1] \end{aligned}$$



a) Hermit polynomials b) Ferguson cubic curve

Ferguson cubic curve

- ***properties***:
  - the initial point **P(0)** is the given point **A**
  - the initial point **P(1)** is the given point **B**
  - tangent vector **P´(0)** is the given tangent vector **a**
  - tangent vector **P´(1)** is the given tangent vector **b**

Example 2.1 – Ferguson cubic curve. Ferguson cubic curve $\mathbf{P}(t)$, $t \in [0,1]$ is given by definition points $\mathbf{A} = (1,0)$ and $\mathbf{B} = (4,2)$ and tangent vectors $\mathbf{a} = (-1,3)$ and $\mathbf{b} = (2,-2)$ at these points.

Find the parametric expression and vector equation of Ferguson cubic curve $\mathbf{P}(t)$ and its tangent vector $\mathbf{P}'(t)$. At $t = 0$, $\frac{1}{4}$, $\frac{1}{2}$, $\frac{3}{4}$ and 1, calculate the coordinates of points on Ferguson cubic curve and tangent vectors at these points. Draw all calculated points and construct tangent vectors at these points. Sketch Ferguson cubic curve.

Ferguson cubic curve

Exercise 2.1 Ferguson cubic curve $\mathbf{P}(t)$, $t \in [0,1]$ is given by definition points $\mathbf{A} = (0,0)$ and $\mathbf{B} = (2,0)$ and tangent vectors $\mathbf{a}$ and $\mathbf{b}$ at these points.

Find the parametric expression and vector equation of Ferguson cubic curve $\mathbf{P}(t)$ and its tangent vector $\mathbf{P}'(t)$.At $t = 0$, $t = \frac{1}{2}$ and $t = 1$, calculate the coordinates of points on Ferguson cubic curve and tangent vectors at these points. Draw all calculated points and construct tangent vectors at these points. Sketch Ferguson cubic curve. Consider the following input data:

a) $\mathbf{a} = (0,1)$, $\mathbf{b} = (0,-1)$, b) $\mathbf{a} = (-1,1)$, $\mathbf{b} = (1,-1)$,

c) $\mathbf{a} = (1,1)$, $\mathbf{b} = (1,1)$, d) $\mathbf{a} = (-1,-1)$, $\mathbf{b} = (-1,-1)$.

Ferguson cubic curve

Exercise 2.2 Piecewise interpolation curve k consisting of individual Ferguson cubic curves is given by the following sequence of definition points and tangent vectors at these points:

$$\mathbf{Q}_0 = (0,0),\ \mathbf{Q}_1 = (6,0),\ \mathbf{Q}_2 = (0,0),\ \mathbf{Q}_3 = (-6,0),\ \mathbf{Q}_4 = (0,0),$$
$$\mathbf{q}_0 = (9,9),\ \mathbf{q}_1 = (0,-9),\ \mathbf{q}_2 = (-9,9),\ \mathbf{q}_3 = (0,-9),\ \mathbf{q}_4 = (9,9).$$

Draw the given definition points, construct the given tangent vectors and sketch curve k. How many individual Ferguson cubic curves create curve k? Determine the continuity along curve k without calculating it. Find vector equations of all Ferguson cubic curves and verify the assumed continuity.