Smoothed Particle Hydrodynamics - a mesfree particle method

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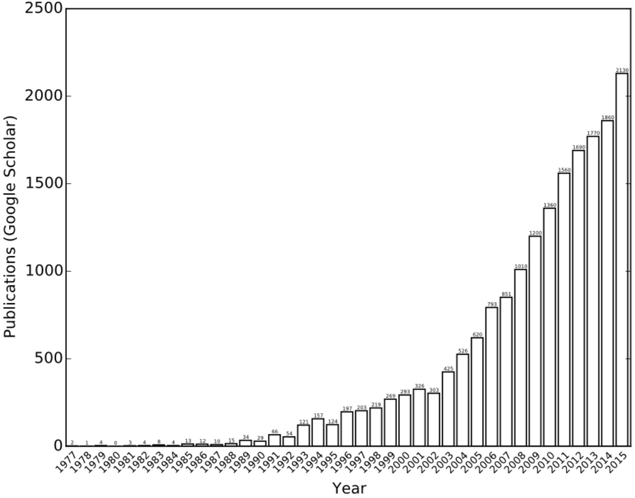
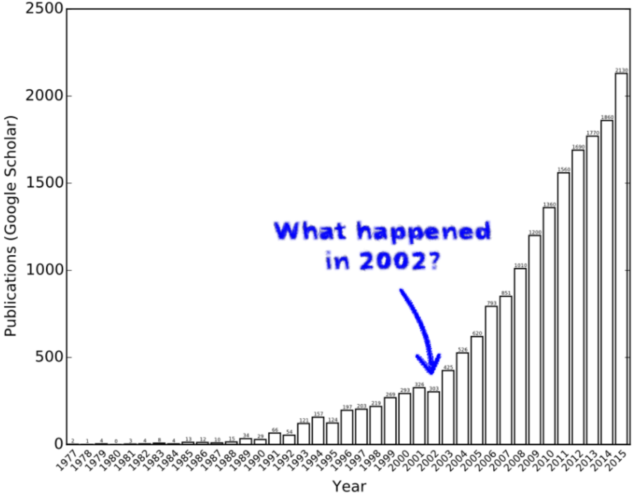
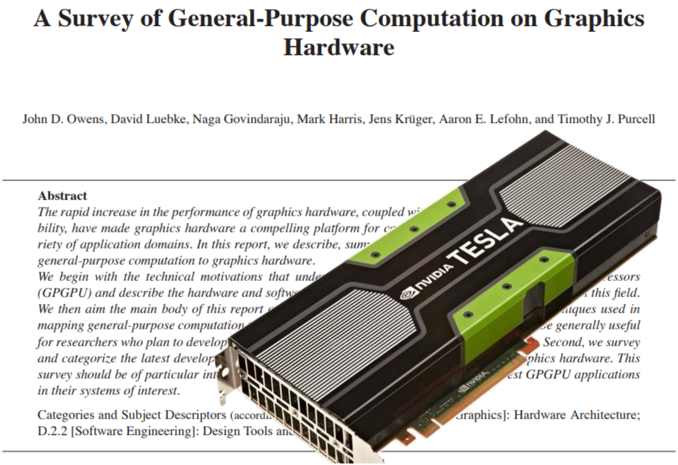
Smoothed Particle Hydrodynamics



Smoothed Particle Hydrodynamics



Smoothed Particle Hydrodynamics



A Survey of General-Purpose Computation on Graphics Hardware

John D. Owens, David Luebke, Naga Govindaraju, Mark Harris, Jens Krüger, Aaron E. Lefohn, and Timothy J. Purcell

Abstract

The rapid increase in the performance of graphics hardware, coupled w… bility, have made graphics hardware a compelling platform for c… riety of application domains. In this report, we describe, sum… general-purpose computation to graphics hardware.

We begin with the technical motivations that unde… …essors (GPGPU) and describe the hardware and softw… … this field. We then aim the main body of this report … …iques used in mapping general-purpose computation … …e generally useful for researchers who plan to develop … Second, we survey and categorize the latest develop… …phics hardware. This survey should be of particular int… …est GPGPU applications in their systems of interest.

Categories and Subject Descriptors (accord… …raphics]: Hardware Architecture; D.2.2 [Software Engineering]: Design Tools an…

Content

- SPH method - background, basics and principles of method
- application on fluid dynamics (derivation and properties of SPH approximation)

- some of the topics in SPH method (BC, adaptivity)
- alternative formulations and it's benefits (ALE-SPH, R-SPH,...)
- note about implementation (GPU usage, neighbour list)

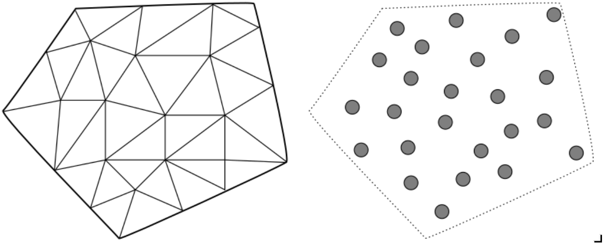
- examples, case studies (free surface flow with 3D complex geometry, wave breaking)
- development, what we are working on

Smoothed Particle Hydrodynamics - a meshfree particle method based on Lagrangian description

Smoothed Particle Hydrodynamics

Smoothed Particle Hydrodynamics - a **meshfree particle** method based on Lagrangian description

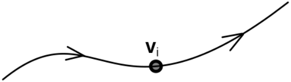
- meshfree particle method
- **no topological connection** between particles



Smoothed Particle Hydrodynamics

Smoothed Particle Hydrodynamics - a meshfree particle method **based on Lagrangian description**

- based on Lagrangian description - particles transport the values of physical quantities



- material body $\mathscr{B} = \{\mathcal{X}\}$
- finite number of particles $\mathscr{B}_h = \{\mathcal{X}\}_h$
- Material description

$$\boldsymbol{\gamma}_t : \mathscr{B} \mapsto \mathbb{E}^3$$
$$\mathcal{X} \mapsto \mathbf{x} = \boldsymbol{\gamma}_t(\mathcal{X}, t). \tag{1}$$

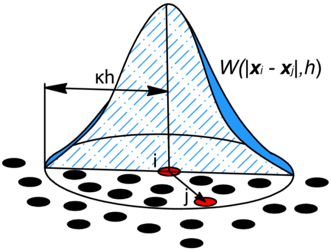
- Referential description: Lagrangian description for $\kappa_0 = \kappa_t(t = 0)$

$$\boldsymbol{\chi}_t : \mathbb{E}^3 \mapsto \mathbb{E}^3$$
$$\mathbf{X} \mapsto \mathbf{x} = \boldsymbol{\chi}(\mathbf{X}, t) \tag{2}$$

Smoothed Particle Hydrodynamics

Smoothed Particle Hydrodynamics - a meshfree particle method based on Lagrangian description

- We use weight function to compute contribution from neighbour particles



- compactly supported, $\mathcal{C}^1(\mathbb{R}^n)$ (at-least), radial and positive

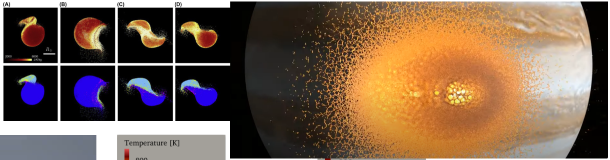
SPH - why all that?

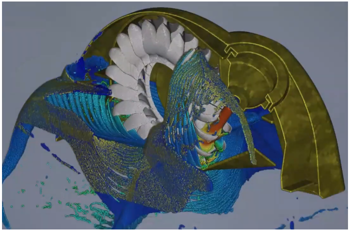
• Applications - whenever we can benefit of meshless lagrangian point of view

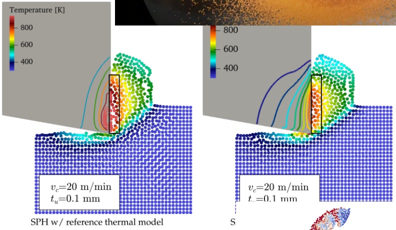
- CFD: free surface flow, massive fluid-air mixing, hight velocity impact, impact of waves, floating objects, fluid structure interaction
- CSM: large deformations, material fragmentation, cracks, penetrations
- complex multiphysics simulations, astrophysical phenomena, explosion phenomena

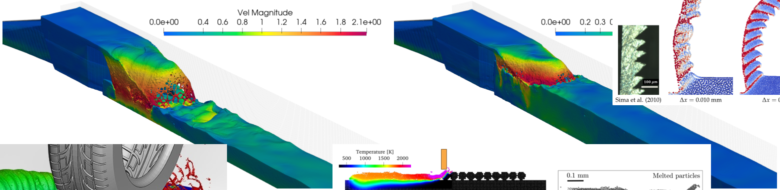
• new formulations - possibility of high order method

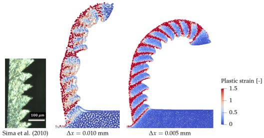
SPH - why all that?

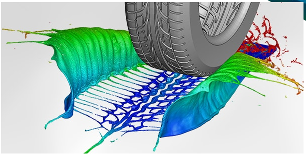


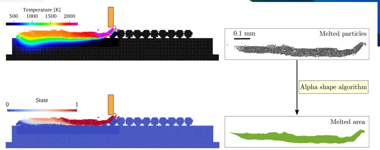












SPH method - construction

- let's start with Dirack delta distribution $\delta_{\mathbf{x}}[\varphi] = \varphi(\mathbf{x})$

$$f(\mathbf{x}) = \int_{\Omega} f(\mathbf{x}')\delta(\mathbf{x} - \mathbf{x}')d\mathbf{x}'$$

$$f(\mathbf{x}) \simeq \langle f(\mathbf{x})\rangle = \int_{\Omega} f(\mathbf{x}')W(\mathbf{x} - \mathbf{x}', h)d\mathbf{x}'$$

- if we substitute $\mathcal{D}f((x))$ instead of $f((x))$

$$\langle \mathcal{D}f(\mathbf{x})\rangle = \int_{\Omega} \mathcal{D}f(\mathbf{x}')W(\mathbf{x} - \mathbf{x}', h)d\mathbf{x}'$$

$$\langle \mathcal{D}f(\mathbf{x})\rangle = -\int_{\Omega} f(\mathbf{x}') \cdot \nabla W(\mathbf{x} - \mathbf{x}', h)d\mathbf{x}'.$$

SPH method - construction - Weight function W

$$f(\mathbf{x}) = \int_\Omega f(\mathbf{x}')\delta(\mathbf{x}-\mathbf{x}')d\mathbf{x}' \quad \to \quad \langle f(\mathbf{x})\rangle = \int_\Omega f(\mathbf{x}')W(\mathbf{x}-\mathbf{x}',h)d\mathbf{x}'$$

$$\langle \mathcal{D}f(\mathbf{x})\rangle = \int_\Omega \mathcal{D}f(\mathbf{x}')W(\mathbf{x}-\mathbf{x}',h)d\mathbf{x}' \quad \to \quad \langle \mathcal{D}f(\mathbf{x})\rangle = -\int_\Omega f(\mathbf{x}')\cdot\nabla W(\mathbf{x}-\mathbf{x}',h)d\mathbf{x}'$$

Properties of weight function:

- compactly supported $W(\mathbf{x}-\mathbf{x}',h) = 0$ for $|\mathbf{x}-\mathbf{x}| > \kappa h$
- normalized $\int_\Omega W(\mathbf{x}-\mathbf{x}',h)d\mathbf{x}' = 1$ and $W(+\infty) = 0$
- $\lim_{h\to 0^+} W(\mathbf{x}-\mathbf{x}',h) = \delta(\mathbf{x}-\mathbf{x}')$ in distributive (weak) sense

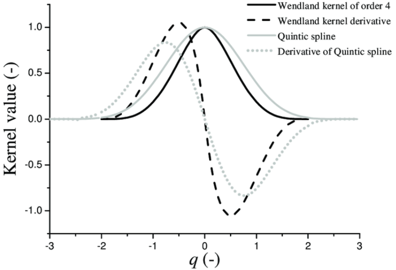
- radial (i. e $W(\mathbf{x}-\mathbf{x}',h) = W(r)$), decreasing with r
- positive (for meaningul representation of physical quantities, mathematically not necessary)
- smooth enough - $\mathcal{C}^1(\mathbb{R}^n)$ at least

SPH method - construction - Weight function W

Usually we use bell (bump-like) shaped functions:

- Polynomial type (Wendland kernel)

$$W(r,h) = \alpha_d \left(1 - \frac{r}{2}\right)^4 (2r+1); \quad 0 \leq r \leq 2$$



- Gaussian type[1]

$$W(r,h) = \alpha_d e^{-r^2}$$

- Spline kernles

$$W(r,h) = \alpha_d \begin{cases} (3-r)^5 - 6(2-r)^5 + 15(1-r)^5, & 0 \leq r \leq 1, \\ (3-r)^5 - 6(2-r)^5, & 1 \leq r \leq 2, \\ (3-r)^5, & 2 \leq r \leq 3, \\ 0, & r \geq 3 \end{cases}$$

[1] Without compact suppot, but decreasing fast, cut-off radius usually defined

SPH method - particle discretization

- the domain is discretized in finite number of particles that represents elementary volumes ΔV_i and transport the main physicial quantitis

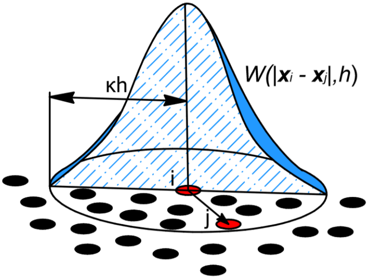
$$\langle f(\mathbf{x})\rangle = \int_{\Omega} f(\mathbf{x}')W(\mathbf{x}-\mathbf{x}',h)d\mathbf{x}' \to \langle f(\mathbf{x})\rangle_i = \sum_{i=1}^{N} f_i\, W(\mathbf{x}-\mathbf{x}_i,h)\Delta V_i$$

$$\langle \mathcal{D} f(\mathbf{x})\rangle = -\int_{\Omega} f(\mathbf{x}')\cdot \nabla W(\mathbf{x}-\mathbf{x}',h)d\mathbf{x}' \to \langle \mathcal{D} f(\mathbf{x})\rangle_i = -\sum_{i=1}^{N} f_i\, \nabla W(\mathbf{x}-\mathbf{x}_i,h)\Delta V_i$$

$$\Delta V_i = \frac{m_i}{\rho_i}$$

$$\rho(\mathbf{x}) = \sum_{j=1}^{N} m_j W(\mathbf{x}-\mathbf{x}_j,h).$$

$$\langle f\rangle_i \xrightarrow{\Delta x/h\to 0} \langle f(\mathbf{x}_i)\rangle \xrightarrow{h\to 0} f(\mathbf{x}).$$



SPH method - convergence of operators

$$\langle f \rangle_i \xrightarrow{\Delta x/h \to 0} \langle f(\mathbf{x}_i) \rangle \xrightarrow{h \to 0} f(\mathbf{x}).$$

- from discrete to continuous: increasing the number of particles in Ω i.e. $\Delta x/h \to 0$
- from smoothed to exact: $h \to 0$

SPH method - convergence of operators

$$\langle f \rangle_i \xrightarrow{\Delta x/h \to 0} \langle f(\mathbf{x}_i) \rangle \xrightarrow{h \to 0} f(\mathbf{x}).$$

- from discrete to continuous: increasing the number of particles in Ω i.e. $\Delta x/h \to 0$
- from smoothed to exact: $h \to 0$

$$\langle f \rangle_i \xrightarrow{\Delta x/h \to 0} \langle f(\mathbf{x}_i) \rangle \xrightarrow{h \to 0} f(\mathbf{x}).$$

- **for regular particle distribution!**

- convergence strongly depends on particles spatial distribution
- even for regular spatial distribution, convergence is between 1st and 2nd order
- theoretical 2nd order convergence for h

SPH method - how to discretize differential operator

$$\langle 1 \rangle_i = \sum_{i=1}^{N} W(\mathbf{x} - \mathbf{x}_i, h)\Delta V_i, \quad \langle \nabla 1 \rangle_i = -\sum_{i=1}^{N} \nabla W(\mathbf{x} - \mathbf{x}_i, h)\Delta V_i \neq 0$$

- we can use that - if we recall $h(\mathbf{x}) = 1$ and $\nabla h(\mathbf{x}) = 0 \;\; \forall \mathbf{x} \in \Omega$:

$$\langle \nabla f(\mathbf{x}) h + \nabla h f(\mathbf{x}_i) \rangle = \int_{\Omega_i} (\nabla f(\mathbf{x}') + \nabla h f(\mathbf{x}_i)) W(\mathbf{x} - \mathbf{x}', h) d\mathbf{x}',$$

$$\int_{\Omega} \left[\nabla f(\mathbf{x}') + \nabla h f(\mathbf{x}_i)\right] W(\mathbf{x} - \mathbf{x}', h) d\mathbf{x}'$$

$$= \int_S W(\mathbf{x} - \mathbf{x}', h) f(\mathbf{x}') \mathbf{n} dS' - \int_{\Omega} f(\mathbf{x}') \nabla W(\mathbf{x} - \mathbf{x}', h) d\mathbf{x}'$$

$$+ \int_S h(\mathbf{x}') f(\mathbf{x}_i) W(\mathbf{x} - \mathbf{x}', h) \mathbf{n} dS' - \int_{\Omega} h(\mathbf{x}') f(\mathbf{x}_i) \nabla W(\mathbf{x} - \mathbf{x}', h) d\mathbf{x}'$$

$$\langle \nabla f + \nabla h f_i \rangle_i = -\sum_{j=1}^{N} \nabla_j W_{ij} f_j \Delta V_j - \sum_{j=1}^{N} \nabla_j W_{ij} f_i \Delta V_j.$$

SPH method - how to discretize differential operator

$$\langle \nabla f + \nabla h f_i \rangle_i = -\sum_{j=1}^{N} \nabla_j W_{ij} f_j \Delta V_j - \sum_{j=1}^{N} \nabla_j W_{ij} f_i \Delta V_j.$$

- with that we obtain symmetric approximation of derivative

$$\langle \mathcal{D} f(\mathbf{x}) \rangle_i = \sum_{j=1}^{N} (f_j - f(\mathbf{x}))\, \nabla W(\mathbf{x} - \mathbf{x}_j, h) \Delta V_j$$

$$\langle \mathcal{D} f \rangle_i = \sum_{j=1}^{N} (f_j - f_i)\, \nabla W(\mathbf{x}_i - \mathbf{x}_j, h) \Delta V_j.$$

- **symmetry is needed for conservation**
- concept of $\nabla 1 = 0$ is widely employed in SPH for various approximation

SPH method - how to discretize differential operator

- we can have even more advanced approximation - from identity:

$$\frac{\nabla f}{\rho} = \frac{f}{\rho^q}\nabla\left(\frac{1}{\rho^{1-q}}\right) + \frac{1}{\rho^{2-q}}\nabla\left(\frac{f}{\rho^{q-1}}\right), \quad q \in \mathbb{N}$$

$$\left\langle \frac{\nabla f}{\rho} \right\rangle_{i,q} = \sum_{j=1}^{N} \left(\frac{f_i}{\rho_i^{2-q}\rho_j^{q}} + \frac{f_j}{\rho_i^{q}\rho_j^{2-q}} \right) \nabla_i W_{ij} m_j .$$

- usually used with $q = 1$,

$$\left\langle \frac{\nabla f}{\rho} \right\rangle_{i,1} = \sum_{j=1}^{N} \left(\frac{f_i + f_j}{\rho_i \rho_j} \right) \nabla_i W_{ij} m_j$$

- or with $q = 2$

$$\left\langle \frac{\nabla f}{\rho} \right\rangle_{i,2} = \sum_{j=1}^{N} \left(\frac{f_i}{\rho_i^2} + \frac{f_j}{\rho_j^2} \right) \nabla_i W_{ij} m_j .$$

SPH - 2nd order derivatives

$$\langle \nabla^2 f \rangle_i = \sum_j f_j \nabla_i^2 W_{ij} \Delta V_j$$

• sensitive to particle disorder moreover don't guarantee right direction of physical processes
• we define $F(|\mathbf{x}_{ij}|)$ that $\nabla_i W(\mathbf{x}_{ij}, h) = -\mathbf{x}_{ij} F(|\mathbf{x}_{ij}|)$, moreover we define integral $I(\mathbf{x})$ (for the clarity we start in 1D)

$$I(x) = \int (f(x') - f(x)) F\left(\left|x - x'\right|\right) dx'.$$

$$I(x) \simeq \int \left(f(x) + \frac{df}{dx}(x' - x) + \frac{1}{2}\frac{d^2 f}{dx}(x' - x)^2 - f(x) \right) F\left(\left|x - x'\right|\right) dx'$$
$$= \frac{1}{2}\frac{d^2 f}{dx^2} \int (x' - x)^2 F\left(\left|x - x'\right|\right) dx' = \frac{1}{2}\frac{d^2 f}{dx^2}$$

$$\left\langle \frac{d^2 f}{dx^2} \right\rangle_i = 2 \sum_{j=1}^{N} \frac{f_j - f_i}{x_j - x_i} \frac{dW(x_i - x_j)}{dx} \Delta V_j.$$

$$\boxed{\langle \nabla^2 f \rangle_i^{\text{Mo}} = 2 \sum_{j=1}^{N} (f_j - f_i) \frac{\mathbf{x}_{ij} \cdot \nabla_a W_{ij}}{\|\mathbf{x}_{ij}\|^2} \Delta V_j}$$

SPH - let's look on fluid dynamics

We are interested in incompressible viscid flows, mosly with free surfaces.

ISPH - incompressible SPH

$$\nabla \cdot \mathbf{v} = 0$$

$$\frac{\mathrm{D}\mathbf{v}}{\mathrm{D}t} = -\frac{1}{\rho}\nabla p + \nu \nabla^2 \mathbf{v} + \mathbf{f}$$

- projection method (solving a pressure Poisson equation)
- implicit scheme

WCSPH - weakly compressible SPH

$$\frac{\mathrm{D}\rho}{\mathrm{D}t} = -\rho \mathrm{div}(\mathbf{v})$$

$$\frac{\mathrm{D}\mathbf{v}}{\mathrm{D}t} = -\frac{1}{\rho}\nabla p + \nu \nabla^2 \mathbf{v} + \mathbf{f}$$

$$p = p(\rho)$$

$$P = b\left[\left(\frac{\rho}{\rho_0}\right)^{\gamma} - 1\right] + p_0$$

- explicit scheme

SPH - Weakly-compressible SPH (WCSPH)

$$\frac{\mathsf{D}\rho_i}{\mathsf{D}t} = -\left\langle \rho \mathsf{div}(\mathbf{v}) \right\rangle_i = \rho_i \sum_{j=1}^{N} \frac{m_j}{\rho_j} \mathbf{v}_{ij} \cdot \nabla W_{ij}$$

$$\frac{\mathsf{D}\mathbf{v}_i}{\mathsf{D}t} = -\left\langle \nabla p \right\rangle_{i,1} + \left\langle \nabla \cdot \tau \right\rangle_i + \mathbf{f}_i = -\sum_{j=1}^{N} \left(\frac{p_i + p_j}{\rho_i \rho_j} + \Pi_{ij} \right) \nabla_i W_{ij} m_j + \mathbf{f}_i$$

$$\frac{\mathsf{D}\mathbf{v}_i}{\mathsf{D}t} = -\left\langle \nabla p \right\rangle_{i,2} + \left\langle \nabla \cdot \tau \right\rangle_i + \mathbf{f}_i = -\sum_{j=1}^{N} \left(\frac{p_i}{\rho_i^2} + \frac{p_j}{\rho_j^2} + \Pi_{ij} \right) \nabla_i W_{ij} m_j + \mathbf{f}_i$$

SPH - SPH as Hamiltonian system

• Let's take completely different approach - Lagrangian particles forms Hamiltionian system which preserves conservation properties (mass, momentum, energy)

$$\mathcal{L}(\mathbf{x}, v) = \int_\Omega \rho\Big(\frac{1}{2}v^2 - e(\rho, p) - \phi(\mathbf{x})\Big)\mathsf{d}\mathbf{x}$$

$$\langle\mathcal{L}(\mathbf{x}, v)\rangle_i = \sum_{j=1}^{N}\left[\frac{1}{2}m_j v_j^2 - m_j U(t, \mathbf{x}_j) - m_j e(\rho_j, s_j)\right]$$

$$\frac{\mathsf{d}}{\mathsf{d}t}\Big(\frac{\partial\mathcal{L}}{\partial\mathbf{v}_j}\Big) - \frac{\partial\mathcal{L}}{\partial\mathbf{x}_j} = \mathcal{Q}_j^V + \mathcal{Q}_i$$

$$p_i = -\frac{\partial e_i}{\partial v_i}\bigg|_s = \rho_i^2\frac{\partial e_i}{\partial\rho_i}\bigg|_s \qquad v_i := \frac{1}{\rho_i} \qquad T_i = \frac{\partial e_i}{\partial s_i}\bigg|_V$$

$$m_i\frac{\mathsf{d}\mathbf{v}_i}{\mathsf{d}t} - m_i\mathbf{f}_i + \sum_j m_j\frac{p_j}{\rho_j^2}\frac{\partial\rho_j}{\partial\mathbf{x}_i} = \mathcal{Q}_j^V + \mathcal{Q}_i$$

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$$\langle \mathcal{L}(\mathbf{x}, v)\rangle_i = \sum_{j=1}^{N}\Big[1/2 m_j v_j^2 - m_j U(t, \mathbf{x}_j) - m_j e(\rho_j, s_j)\Big]$$

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- to get particle acceleration we need to model variantion of density with position

SPH - SPH as Hamiltonian system

$$m_i \frac{\mathrm{d}\mathbf{v}_i}{\mathrm{d}t} - m_i \mathbf{f}_i + \sum_j m_j \frac{p_j}{\rho_j^2} \frac{\partial \rho_j}{\partial \mathbf{x}_i} = \mathcal{Q}_j^V + \mathcal{Q}_i$$

- to get particle acceleration we need to model variantion of density with position

$$\rho(\mathbf{x}) = \sum_{j=1}^{N} m_j W(\mathbf{x} - \mathbf{x}_j, h)$$

$$m_i \frac{\mathrm{d}\mathbf{v}_i}{\mathrm{d}t} - m_i \mathbf{f}_i + \left[\sum_j m_j \frac{p_j}{\rho_j^2} m_j \nabla W_{ij} + m_i \frac{p_i}{\rho_i^2} m_i \sum_j \nabla W_{ij} \right] = \mathcal{Q}_j^V + \mathcal{Q}_i$$

$$\frac{\mathrm{D}\mathbf{v}_i}{\mathrm{D}t} = -\left\langle \nabla p \right\rangle_{i,2} + \left\langle \nabla \cdot \tau \right\rangle_i + \mathbf{f}_i = -\sum_{j=1}^{N} \left(\frac{p_i}{\rho_i^2} + \frac{p_j}{\rho_j^2} + \Pi_{ij} \right) \nabla_i W_{ij} m_j + \mathbf{f}_i$$

SPH - SPH as Hamiltonian system - density sumation

- actually, we have several ways how to determine the density

$$\rho_i = \frac{m_i}{V_i}, \quad \langle\rho\rangle_i = \sum_{j=1}^{N} m_j W(\mathbf{x}_i - \mathbf{x}_j, h)$$

$$V_i = 1/\sum_{i}^{N} W(\mathbf{x}_i - \mathbf{x}_j, h), \qquad \Delta\mathbf{x}_i = \sqrt[n]{V_i}$$

$$\langle\rho\rangle_i = m_i \sum_{j=1}^{N} W(\mathbf{x}_i - \mathbf{x}_j, h)$$

$$\frac{\mathrm{D}W_{ij}}{\mathrm{D}t} = (\mathbf{v}_i - \mathbf{v}_j)\nabla_i W_{ij}, \quad \frac{\mathrm{D}\rho}{\mathrm{D}t} = \sum_{j=1}^{N} (\mathbf{v}_i - \mathbf{v}_j) \cdot \nabla W_{ij} m_j, \; p_i = f(\rho_i)$$

$$\rho_i = \frac{m_i}{V_i} = m_i \sum_{j} W_{ij}, \quad V_i = 1/\sum_{i}^{N} W(\mathbf{x}_i - \mathbf{x}_j, h)$$

$$\frac{\mathrm{D}V_i}{\mathrm{D}t} = V_i^2 \sum_{j=1}^{N} (\mathbf{v}_i - \mathbf{v}_j) \cdot \nabla W_{ij}, \; p_i = f(\rho_i)$$

SPH - schemes with different density sumations

$$(\mathbf{r}_{i0}, \mathbf{v}_{i0}),\ \rho_{i0} = f^{-1}(p_{i0}),\ \rho_{i0} = \sum_j m_j W_{ij}$$

$$\rho_i = \sum_j m_j W_{ij},\ p_i = f(\rho_i)$$

$$\frac{\mathsf{D}\mathbf{v}}{\mathsf{D}t} = -\sum_{j=1}^{N}\left(\frac{p_i}{\rho_i^2} + \frac{p_j}{\rho_j^2} + \Pi_{ij}\right)\nabla_i W_{ij} m_j$$

$$(\mathbf{r}_{i0}, \mathbf{v}_{i0}),\ \rho_{i0} = f^{-1}(p_{i0}),\ m_i = \rho_{i0} V_{i0}$$

$$V_i = 1/\sum W_{ij},\ \rho_i = \frac{m_i}{V_i},\ p_i = f(\rho_i)$$

$$m_i \frac{\mathsf{D}\mathbf{v}_i}{\mathsf{D}t} = -\sum_j^N (p_i V_i^2 + p_j V_j^2)\nabla W_{ij}$$

$$(\mathbf{r}_{i0}, \mathbf{v}_{i0}, \rho_{i0}),\ \rho_{i0} = f^{-1}(p_{i0}),\ m_i = \rho_{i0} V_{i0}$$

$$V_i(t) = m_i/\rho_i(t)$$

$$\frac{\mathsf{D}\rho}{\mathsf{D}t} = \sum_{j=1}^{N}(\mathbf{v}_i - \mathbf{v}_j)\cdot\nabla W_{ij} m_j,\ p_i = f(\rho_i)$$

$$\frac{\mathsf{D}\mathbf{v}}{\mathsf{D}t} = -\sum_{j=1}^{N}\left(\frac{p_i}{\rho_i^2} + \frac{p_j}{\rho_j^2} + \Pi_{ij}\right)\nabla_i W_{ij} m_j$$

$$(\mathbf{r}_{i0}, \mathbf{v}_{i0}, V_{i0}),\ \rho_{i0} = f^{-1}(p_{i0}),\ m_i = \rho_{i0} V_{i0}$$

$$\frac{\mathsf{D}V_i}{\mathsf{D}t} = V_i^2 \sum_{j=1}^{N}(\mathbf{v}_i - \mathbf{v}_j)\cdot\nabla W_{ij},\ p_i = f(\rho_i)$$

$$m_i \frac{\mathsf{D}\mathbf{v}_i}{\mathsf{D}t} = -\sum_j^N (p_i V_i^2 + p_j V_j^2)\nabla W_{ij}$$

SPH - Weakly-compressible SPH (WCSPH)

- back to our scheme obtained from conservation laws

$$\frac{\mathsf{D}\rho_i}{\mathsf{D}t} = -\left\langle \rho \mathsf{div}(\mathbf{v}) \right\rangle_i = \rho_i \sum_{j=1}^{N} \frac{m_j}{\rho_j} \mathbf{v}_{ij} \cdot \nabla W_{ij}$$

$$\frac{\mathsf{D}\mathbf{v}_i}{\mathsf{D}t} = -\left\langle \nabla p \right\rangle_{i,1} + \left\langle \nabla \cdot \tau \right\rangle_i + \mathbf{f}_i = -\sum_{j=1}^{N} \left(\frac{p_i + p_j}{\rho_i \rho_j} + \Pi_{ij} \right) \nabla_i W_{ij} m_j + \mathbf{f}_i$$

$$\frac{\mathsf{D}\mathbf{v}_i}{\mathsf{D}t} = -\left\langle \nabla p \right\rangle_{i,2} + \left\langle \nabla \cdot \tau \right\rangle_i + \mathbf{f}_i = -\sum_{j=1}^{N} \left(\frac{p_i}{\rho_i^2} + \frac{p_j}{\rho_j^2} + \Pi_{ij} \right) \nabla_i W_{ij} m_j + \mathbf{f}_i$$

Viscous terms still need to be resolved.

SPH - WCSPH - viscous terms

$$\frac{\mathsf{D}\mathbf{v}_i}{\mathsf{D}t} = -\left\langle \nabla p \right\rangle_{i,2} + \left\langle \nabla \cdot \tau \right\rangle_i + \mathbf{f}_i = -\sum_{j=1}^{N} \left(\frac{p_i}{\rho_i^2} + \frac{p_j}{\rho_j^2} + \Pi_{ij} \right) \nabla_i W_{ij} m_j + \mathbf{f}_i$$

- artificial viscosity (combination of Neumann & Richtmyer and Landau, Landshoff, Lifshitz nunumerical viscosity)

$$\Pi_{ij} = \begin{cases} \dfrac{-\alpha c_{ij} \mu_{ij} + \beta \mu_{ij}^2}{\bar{\rho}_{ij}} & \mathbf{v}_{ij} \cdot \mathbf{x}_{ij} < 0 \\ 0 & \mathbf{v}_{ij} \cdot \mathbf{x}_{ij} \geq 0 \end{cases} , \quad \mu_{ij} = \frac{h \mathbf{v}_{ij} \cdot \mathbf{x}_{ij}}{|\mathbf{x}_{ij}|^2 + \epsilon h^2}.$$

α, β are adjustable constants (usually $\beta = 0$ for free surface flows), ϵ prevents singularities

- direct discretization of viscous term

$$\Pi_{ij}^{\eta} = -\eta K \frac{\mu_{ij}}{h \rho_i \rho_j}, \quad \mu_{ij} = \frac{h \mathbf{v}_{ij} \cdot \mathbf{x}_{ij}}{|\mathbf{x}_{ij}|^2 + \epsilon h^2}.$$

η is kinematic viscosity and K is dimension coeficient ($K = 6, 8$ and 15 in 1D,2D and 3D)

SPH - WCSPH - artificial viscosity

One more note about artificial viscosity.

$$\Pi_{ij} = \begin{cases} \dfrac{-\alpha c_{ij}\mu_{ij} + \beta\mu_{ij}^2}{\bar{\rho}_{ij}} & \mathbf{v}_{ij}\cdot\mathbf{x}_{ij} < 0 \\ 0 & \mathbf{v}_{ij}\cdot\mathbf{x}_{ij} \geq 0 \end{cases} \quad , \quad \mu_{ij} = \frac{h\mathbf{v}_{ij}\cdot\mathbf{x}_{ij}}{|\mathbf{x}_{ij}|^2 + \epsilon h^2}.$$

- artificial viscosity parameter α (with $\beta = 0$) is often related to the physical viscosity

$$\nu = \frac{\alpha h c_0}{d(d+2)}$$

- later in dam-bream example

$$\text{Re}_{\text{real}} = \frac{uL}{\nu} \simeq 10^6$$

meanwhile

$$\text{Re}_{\text{numerical}} = \frac{8uL}{\alpha h c_0} = 16000 \quad \text{for } 2\cdot 10^5 \text{particles}$$

- **we would need to 800** $\cdot 10^6$ **of particles to get the real Re!**

SPH - WCSPH scheme

$$\frac{\mathrm{D}\mathbf{x}}{\mathrm{D}t} = \mathbf{v}_i$$

$$\frac{\mathrm{D}\rho_i}{\mathrm{D}t} = -\left\langle \rho \mathrm{div}(\mathbf{v}) \right\rangle_i = \rho_i \sum_{j=1}^{N} \frac{m_j}{\rho_j} \mathbf{v}_{ij} \cdot \nabla W_{ij}$$

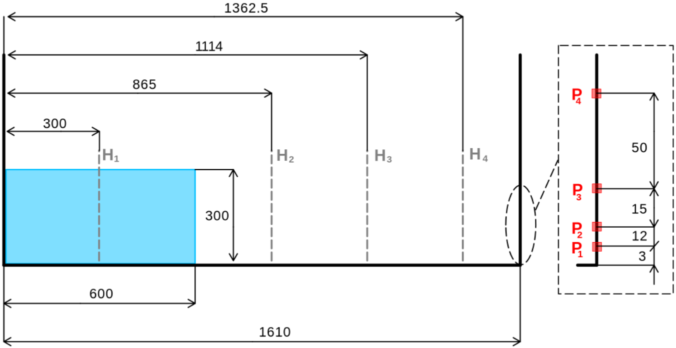
$$\frac{\mathrm{D}\mathbf{v}_i}{\mathrm{D}t} = -\left\langle \nabla p \right\rangle_{i,2} + \left\langle \nabla \cdot \tau \right\rangle_i + \mathbf{f}_i = -\sum_{j=1}^{N} \left(\frac{p_i}{\rho_i^2} + \frac{p_j}{\rho_j^2} + \Pi_{ij} \right) \nabla_i W_{ij} m_j + \mathbf{f}_i$$

$$\Pi_{ij} = \begin{cases} \dfrac{-\alpha c_{ij} \mu_{ij} + \beta \mu_{ij}^2}{\bar{\rho}_{ij}} & \mathbf{v}_{ij} \cdot \mathbf{x}_{ij} < 0 \\ 0 & \mathbf{v}_{ij} \cdot \mathbf{x}_{ij} \geq 0 \end{cases} \quad , \quad \mu_{ij} = \frac{h \mathbf{v}_{ij} \cdot \mathbf{x}_{ij}}{|\mathbf{x}_{ij}|^2 + \epsilon h^2}.$$

$$p = b \left[\left(\frac{\rho}{\rho_0} \right)^{\gamma} - 1 \right] + p_0$$

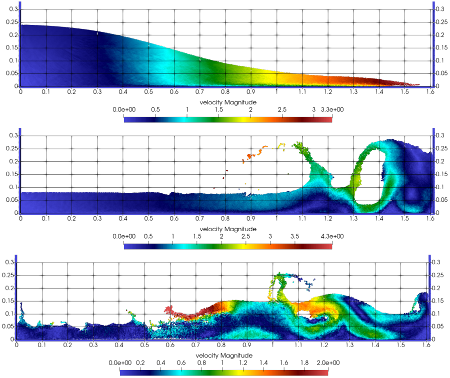
SPH - WCSPH scheme

- simple dam-break testcase to check our scheme

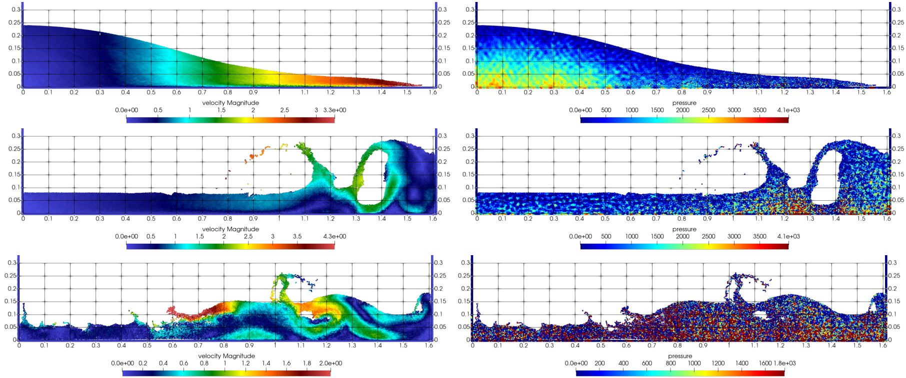


- block of water separated by gate, in $t = 0$ gate is removed, water starts to move due to gravitational force and collides with opposite wall

SPH - WCSPH scheme



SPH - WCSPH scheme



Where is the problem?

SPH - diffusive terms, δ-SPH

- we have central explicit scheme

$$\frac{\mathsf{D}\rho_i}{\mathsf{D}t} = -\left\langle \rho \mathsf{div}(\mathbf{v}) \right\rangle_i = \rho_i \sum_{j=1}^{N} \frac{m_j}{\rho_j} \mathbf{v}_{ij} \cdot \nabla W_{ij}$$

- we use numerical speed of sound $\Delta t \sim h/c_0$ (we are not able to use physical speed of sound due to too small time step)

$$P = b\left[\left(\frac{\rho}{\rho_0}\right)^{\gamma} - 1\right] + p_0$$

- weakly-compressible model (assumption) $\left|\Delta\rho/\rho\right| \leq 0.01$

SPH - diffusive terms, δ-SPH

- Additional diffusion term in continuity equations

$$\frac{\mathrm{D}\rho_i}{\mathrm{D}t} = \sum_{j=1}^{N} m_j \mathbf{v}_{ij} \cdot \nabla_i W_{ij} + \delta_\psi h c_0 \mathscr{D}$$

$$\mathscr{D} = 2 \sum_{j=1}^{N} \psi_{ij} \cdot \frac{\mathbf{x}_{ij} \cdot \nabla_j W_{ij}}{\left\|\mathbf{x}_{ij}\right\|^2} \frac{m_j}{\rho_j}$$

- approximation of even density derivative
- mass conservation consistency $\lim_{h\to 0}\left(\lim_{\Delta x/h \to 0}\left\langle \frac{d\rho_{\mathscr{D}}}{dt} \right\rangle_i\right) = 0$
- global mass conservation $\sum_{i=1}^{N}\left\langle \frac{d\rho_{\mathscr{D}}}{dt} \right\rangle_i = 0$

SPH - diffusive terms, δ-SPH

$$\frac{\mathsf{D}\rho_i}{\mathsf{D}t} = -\left\langle \rho \mathsf{div}(\mathbf{v}) \right\rangle_i = \sum_{j=1}^{N} m_j \mathbf{v}_{ij} \cdot \nabla_i W_{ij} + \delta_\psi h c_0 \mathscr{D}$$

$$\mathscr{D} = 2 \sum_{j=1}^{N} \psi_{ij} \cdot \frac{\mathbf{x}_{ij} \cdot \nabla_j W_{ij}}{\left\| \mathbf{x}_{ij} \right\|^2} \frac{m_j}{\rho_j}, \quad \psi_{ij}^{\mathsf{Mo}} = \rho_j - \rho_i$$

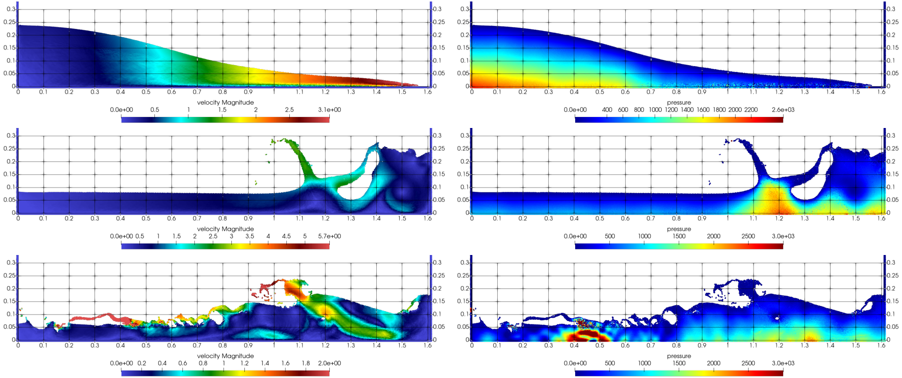
$$\frac{\mathsf{D}\mathbf{v}_i}{\mathsf{D}t} = -\left\langle \nabla p \right\rangle_{i,2} + \left\langle \nabla \cdot \tau \right\rangle_i + \mathbf{f}_i = -\sum_{j=1}^{N} \left(\frac{p_i}{\rho_i^2} + \frac{p_j}{\rho_j^2} + \Pi_{ij} \right) \nabla_i W_{ij} m_j + \mathbf{f}_i$$

$$\frac{\mathsf{D}\mathbf{x}}{\mathsf{D}t} = \mathbf{v}_i$$

$$P = b \left[\left(\frac{\rho}{\rho_0} \right)^\gamma - 1 \right] + p_0$$

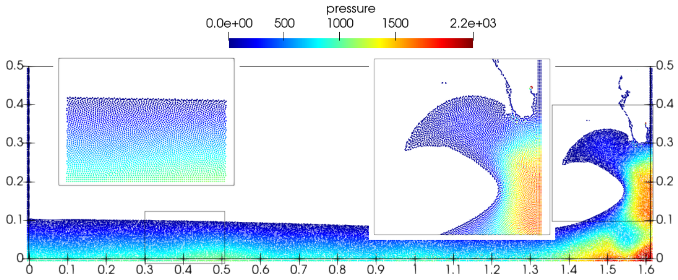
ψ_{ij}^{Mo} is mostly used diffusion term, but doesn't satisfy hydorstatic solution. For hydrostatic cases it causes spurious water level raise, with the presence of dynamics works fine, although we have variants suitable for mostly hydrostatic problems also.

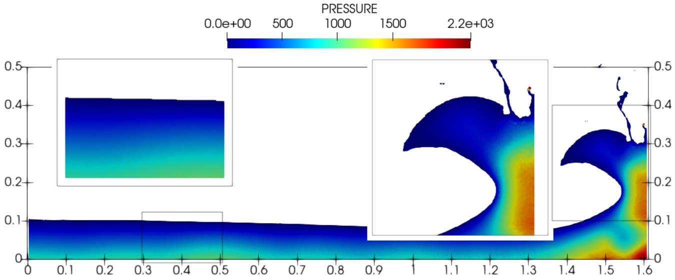
SPH - diffusive terms, δ-SPH



It's better but far from perfect.

SPH - Smoothing: result interpolation





SPH - particles shifting techniques (PST)

• in terms to obtain better approximation results, we are trying to arrange particles to obtain more regular spatial distribution

• based on diffusion law and particle concentration, small adjustments in particles positions are done

$$\mathbf{J} = -D_F \nabla C.$$

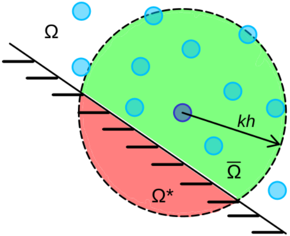
$$\langle \nabla C \rangle_i = \sum_{j=1}^{N} \frac{m_j}{\rho_j} \nabla_i W_{ij}$$

$$\delta \mathbf{x}_s = -D \nabla C.$$

• some cases are not possible to solve without shifting techniques! *(tensile instabilities)*

SPH - Boundary conditions

- kernel truncation for particles near boundary

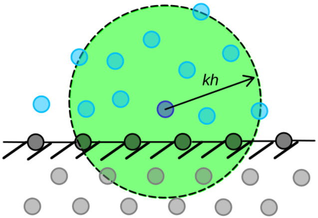


- we need to ensure particles don't go through boundary
- we need to solve deformed support (i.e. overlaping of effective area and boundary)[2]
- enforce predescribed boundary condition

[2] strictly taken: compact support cannot overlap boundary and deform itself towards boundary, due to this reason we talk about effective area which can overlap boundary

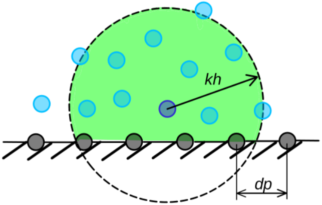
SPH - Boundary conditions

Additional particles



- predefined layers of boundary particles *(virtual particles)*
- mirroring of fluid particles near to boundary *(ghost particles)*

Kernel corrections

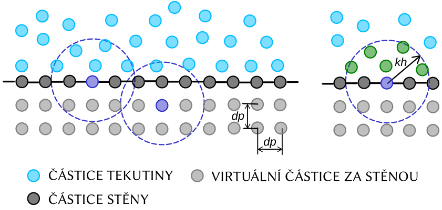


- Kernel renormalisation in order to compensate missing neighbours

SPH - BC: dynamic boundary conditions (DBC)

$$\langle f \rangle_i = \sum_{j \in \mathcal{F}} f_j W(\mathbf{x}_i - \mathbf{x}_j, h) \frac{m_j}{\rho_j} + \sum_{j \in \mathcal{G}} f_j W(\mathbf{x}_i - \mathbf{x}_j, h) \frac{m_j}{\rho_j}$$

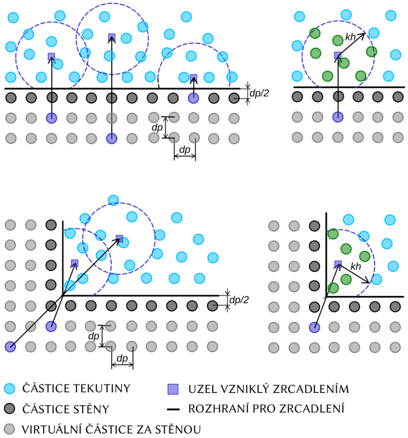
$$\langle \mathcal{D} f \rangle_i = - \sum_{j \in \mathcal{F}} f_j \nabla W(\mathbf{x}_i - \mathbf{x}_j, h) \frac{m_j}{\rho_j} - \sum_{j \in \mathcal{G}} f_j \nabla W(\mathbf{x}_i - \mathbf{x}_j, h) \frac{m_j}{\rho_j}$$



SPH - BC: modified dynamic boundary conditions (mDBC)

$$\langle f \rangle_i = \sum_{j \in \mathcal{F}} f_j W(\mathbf{x}_i - \mathbf{x}_j, h) \frac{m_j}{\rho_j} + \sum_{j \in \mathcal{G}} f_j W(\mathbf{x}_i - \mathbf{x}_j, h) \frac{m_j}{\rho_j}$$

$$\langle \mathcal{D} f \rangle_i = - \sum_{j \in \mathcal{F}} f_j \nabla W(\mathbf{x}_i - \mathbf{x}_j, h) \frac{m_j}{\rho_j} - \sum_{j \in \mathcal{G}} f_j \nabla W(\mathbf{x}_i - \mathbf{x}_j, h) \frac{m_j}{\rho_j}$$



SPH - Boundary conditions - kernel corrections

- kernel correction in order to compensate lack of neighbors

$$\nabla W(\mathbf{x}_i - \mathbf{x}_j, h) \quad \rightarrow \quad \nabla W^{\mathcal{L}}(\mathbf{x}_i - \mathbf{x}_j, h) = \mathcal{L}(\mathbf{x}_i) \nabla W(\mathbf{x}_i - \mathbf{x}_j, h)$$

- Shepard renormalisation factor

$$\mathcal{L}(\mathbf{x}_i) = \frac{1}{\gamma(\mathbf{x}_i)} \mathbf{I}$$

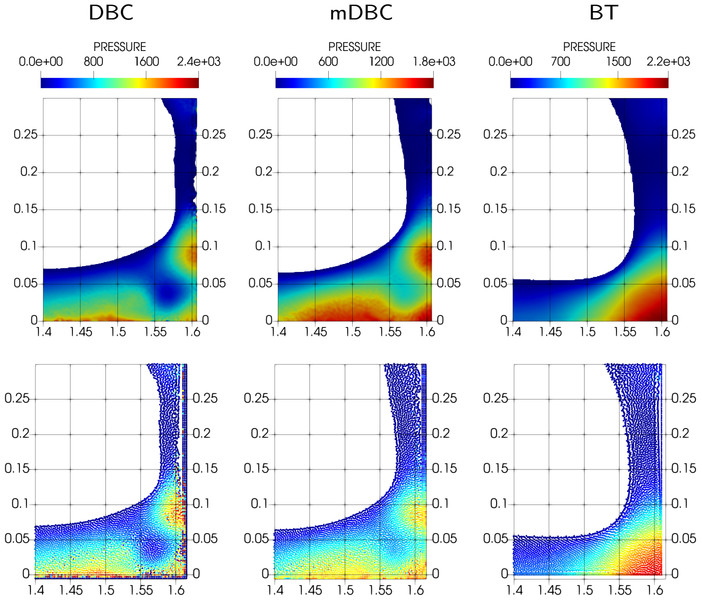
$$\gamma(\mathbf{x}) = \int_{\Omega} W(\mathbf{x} - \mathbf{x}', h) d\mathbf{x}', \quad \langle \gamma \rangle_i = \sum_{j \in \mathcal{F}} W(\mathbf{x}_i - \mathbf{x}_j, h) \frac{m_j}{\rho_j}$$

- MLS (*moving least-squares*)

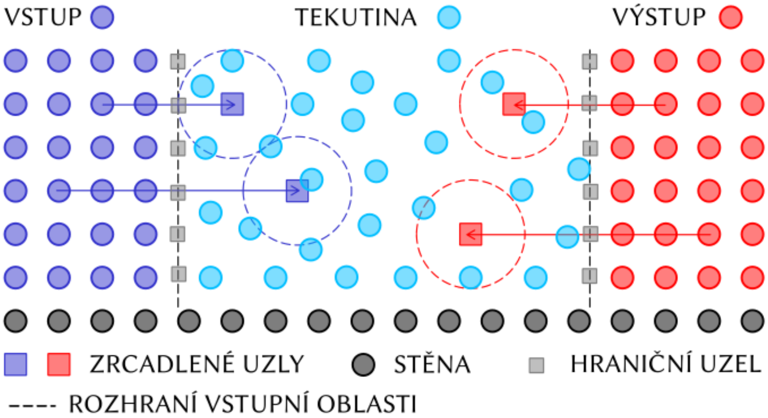
$$\mathcal{L}(\mathbf{x}_i) = \left[\sum_{j \in \mathcal{F}} (\mathbf{x}_i - \mathbf{x}_j) \otimes \nabla W(\mathbf{x}_i - \mathbf{x}_j, h) \frac{m_j}{\rho_j} \right]^{-1}$$

- repulsive mechanism needed (usually with one layer of boundary particles)

Differences between boundaries

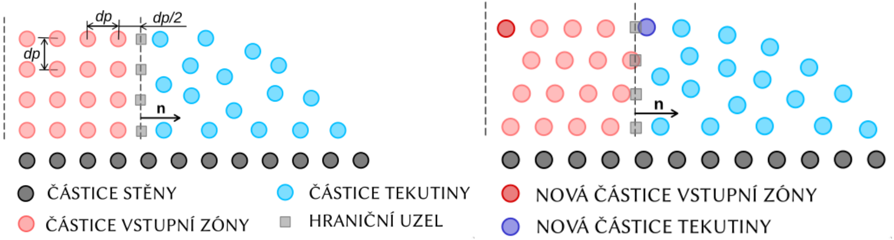


SPH - open boundaries - inlet and outlet



SPH - principle of inlet condition

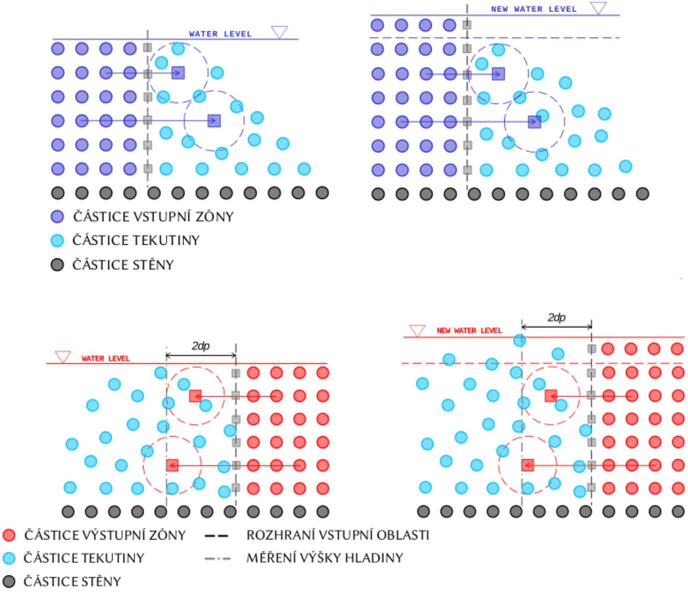
- we add new particles (discretisation points) into domain



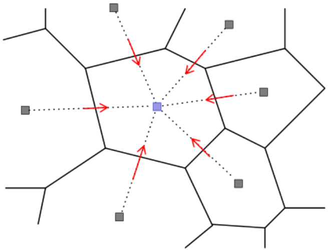
- buffer zone of particles with predescribed velocity
- position of buffer particles are updated and new particles are created in buffer zone

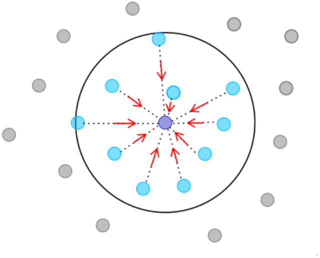
$$\mathbf{x}_{nb} = \mathbf{x_b} + [(\mathbf{x}_f - \mathbf{x}_b) \cdot \mathbf{n}_b - L_b]\mathbf{n}_b.$$

SPH - open boundaries - water level adjustment



SPH: ALE-SPH (R-ALE-SPH) variants





- generalized transport velocity formulation (with some inspiration in FVM method)

$$L_{v_0}(\mathbf{\Phi}) + \sum_{\alpha=1,d} \frac{\partial}{\partial x_\alpha}(\mathbf{F}_E^\alpha - v_0^\alpha \mathbf{\Phi})) = \mathbf{S}$$

$$L_{v_0}(\mathbf{\Phi}) = \frac{\partial \mathbf{\Phi}}{\partial t} + \sum_{\alpha=1,d} \frac{\partial (v_0^\alpha \mathbf{\Phi})}{\partial x_\alpha}$$

$$P = b\left[\left(\frac{\rho}{\rho_0}\right)^\gamma - 1\right]$$

SPH: ALE-SPH (R-ALE-SPH) variants

$$\mathbf{F} = \mathbb{F} - \mathbf{v_0} \otimes \mathbf{\Phi}$$

$$\frac{d}{dt}(\omega_i \mathbf{\Phi}_i) + \omega_i \langle \nabla \cdot \mathbf{F}_i \rangle_i = \mathbf{S}_i$$

$$\frac{d}{dt}(\omega_i \mathbf{\Phi}_i) = -\omega_i \sum_{j \in \mathcal{F}} (\mathbf{F}_j + \mathbf{F}_i) \cdot \nabla_i W_{ij} \omega_j - \omega_i \sum_{k \in \mathcal{B}} W_{ik} (\mathbf{F}_k + \mathbf{F}_i) \cdot \mathbf{n}_k s_k + \mathbf{S}_i$$

$$(\mathbf{F}_j + \mathbf{F}_i) \simeq 2\mathbf{G}_E(\mathbf{\Phi}_i, \mathbf{\Phi}_j)$$

- moving Riemann problem between particles i and j

$$\mathbf{G}_E(\mathbf{\Phi}_i, \mathbf{\Phi}_j) = \mathbf{F}_E(\mathbf{\Phi}_{ij}(\lambda_{0,ij})) - \mathbf{v}_0(\mathbf{x}_{ij}, t) \otimes \mathbf{\Phi}_{ij}(\lambda_{0,ij})$$

$$\mathbf{\Phi}_{ij}(\lambda_{0,ij}) = \mathbf{\Phi}_E(\lambda_{0,ij}, \mathbf{\Phi}_i, \mathbf{\Phi}_j)$$

$$\lambda_{0,ij} = \mathbf{v}_0(\mathbf{x}_{ij}, t) \cdot \mathbf{n}_{ij}$$

$$\mathbf{x}_{ij} = \frac{\mathbf{x}_i + \mathbf{x}_j}{2}$$

SPH: ALE-SPH (R-ALE-SPH) variants

- resulting scheme:

$$\frac{\mathrm{d}\mathbf{x}_i}{\mathrm{d}t} = \mathbf{v}_{0,i}$$

$$\frac{\mathrm{d}\omega_i}{\mathrm{d}t} = \omega_i \sum_{j \in D_i} \nabla_i W_{ik} \cdot (\mathbf{v}_{0,j} - \mathbf{v}_{0,i})\omega_j$$

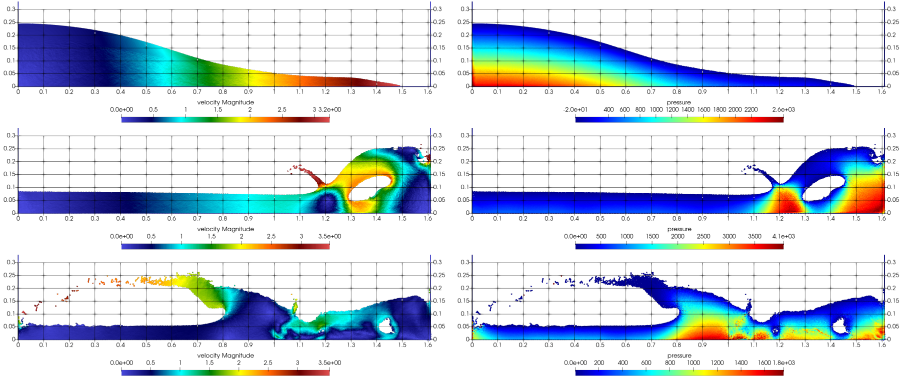
$$\frac{\mathrm{d}\omega_i \boldsymbol{\Phi}_\mathbf{i}}{\mathrm{d}t} = -\omega_i \sum_{j \in D_i} \mathbf{G}_e(\boldsymbol{\Phi}_\mathbf{i}, \boldsymbol{\Phi}_j) \cdot \nabla_i W_{ij} \omega_j + \mathbf{S}$$

+ appropriate approximate Riemann solver for $\mathbf{G}_E$

$$\boldsymbol{\Phi} = \begin{pmatrix} \rho \\ \rho u \\ \rho v \\ \rho w \end{pmatrix} \quad \mathbf{G}_E^x = \begin{pmatrix} \rho_E(u_E - u_0) \\ \rho_E u_E(u_E - u_0) + p_E \\ \rho_E u_E(v_E - v_0) \\ \rho_E u_E(w_E - w_0) \end{pmatrix}, \quad \mathbf{G}_E^y = \begin{pmatrix} \rho_E(v_E - v_0) \\ \rho_E v_E(u_E - u_0) \\ \rho_E v_E(v_E - v_0) + p_E \\ \rho_E v_E(w_E - w_0) \end{pmatrix},$$

$$\mathbf{G}_E^z = \begin{pmatrix} \rho_E(w_E - w_0) \\ \rho_E w_E(u_E - u_0) \\ \rho_E w_E(v_E - v_0) \\ \rho_E w_E(w_E - w_0) + p_E \end{pmatrix}, \quad \mathbf{S} = \begin{pmatrix} 0 \\ \rho\omega_i g_x \\ \rho\omega_i g_y \\ \omega_i g_z \end{pmatrix}$$

SPH - R-ALE-SPH-variant



SPH - R-SPH variant

- we can use Riemann solvers even with classical formulation

$$\frac{\mathrm{D}\mathbf{x}}{\mathrm{D}t} = \mathbf{v}_i$$

$$\frac{\mathrm{D}\rho_i}{\mathrm{D}t} = \sum_{j=1}^{N} m_j \mathbf{v}_{ij} \cdot \nabla_i W_{ij} \quad \rightarrow \quad \frac{\mathrm{D}\rho_i}{\mathrm{D}t} = 2\rho_i \sum_{j=1}^{N} (\mathbf{v}_i - \mathbf{v}^*) \cdot \nabla_i W_{ij} \frac{m_j}{\rho_j}$$

$$\frac{\mathrm{D}\mathbf{v}_i}{\mathrm{D}t} = -\sum_{j=1}^{N} \left(\frac{p_i + p_j}{\rho_i \rho_j} + \Pi_{ij} \right) \nabla_i W_{ij} m_j \quad \rightarrow \quad \frac{\mathrm{D}\mathbf{v}_i}{\mathrm{D}t} = -2 \sum_{j=1}^{N} \left(\frac{p^*}{\rho_i \rho_j} \right) \nabla_i W_{ij} m_j + \mathbf{f}_i$$

$$p = b \left[\left(\frac{\rho}{\rho_0} \right)^{\gamma} - 1 \right] + p_0$$

+ appropriate approximate Riemann solver for $\mathbf{v}^*$ and p^*

SPH - R-SPH variant

- it's almost for free!

```
ps = pRiemannLinearized(rhoL, rhoR, vL, vR,...);

ac_sum.x -= 2.*dW.x*(ps/(arho*nrho))*m;
ac_sum.y -= 2.*dW.y*(ps/(arho*nrho))*m;
```

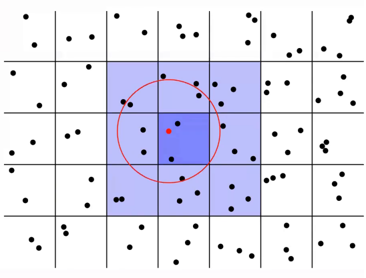
instead of

```
p_temp = (ap + np)/(arho*nrho);
visco = Artificial_Viscosity(h, drs, drdv,...);

ac_sum.x -= dW.x*(p_temp + visco)*m;
ac_sum.y -= dW.y*(p_temp + visco)*m;
```

SPH - implementation and GPUs

- GPGPU - general-purpose computing on graphics processing units
- SPH method is suitable for GPU implementation *(suitable, not perfect)*



- **effective way to find co-interacting particles is the crucial thing in whole SPH!**
- **linked-list algorithm**, octree
- beside that, GPU programming has some of its own specific and let's not forget we have elements without topological connections, mixing with each other

Quick note - SPHERIC

SPHERIC is the international organisation representing the community of researchers and industrial users of Smoothed Particle Hydrodynamics (SPH).

- GC1: Convergence, consistency and stability
- GC2: Boundary conditions
- GC3: Adaptivity
- GC4: Coupling to other models
- GC5: Applicability to industry

GC#2: Boundary conditions

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In order to close the fluid dynamics equations (Euler and Navier-Stokes, both compressible or incompressible) initial (ICs) and boundary conditions (BCs) are necessary. They can be classified as:

1. solid boundaries (free slip, no slip, pressure normal derivative)
2. free surface,
3. inlet/oulet,
4. initial conditions,
5. coupling with other models.

To include these boundaries in an SPH simulation, researchers use various techniques depending on the type of condition considered. Let's try to summarize the most relevant references. This list is of course open to discussion and is also a living one, since new relevant references can be incorporated. It is relevant to mention that in the most important SPH review papers, [Monaghan, 2012, 2005a, Gomez- Gesteira et al., 2010], there are already specific review sections on BCs. There are some key issues that remain to be addressed:

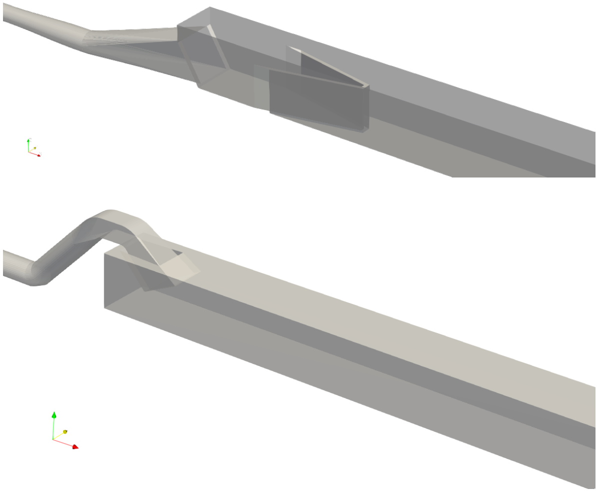
1. How to include BCs without loosing intrinsic SPH conservation properties?
2. How to include BCs consistently?
3. How to include solid wall BCs for real geometries with complex shapes (2D, 3D)?
4. How to provide an initial distribution of particles which avoids the onset of shocks once the time-integration starts?
5. How to treat back flows when implementing inlet/oulet boundary conditions?
6. etc.

The references follow:

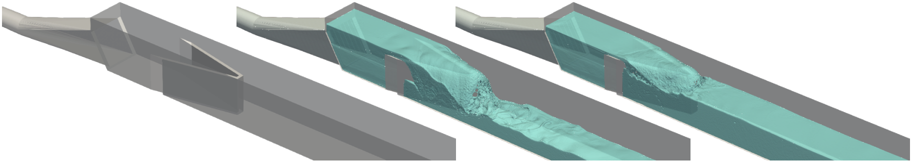
A. Amicarelli, G. Agate, and R. Guandalini. A 3D Fully Lagrangian Smoothed Particle Hydrodynamics model with both volume and surface discrete elements. International Journal for Numerical Methods in Engineering, 95:419–450, 2013.
URL http://doi.org/10.1002/nme.4514

S. Attaway, M. Heinstein, and J. Swegle. Coupling of smooth particle hydrodynamics with the finite element method. Nuclear Engineering and Design, 150 (2–3):199–205, 1994. URL https://doi.org/10.1016/0029-5493(94)90136-8

SPH - case study 1

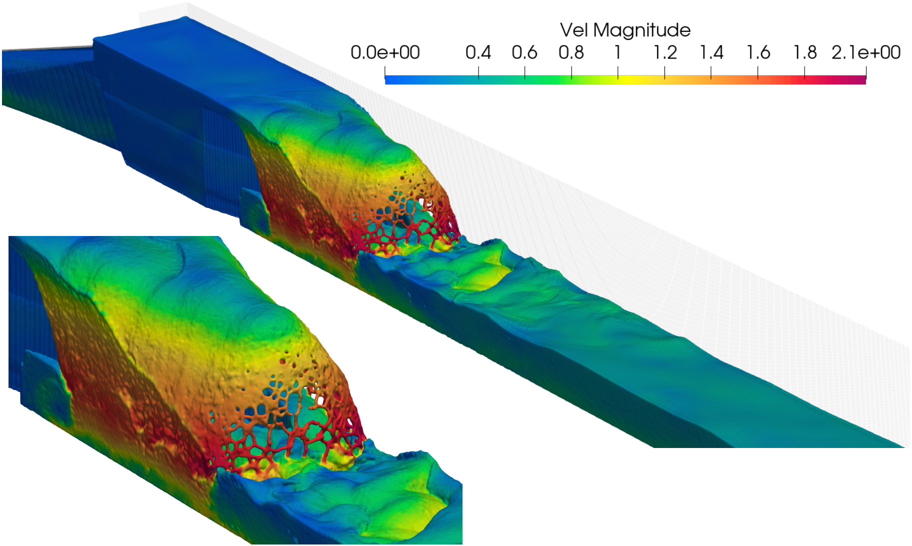


SPH - case study 1

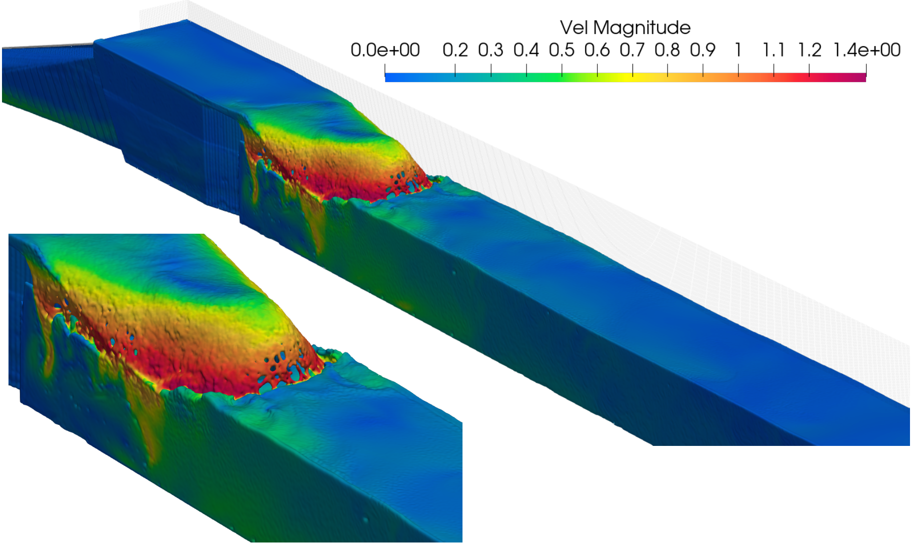


- high water level mode with $\dot{V}_H = 10.8$ l/s and low water level mode with $\dot{V}_H = 11.4$ l/s
- 8 $\cdot 10^6$ particles (initial spacing $d_p = 2.5$ cm)
- Wendland kernel, artificial viscosity with $\alpha = 0.01$, Molteni diffusive term wih $\delta = 0.1$, referential density $\rho_0 = 1000$
- open-source code DualSPHysics

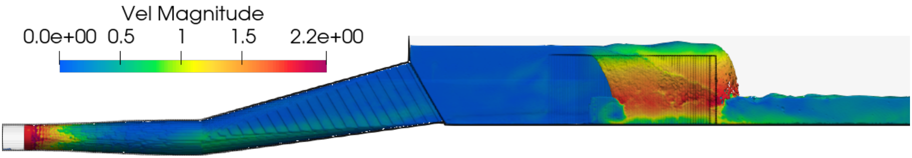
SPH - case study 1



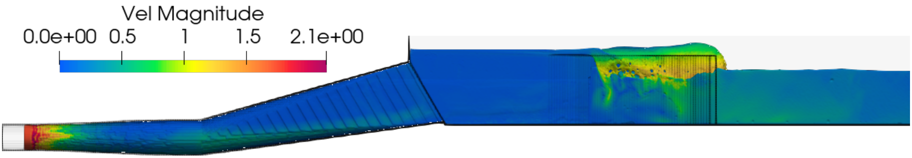
SPH - case study 1 - V shape geometry



SPH - case study 1 - V shape geometry

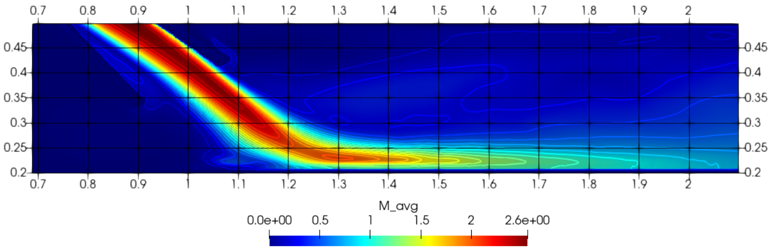


LWL

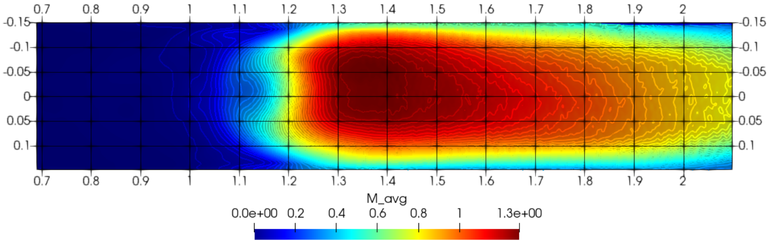


HWL

SPH - case study 1 - siphon geometry

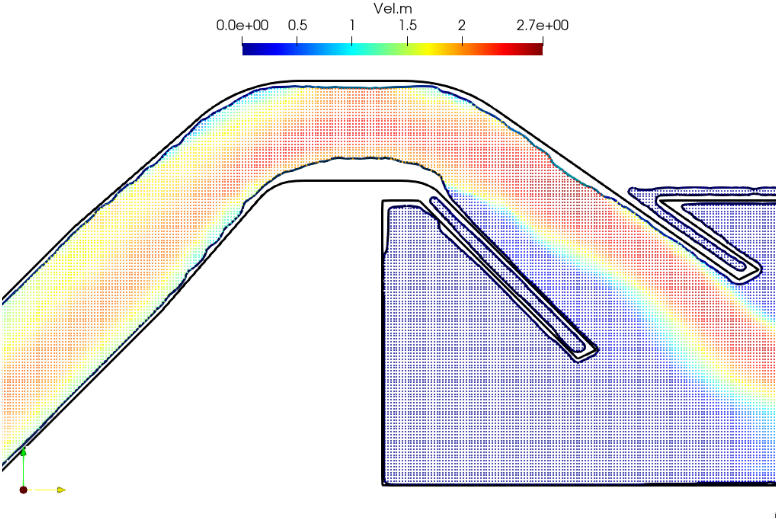


cut in axis of channel

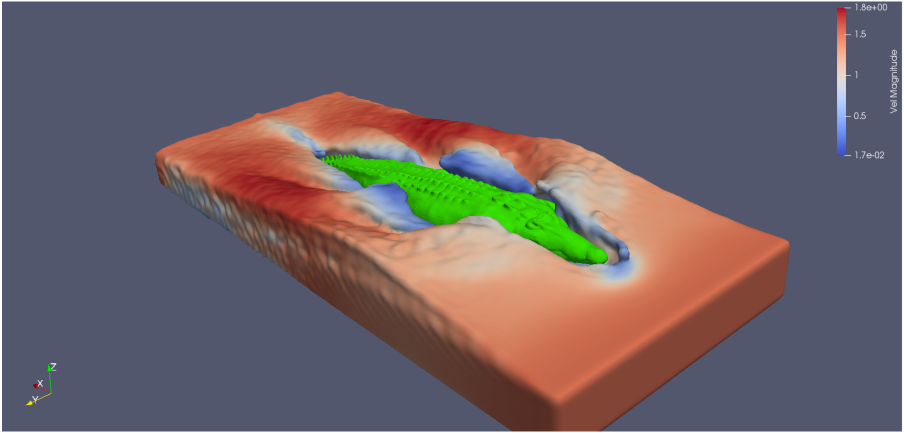


cut 10 cm above bottom

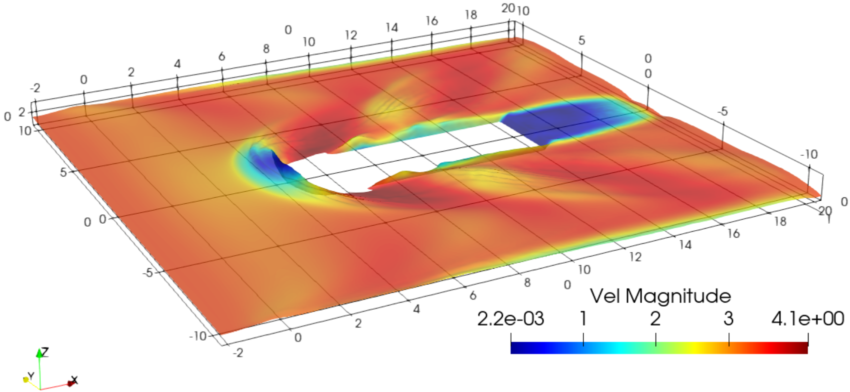
SPH - case study 1 - siphon geometry



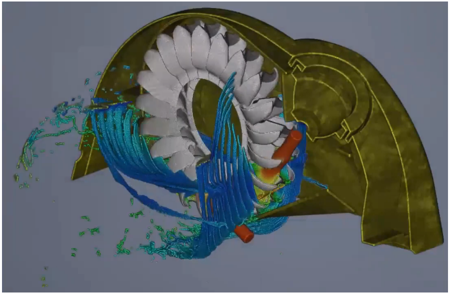
SPH - case study 1.5

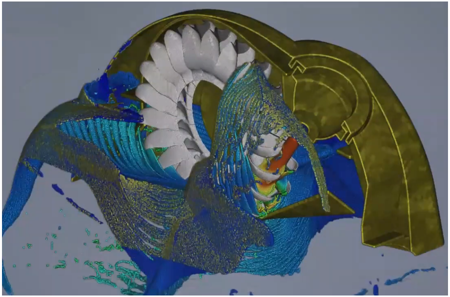


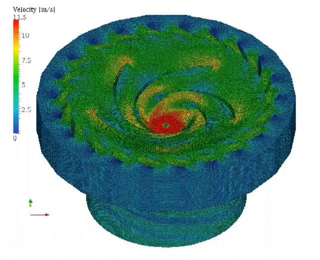
SPH - case study 2

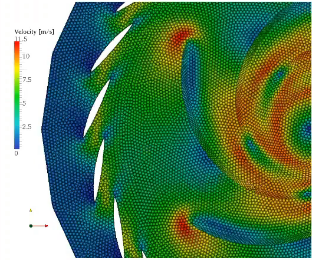


SPH - a few more exmaples



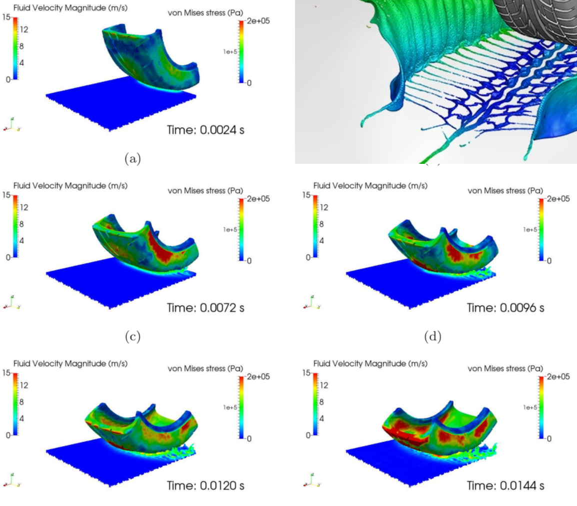




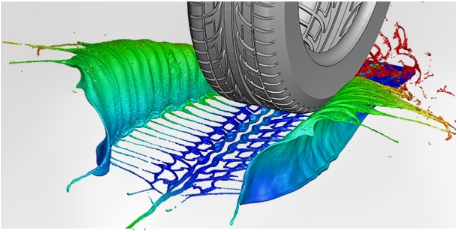


J. Ch. Marongiu (2016), ANDRITZ Hydro

SPH - a few more exmaples

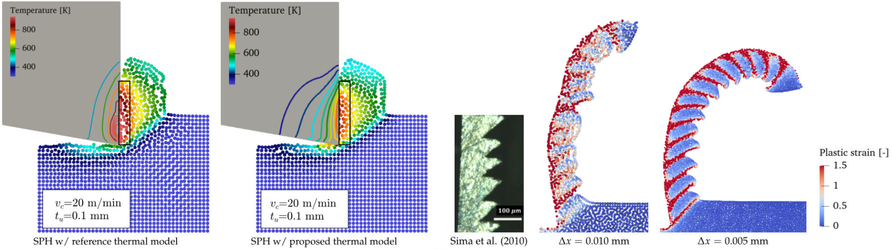


(a) (c) (d) (e) (f)



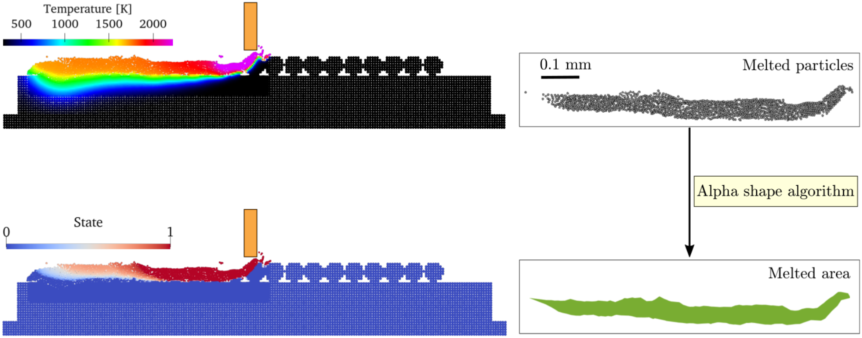
SPH flow, NextFlow

SPH - a few more exmaples



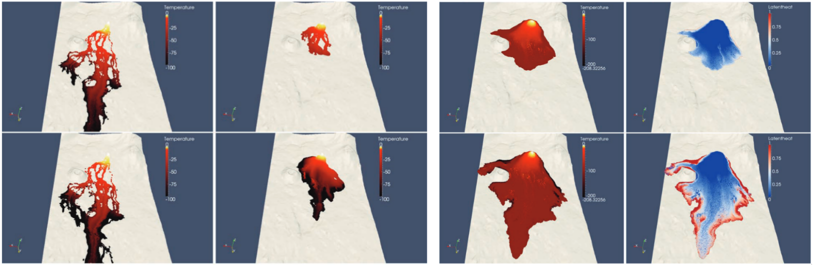
Afrasiabi et al. (2021) - steel cutting with heat transfer

SPH - a few more exmaples



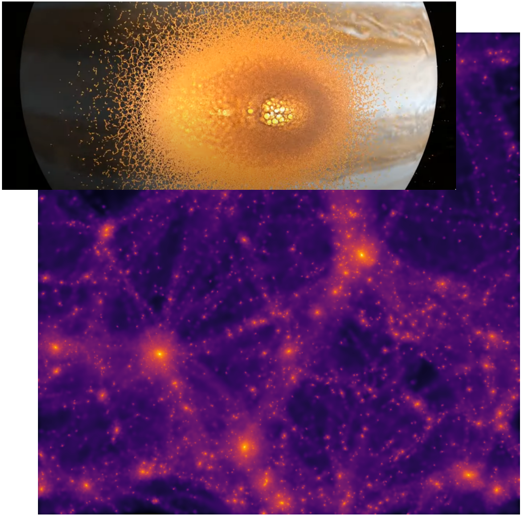
Afrasiabi et al. (2021) - heat melting process

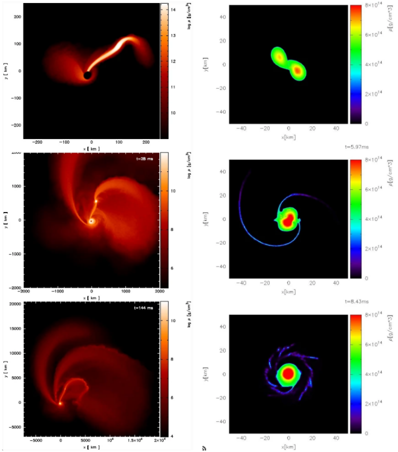
SPH - a few more exmaples



GPUSPH and, well, LOTR

SPH - a few more exmaples





SWIFT code, openSPH, Rosswog (2015)

SPH - a meshfree particle method

Thank you for your attention!