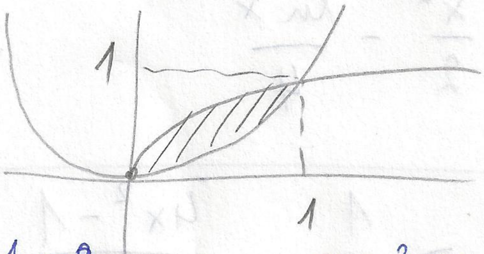
dalsí př. (dom. cv)

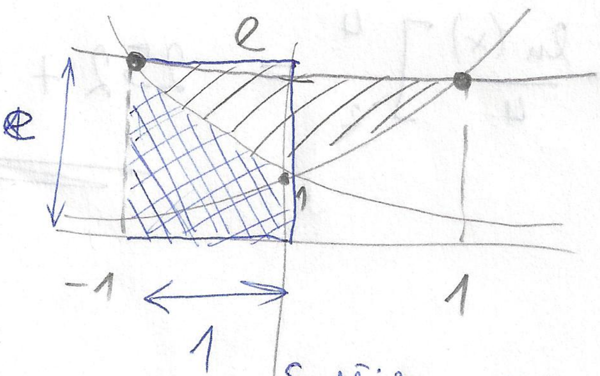
1025 (ii)



$$S = \int_0^1 \sqrt{x}\,dx - \int_0^1 x^2\,dx = -\left[\frac{x^3}{3}\right]_0^1 + \left[\frac{2}{3}\sqrt{x^3}\right]_0^1 =$$

$$= -\frac{1}{3} + \frac{2}{3} = \frac{1}{3}$$

1027 (ii)



$$S = \overset{\text{obd.}}{1\cdot e} - \overset{S_{\text{pod křivkou}}}{\int_{-1}^{0} e^{-x}\,dx} + \overset{\text{obd.}}{1\cdot e} - \int_0^1 e^x\,dx =$$

$$= e + \left[e^{-x}\right]_{-1}^{0} + e - \left[e^x\right]_0^1 = e + (1-e) + e - (e-1)$$

$$= \cancel{e} + 1 - \cancel{e} + \cancel{e} - \cancel{e} + 1 = 2$$

2

1036 (ii) $\quad l = ?$

$$y = \frac{x^2}{2} - \frac{\ln x}{4} \qquad x \in \langle 2,4 \rangle$$

$$y' = x - \frac{1}{4x} = \frac{4x^2-1}{4x}$$

$$l = \int_2^4 \sqrt{1 + \left(\frac{4x^2-1}{4x}\right)^2}\, dx = \int_2^4 \sqrt{\frac{\cancel{16x^2} + 16x^4 \cancel{- 8x^2} + 1}{(4x)^2}}\, dx =$$

$$= \int_2^4 \sqrt{\frac{16x^4 + 8x^2 + 1}{(4x)^2}}\, dx = \int_2^4 \sqrt{\left(\frac{4x^2+1}{4x}\right)^2}\, dx =$$

$$= \int_2^4 \frac{4x^2+1}{4x}\, dx = \int_2^4 x\, dx + \int_2^4 \frac{1}{4x}\, dx =$$

$$= \left[\frac{x^2}{2}\right]_2^4 + \left[\frac{\ln|x|}{4}\right]_2^4 = \frac{16-4}{2} + \frac{1}{4}(\ln 4 - \ln 2) = 6 + \frac{1}{4}\ln 2$$

26.1.

$\frac{1}{x}$ $\frac{1}{\ln x}$

nevlastní určitý $\int$

2. 995 (i)

$$\int_1^e \frac{1}{x\ln x}\,dx = \lim_{k\to 1^+}\int_k^e \text{〃} = \left| \begin{array}{l} t = \ln x \quad \Big| \; t_H = 1 \\ dt = \frac{1}{x}dx \;\Big|\; t_D = \cancel{\frac{1}{k}}\ln k \end{array} \right| =$$

$$= \lim_{k\to 1^+}\int_{\frac{1}{k}}^{1} \frac{1}{t}\,dt = \lim_{k\to 1^+}\Big[\ln|t|\Big]_{\ln k}^{1} =$$

$$= \lim_{k\to 1^+}\left(\ln|\cancel{\ln}\,1| - \ln|\ln k|\right) \overset{\text{spočtu limitu}}{=}$$

$$= 0 - \lim_{k\to 1^+} \ln|\ln k| = \text{"}\infty\text{"} \quad \text{diverguje}$$

$\ln k \to 0$, $\ln|\ln k| \to -\infty$

1. 996 (ii)

$$\int_0^4 \frac{1}{\sqrt{x}}\,dx = \lim_{k\to 0^+}\int_k^4 \frac{1}{\sqrt{x}}\,dx =$$

$$\lim_{k\to 0^+}\left[2\sqrt{x}\right]_k^4 = \lim_{k\to 0^+}\left(4 - 2\sqrt{k}\right) =$$

$$= 4 - 0 = 4$$

999 (iii)

$$\lim_{K\to\infty}\int_1^K \frac{1}{x(x+1)^2}\,dx = \lim_{K\to\infty}\int_1^K\left(\frac{1}{x} - \frac{1}{x+1} - \frac{1}{(x+1)^2}\right)dx =$$

$$= \lim_{K\to\infty}\left[\ln|x| - \ln|x+1| + \frac{1}{x+1}\right]_1^K =$$

$$= \text{—||—}\left(\ln K - \underbrace{\cancel{\ln 1}}_{=0} - \left(\ln(K+1) - \ln 2\right) + \frac{1}{K+1} - \frac{1}{2}\right) =$$

spojité funkce

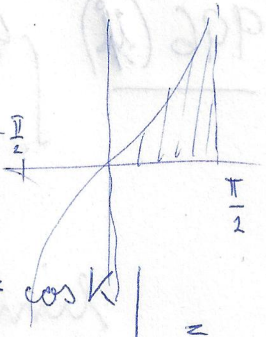
$$= \ln 2 - \frac{1}{2} + \lim_{K\to\infty}\left(\ln K - \ln(K+1) + \frac{1}{K+1}\right) =$$

$$= \text{—||—} + \lim_{K\to\infty}\left(\ln\underbrace{\frac{K}{K+1}}_{\to 1} + \underbrace{\text{—||—}}_{\to 0}\right) =$$

(lim slož. fce) $\to 0$

$$= \ln 2 - \frac{1}{2}$$

1006 (ii)



$$\int_0^{\frac{\pi}{2}} \tan x\,dx = \lim_{K\to\frac{\pi}{2}^-}\int_0^K \tan x\,dx =$$

$$= \lim \int_0^K \frac{\sin x}{\cos x}\,dx \;\left|\begin{matrix} t = \cos x \\ dt = -\sin x\,dx \end{matrix}\right|\begin{matrix} t_H = \cos K \\ t_D = \cos 0 = 1 \end{matrix}\right| =$$

$$= \lim \int_1^{\cos K} \frac{dt}{t} = \lim\left[\ln|t|\right]_1^{\cos K} =$$

$$= 0 - \lim_{K\to\frac{\pi}{2}^-} \underbrace{\ln\underbrace{|\cos K|}_{\to 0}}_{\to -\infty} = \infty \quad \text{diverguje}$$