cv 24/5

554 $\vec{f} = (-y, x, x^2y^2z)$ $Q: x^2+y^2+z^2=R^2$, $z \leq 0$ $(R>0)$

$\phi_Q = ?$ $\vec{n}([0,0,-R]) = -\vec{z}_0$

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kartézsky: $P: \begin{cases} x = u \\ y = v \\ z = -\sqrt{R^2-u^2-v^2} \end{cases}$

$\dot{P}_u = \left(1, 0, \frac{+\cancel{2}u}{\cancel{2}\sqrt{R^2-u^2-v^2}}\right)$ 1 0

$\dot{P}_v = \left(0, 1, \frac{+\cancel{2}v}{\cancel{2}\sqrt{R^2-u^2-v^2}}\right)$ 0 1

$n = (P_u \times P_v) = \left(\frac{-u}{\sqrt{}}, \frac{-v}{\sqrt{}}, 1\right)$

$n([0,0,-R]) = (0,0,1)$ nesouhlasné (orientace $(0,0,-1)$)

$$\phi_Q = -\iint_{u^2+v^2 \leq R^2} \begin{pmatrix} -v \\ u \\ -u^2v^2\sqrt{R^2-u^2-v^2} \end{pmatrix} \cdot \begin{pmatrix} -\frac{u}{\sqrt{}} \\ -\frac{v}{\sqrt{}} \\ 1 \end{pmatrix} du\,dv =$$

$$= -\iint_{u^2+v^2 \leq R^2} \cancel{\frac{+vu}{\sqrt{}}} - \cancel{\frac{vu}{\sqrt{}}} - u^2v^2\sqrt{R^2-u^2-v^2}\, du\,dv =$$

$$= \left| \begin{matrix} u = r\cos\varphi & 0 \leq r \leq R \\ v = r\sin\varphi & 0 \leq \varphi \leq 2\pi \\ J = r & \end{matrix} \right| = +\int_0^R \int_0^{2\pi} r^5 \cos^2\varphi \sin^2\varphi \sqrt{R^2-r^2}\, d\varphi\, dr =$$

$$= \left| \begin{matrix} R^2-r^2 = \xi^2 \\ -\cancel{2}r\,dr = \cancel{2}\xi\, d\xi \\ r^4 = (R^2-\xi^2)^2 \end{matrix} \right| = +\int_0^R \int_0^{2\pi} (R^2-\xi^2)^2 \cos^2\varphi \sin^2\varphi\, \xi^2\, d\varphi\, d\xi \overset{\text{Fubini}}{=}$$

$\frac{1}{4}\sin^2 2\varphi = \frac{1}{8}(1-\cos 4\varphi)$

$$= \frac{1}{8}\int_0^R (R^4\xi^2 - 2R^2\xi^4 + \xi^6) \cdot \int_0^{2\pi} (1-\cos 4\varphi)\, d\varphi =$$

$\left[\frac{1}{4}\sin 4\varphi\right]_0^{2\pi} = 0$

$$= \frac{1}{\cancel{8}_4}\left(\frac{R^7}{3} - \frac{2R^7}{5} + \frac{R^7}{7}\right) \cdot \cancel{2}\pi = \frac{R^7\, 8}{\cancel{4}\, 105}\pi = \boxed{\frac{2\pi R^7}{105}}$$

ve sféř:

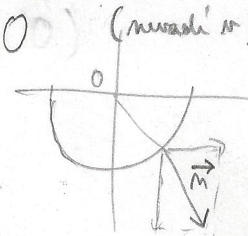
$$P: \begin{cases} x = R \cos u \cos v \\ y = R \sin u \cos v \\ z = R \sin v \end{cases}$$

$$\cancel{P_u = R(-\sin u \cos v, \cos u \cos v, 0)}$$

$$\cancel{P_v = R(-\cos u \sin v, -\sin u \sin v, \cos v)}$$

$$\vec{n} = R^2 \left(\cos u \cos^2 v, \sin u \cos^2 v, \cos v \sin v\right)$$

$\vec{n}\left(\left[0, \frac{\pi}{2}\right]\right) = \vec{0}$ ... ale 1b. ... množina nižší dim. $(\mu_2 = 0)$ (nevadí v ∫)



např.: $\vec{n}\left(\left[0, \frac{\pi}{4}\right]\right) = \left(\frac{\sqrt{2}}{2}, 0, -\frac{1}{2}\right)$ ... vnější normála ✓

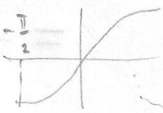
(jako $n([0,0,-R]) = (0,0,-1)$) tj souhlasné

$$f = \left(-R \sin u \cos v, R \cos u \cos v, R^5 \cos^2 u \sin^2 u \cos^4 v \sin v\right)$$

odečtou se

$$\phi_Q = \int_0^{2\pi} \int_{-\frac{\pi}{2}}^{0} R \begin{pmatrix} -\sin u \cos v \\ \cos u \cos v \\ R^4 \cos^2 u \sin^2 u \cos^4 v \sin v \end{pmatrix} \cdot R^2 \begin{pmatrix} \cos u \cos^2 v \\ \sin u \cos^2 v \\ \cos v \sin v \end{pmatrix} dv\, du =$$

$$= R^3 \int_0^{2\pi} \int_{-\frac{\pi}{2}}^{0} R^4 \cos^2 u \sin^2 u \cos^5 v \sin^2 v \, dv\, du \overset{\text{Fubini}}{=}$$



$$= R^7 \frac{2\pi}{8} \int_{-\frac{\pi}{2}}^{0} \underbrace{(1 - \sin^2 v)^2}_{\cos^4 v} \sin^2 v \cos v \, dv \quad \left| \begin{array}{l} t = \sin v \\ dt = \cos v\, dv \end{array} \right|_{-1}^{0} =$$

$$= R^7 \frac{\pi}{4} \int_{-1}^{0} (1 - t^2)^2 t^2 \, dt = R^7 \frac{\pi}{4} \frac{8}{105} = \frac{2\pi R^7}{105}$$