

$$x > -1$$

4)  $y'' + y \ln(1+x) = x + e^x$   $y(0) = -1$   
 $y'(0) = -1$   
 $\Rightarrow x_0 = 0$

$a_0(x) = \ln(1+x) \approx \sum_{k=0}^{\infty} (-1)^k \frac{x^{k+1}}{k+1} = 0 + x - \frac{x^2}{2} + \frac{x^3}{3} - \dots$   
 $\left( \int \frac{1}{1+x} dx \approx \int \sum (-1)^k x^k dx \right)$  Moj.  $|0 < x < 1|$

$f(x) = x + e^x \approx x + \sum_{k=0}^{\infty} \frac{x^k}{k!} = 1 + 2x + \frac{x^2}{2} + \frac{x^3}{6} + \dots$  **3! řes.**

$y \approx \sum c_k x^k$   $x \in (-1, 1)$   $y'(x) = \sum k c_k x^{k-1}$   
 zpp:  $-1 = y(0) \approx c_0 + 0 \Rightarrow c_0 = -1$   $-1 = y'(0) \approx c_1 + 0 \Rightarrow c_1 = -1$

$y'' \approx \sum_{k=0}^{\infty} k(k-1)c_k x^{k-2} = 2c_2 + 6c_3x + 12c_4x^2 + 20c_5x^3 + \dots$

$y \cdot \ln(1+x) \approx (-1 - x + c_2x^2 + c_3x^3 + \dots) \left( 0 + x - \frac{x^2}{2} + \frac{x^3}{6} + \dots \right)$

$\ln(1+x) \backslash y$	$x^0$	$x^1$	$x^2$	$x^3$
$x^0$	0	0	0	0
$x^1$	1	-1	$c_2$	$c_3$
$x^2$	$-\frac{1}{2}$	$+\frac{1}{2}$	$-\frac{c_2}{2}$	
$x^3$	$\frac{1}{6}$	$\frac{1}{6}$		

$y \cdot \ln(1+x) \approx 0 - x + \left(\frac{1}{2} - 1\right)x^2 + \left(c_2 + \frac{1}{2} + \frac{1}{6}\right)x^3 + \dots$   
 $\frac{1}{2} - 1 = -\frac{1}{2}$   
 $\frac{3+1}{6} = \frac{2}{3}$

(7)

$$\text{ree (4) } y'' + y \ln(1+x) = x + e^x$$

$$\begin{aligned} & (2c_2 + 6c_3x + 12c_4x^2 + 20c_5x^3 + \dots) \\ & + \left( 0 - x - \frac{1}{2}x^2 + \left(c_2 + \frac{2}{3}\right)x^3 + \dots \right) = \\ & = 1 + 2x + \frac{x^2}{2} + \frac{x^3}{6} + \dots \end{aligned}$$

$$x^0: \quad 2c_2 = 1 \quad \Rightarrow \quad c_2 = \frac{1}{2}$$

$$x^1: \quad 6c_3 - 1 = 2 \quad \Rightarrow \quad c_3 = \frac{1}{2}$$

$$x^2: \quad 12c_4 - \frac{1}{2} = \frac{1}{2} \quad \Rightarrow \quad c_4 = \frac{1}{12}$$

$$x^3: \quad 20c_5 + \left(\frac{1}{2} + \frac{2}{3}\right) = \frac{1}{6}$$

$$20c_5 + \frac{3+4}{6} = \frac{1}{6}$$

$$20c_5 = -1 \quad \Rightarrow \quad c_5 = -\frac{1}{20}$$

$$y(x) \approx -1 - x + \frac{1}{2}x^2 + \frac{1}{2}x^3 + \frac{1}{12}x^4 - \frac{1}{20}x^5 + o(x^6)$$

$$x \in (-1, 1)$$