Exaktní rce

5. $$\overbrace{x(y^2+1)}^{P}\,dx + \overbrace{\left(\frac{1}{\sqrt{1-y^2}} + x^2y\right)}^{Q}dy = 0$$

(nebo $\left(\frac{1}{\sqrt{1-y^2}} + x^2y\right)y' = x(y^2+1)$ & $y(1)=0$)

$u = u(x,y)$ → fce ve tvaru tot. dif.

(*) $$du = \underbrace{\frac{\partial u}{\partial x}}_{= P}dx + \underbrace{\frac{\partial u}{\partial y}}_{= Q}dy = 0$$

exaktní ?

$$\frac{\partial^2 u}{\partial x \partial y} = \frac{\partial P}{\partial y} \overset{?}{=} \frac{\partial Q}{\partial x} = \frac{\partial^2 u}{\partial y \partial x}$$ (u spoj.

$2xy = 2xy$ ✓

$u = ?$ jako potenciál

$$\frac{\partial u}{\partial x} = x(y^2+1) \quad / \int dx$$

$$u(x,y) = \int x(y^2+1)\,dx = \frac{x^2}{2}(y^2+1) + K(y) \Rightarrow \frac{\partial u}{\partial y} = \frac{x^2}{2}2y + K'(y)$$

& $$\frac{\partial u}{\partial y} = \boxed{\cancel{\frac{x^2}{2}2y} + K'(y) = \underbrace{\frac{1}{\sqrt{1-y^2}} + \cancel{x^2y}}_{Q}} \Rightarrow K'(y) = \frac{1}{\sqrt{1-y^2}}$$

$$K(y) = \int \frac{1}{\sqrt{1-y^2}}\,dy = \arcsin(y) + C_1$$

$$u(x,y) = \frac{x^2}{2}(y^2+1) + \arcsin(y) + C_1$$

(*)⇒ $du = 0 \iff u = C_2$ konst.

$(C = C_2 - C_1)$

O.Ř.: $$\frac{x^2}{2}(y^2+1) + \arcsin(y) = C$$ ... ne nutně

impl. tvar řeš. (∄ expl.)

Cauchy $\frac{1}{2} + 0 = C$

Exaktní rce

$$y(0) = -\frac{1}{2}$$

6. $2xy - 9x^2 + (2y + x^2 + 1)\,y' = 0$

totální diferenciál

$$df = \underbrace{(2xy - 9x^2)}_{P}\,dx + \underbrace{(2y + x^2 + 1)}_{Q}\,dy = 0$$

$$\frac{\partial f}{\partial x} = \text{—〃—} \qquad \frac{\partial f}{\partial y} = \text{—〃—}$$

$$\frac{\partial^2 f}{\partial x \partial y} = 2x \quad = \quad \frac{\partial^2 f}{\partial y \partial x} = 2x \quad \checkmark$$

aby exaktní

mám $\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}$ → hledám f jako potenciál

f izokřivka

$$f = \int \frac{\partial f}{\partial x}\,dx = \int 2xy - 9x^2\,dx = x^2 y - 3x^3 + K(y)$$

Platí

$$\wedge \quad \frac{\partial f}{\partial y} = x^2 + K'(y) = 2y + x^2 + 1$$

$$K'(y) = 2y + 1 \quad /\int$$

$$K(y) = \int 2y + 1\,dy = y^2 + y + (C)$$

$$f = x^2 y - 3x^3 + y^2 + y + (C)$$

$df = 0 \quad /\int$

$f = \text{const}$

(izokřivka)

impl. fce

$$x^2 y - 3x^3 + y^2 + y = C$$

Cauchy:

$$0 - 0 + \frac{1}{4} - \frac{1}{2} = C \Rightarrow C = -\frac{1}{4}$$