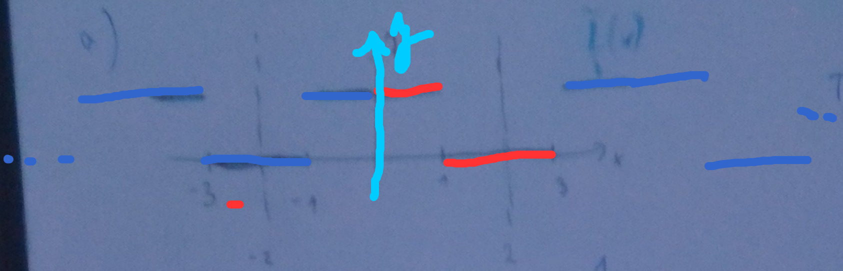
1. (sudé) periodické rozšíření fce → cosinová řada $\Rightarrow$ sudá

$$f(x) = \begin{cases} 1 & \text{pro } x \in \langle 0,1) \\ 0 & \text{pro } x \in (1,3) \end{cases}$$

$\boxed{\langle -2,2 \rangle}$

a)



$T = 4 = 2L$

$L = 2$

b) $a_0 = \frac{1}{2}\int_{-2}^{2} f(x)\,dx = \frac{1}{2}\int_{-1}^{1} 1\,dx + 0 + 0 = \frac{1}{2}[x]_{-1}^{1} = 1$

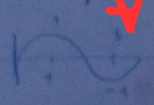
$a_k = \frac{1}{2}\int_{-2}^{2} f(x)\cos\left(\frac{k\pi}{2}x\right)dx = \frac{1}{2}\int_{-1}^{1} 1\cos\left(\frac{k\pi}{2}x\right)dx =$

$\left| \begin{matrix} t = \frac{k\pi}{2}x & \frac{k\pi}{2} \\ \frac{2}{k\pi}\,dt = dx & -\frac{k\pi}{2} \end{matrix} \right| = \frac{1}{2}\frac{2}{k\pi}\int_{-\frac{k\pi}{2}}^{\frac{k\pi}{2}} \cos t\,dt =$

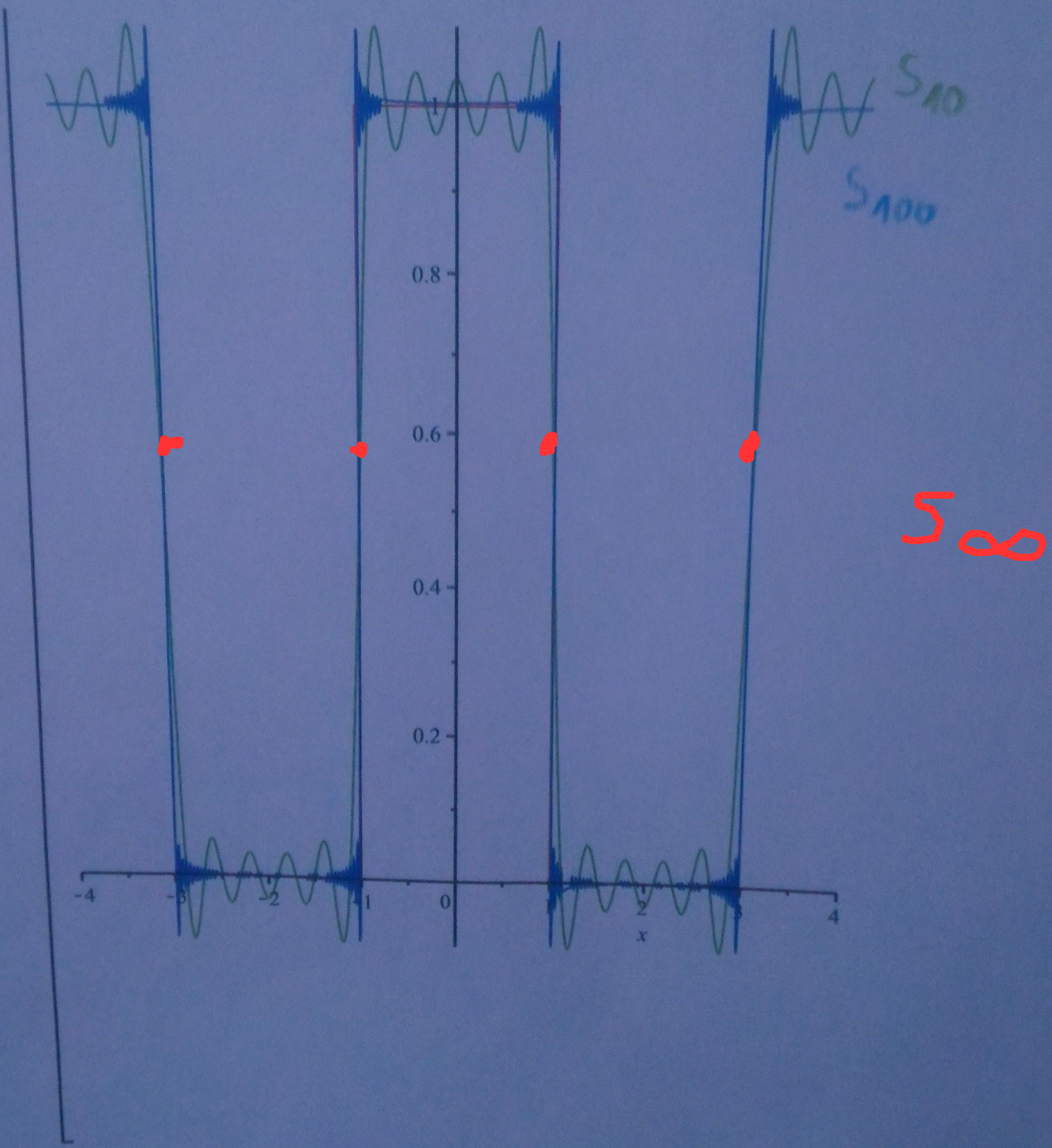
$= \frac{1}{k\pi}[\sin t]_{-\frac{k\pi}{2}}^{\frac{k\pi}{2}} = \frac{1}{k\pi}\left(\sin\frac{k\pi}{2} + \sin\frac{k\pi}{2}\right) =$

$b_k = 0$ (sudá fce) $\qquad = \frac{2}{k\pi}\sin\left(\frac{k\pi}{2}\right)$ číslo $\forall k$

c) $f(x) \approx \frac{1}{2} + \sum_{k=1}^{\infty} \underbrace{\frac{2}{k\pi}\sin\left(\frac{k\pi}{2}\right)}_{a_k} \cos\left(\frac{k\pi}{2}x\right) =$



$= \frac{1}{2} + \frac{2}{\pi}\cos\left(\frac{1}{2}\pi x\right) - \frac{2}{3\pi}\cos\left(\frac{3}{2}\pi x\right) + \ldots$



S_{10}
S_{100}
S_∞
1
0.8
0.6
0.4
0.2
-4
-3
-2
-1
0
1
2
3
4
x

2) $f(x) = -\frac{1}{2}x + \frac{\pi}{2}$ pro $x \in (0, 2\pi)$ → vytvořte sinovou Ř. (liché rozšíření)

$T = 2\pi$

$\underline{\underline{L = \pi}}$

$a_0 = 0 = a_k$

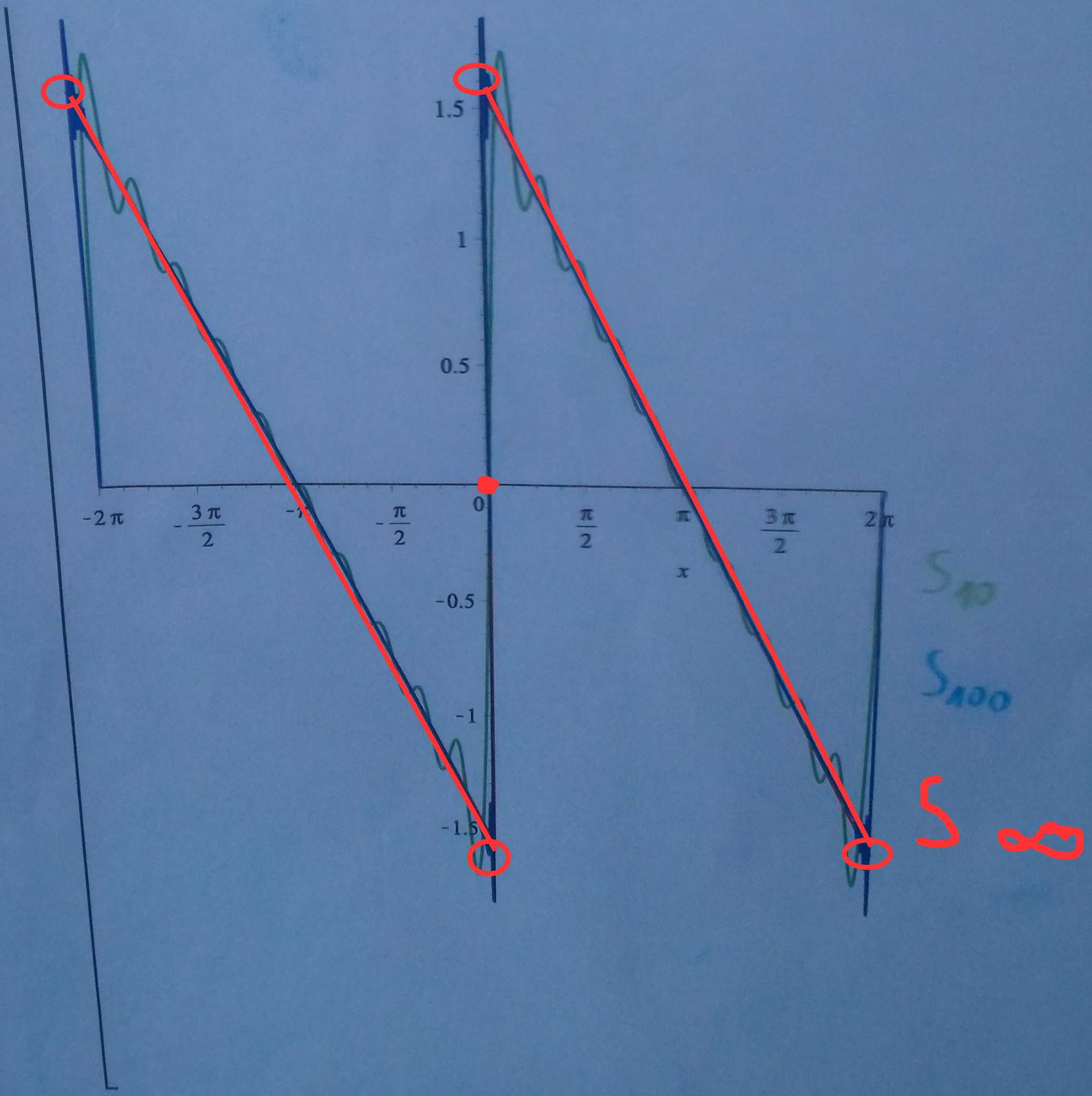
$$b_k = \frac{1}{\pi}\int_0^{2\pi}\left(-\frac{x}{2} + \frac{\pi}{2}\right)\sin\left(\frac{k\pi}{\pi}x\right)dx = -\frac{1}{2\pi}\int_0^{2\pi}\overset{u}{(x-\pi)}\,\overset{v'}{\sin(kx)}\,dx =$$

$$\left|\begin{array}{ll} u = x - \pi & v' = \sin kx \\ u' = 1 & v = \frac{1}{k}(-\cos kx)\end{array}\right| = -\frac{1}{2\pi}\left(\left[\overset{u}{(x-\pi)}\,\overset{v}{\frac{1}{k}(-\cos kx)}\right]_0^{2\pi} + \right.$$

$$\left. + \int_0^{2\pi}\overset{u'v}{\frac{1}{k}\cos kx}\,dx\right) = -\frac{1}{2\pi}\left(\frac{\pi}{k}(-\cos 2k\pi) + \left(+\frac{\pi}{k}(-\cos 2k\pi\right)\right)$$

$$+ \underbrace{\cancel{\frac{1}{k^2}\left[\sin kx\right]_0^{2\pi}}}_{=0} = -\frac{1}{2\pi}\left(-\frac{\pi}{k} - \frac{\pi}{k}\right) = \underline{\underline{\frac{1}{k}}}$$

c) $f(x) \approx \sum_{k=1}^{\infty} \underbrace{\frac{1}{k}}_{b_k} \sin\left(\frac{k\pi}{\pi}x\right) = \sin x + \frac{1}{2}\sin 2x + \frac{1}{3}\sin 3x$



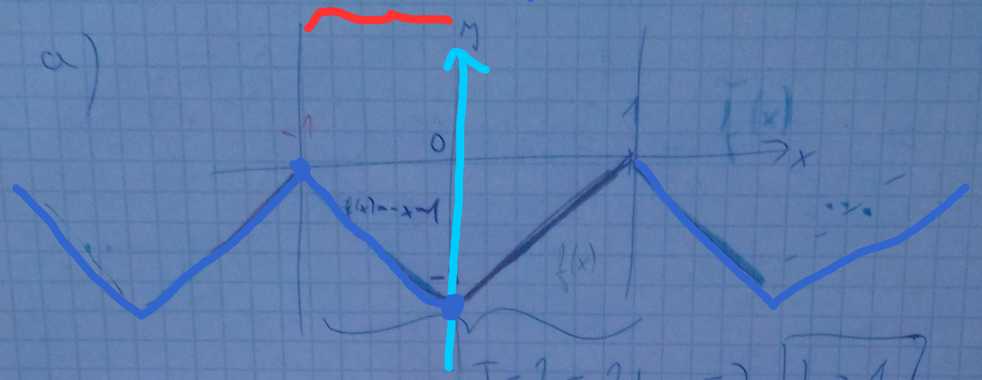
1.5
1
0.5
0
-0.5
-1
-1.5
-2 π
3 π / 2
-π
π / 2
π / 2
π
3 π / 2
2 π
x
S40
S100
S∞

3.) $f(x) = (x-1)$ pro $x \in \langle 0,1 \rangle$

$\rightarrow$ cosinový rozvoj

$\Rightarrow$ sudé rozšíření

a)



$T = 2 = 2L \Rightarrow \boxed{L = 1}$

b) $b_k = 0$ (sudá)

$$a_0 = \frac{1}{1}\int_{-1}^{1} f(x)\,dx \overset{\text{sudá fce}}{=} 2\int_0^1 (x-1)\,dx =$$

$$= 2\left[\frac{x^2}{2} - x\right]_0^1 = 2\left(\frac{1}{2} - 1\right) = -1$$

$$\boxed{\frac{a_0}{2} = -\frac{1}{2}}$$

$$a_k = \frac{1}{1}\int_{-1}^{1} f(x)\cos\left(\frac{k\pi x}{1}\right) \overset{\text{sudá} \cdot \text{sudá} = \text{sudá fce}}{=} 2\int_0^1 (x-1)\cos(k\pi x) =$$

$$\left|\ \text{pp}\ \begin{array}{ll} u = (x-1) & v' = \cos(k\pi x) \\ u' = 1 & v = \frac{1}{k\pi}\sin(k\pi x) \end{array}\right| =$$

$$= 2\left[\cancel{(x-1)\frac{1}{k\pi}\sin(k\pi x)}\right]_0^1 - 2\int_0^1 \frac{1}{k\pi}\sin(k\pi x)\,dx =$$

$$= +\frac{2}{k^2\pi^2}\left[+\cos(k\pi x)\right]_0^1 = \frac{2}{k^2\pi^2}\left(\cos(k\pi) - 1\right) =$$

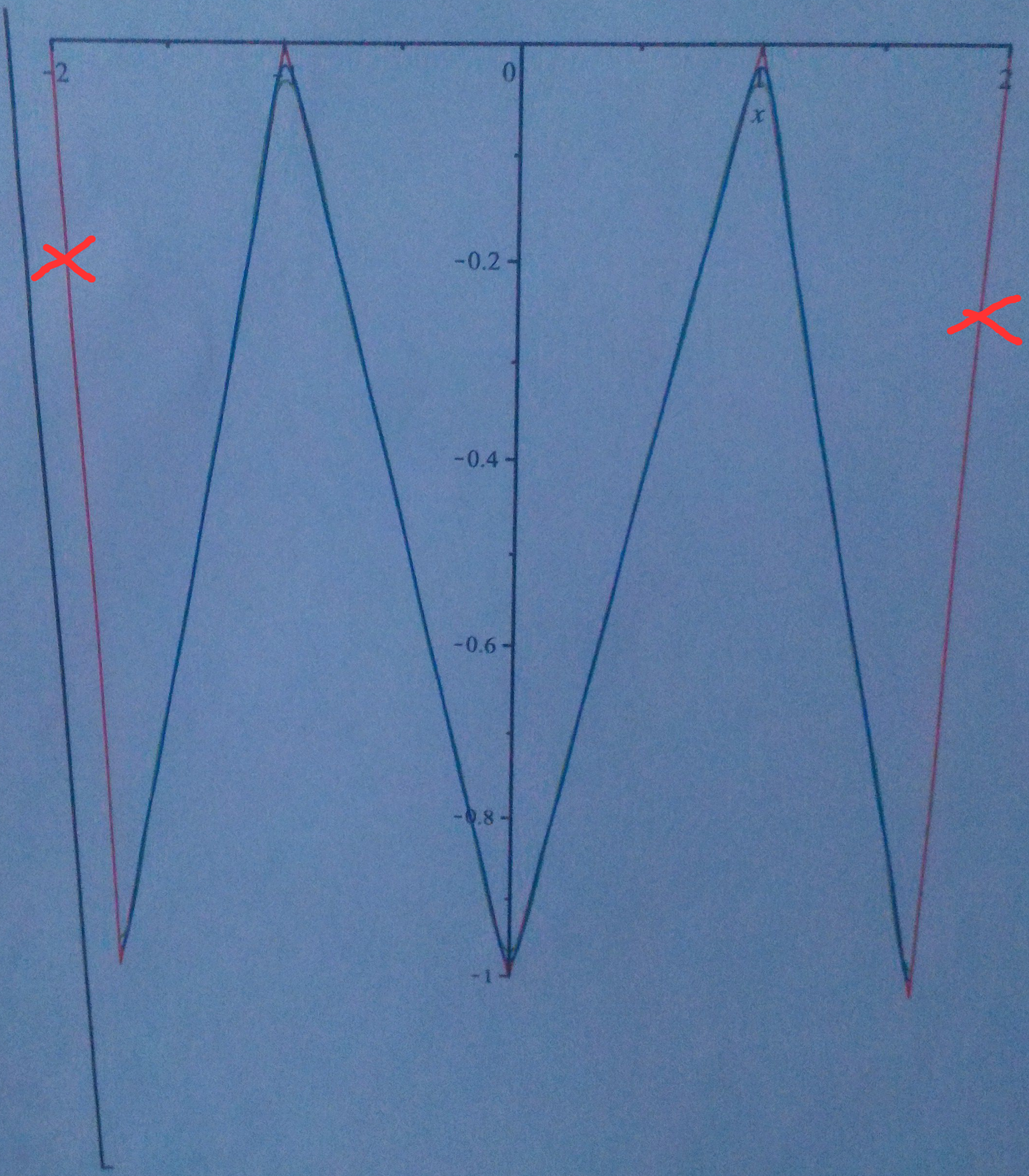
$$= \frac{2}{k^2\pi^2}\left((-1)^k - 1\right) = \text{číslo } \forall k$$

c) $$f(x) = \overbrace{-\frac{1}{2}}^{a_0/2} + \sum_{k=1}^{\infty} \overbrace{\frac{2}{k^2\pi^2}\left((-1)^k - 1\right)}^{a_k} \cos\left(\frac{k\pi x}{1}\right) =$$

$$\left((-1)^k - 1\right) = \begin{cases} 0 & k = 2n \\ -2 & k = 2n-1 \end{cases}$$

pozn $$= -\frac{1}{2} + \sum_{n=1}^{\infty} \frac{-4}{(\underbrace{2n-1}_{K})^2\pi^2} \cos\left((\underbrace{2n-1}_{K})\pi x\right)$$

lichá



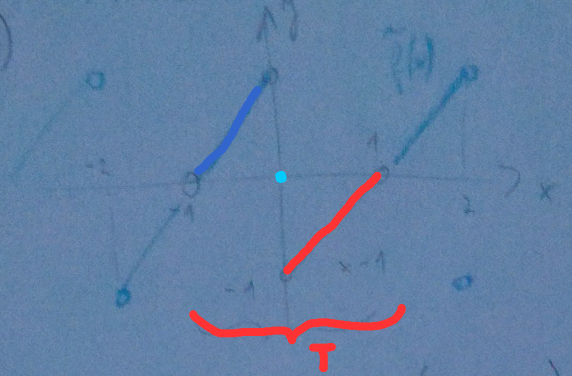
-2
-1
0
1
2
x
-0.2
-0.4
-0.6
-0.8
-1

4) $f(x) = (x-1)$ pro $x \in (0,1)$

→ sinový rozvoj

→ liché rozšíření

a)



$T = 2 = 2L$

b) $a_k = 0$ $(a_0 = 0)$ lichá fce

$$b_k = \frac{1}{1}\int_{-1}^{1} \tilde{f}(x) \sin\left(\frac{k\pi}{1}x\right) dx = 2\int_0^1 \underbrace{(x-1)}_{u} \underbrace{\sin(k\pi x)}_{v'} dx =$$

lichá · lichá = sudá

$$\left| \begin{array}{ll} u = (x-1) & u' = 1 \\ v' = \sin(\,) & v = -\frac{1}{k\pi}\cos(\,) \end{array} \right|$$

$$= 2\left[-\frac{1}{k\pi}\underbrace{(x-1)}_{u}\underbrace{\cos(k\pi x)}_{v}\right]_0^1 + \underbrace{\frac{2}{k\pi}\int_0^1 \cos(k\pi x)\, dx}_{\int u'v} =$$

$$= 0 + \frac{2}{k\pi}(-1)\cdot 1 + \cancel{\frac{2}{k^2\pi^2}\left[\sin k\pi x\right]_0^1} = -\frac{2}{k\pi}$$
$= 0$

c) $$f(x) \approx \sum_{k=1}^{\infty} \overbrace{-\frac{2}{k\pi}}^{b_k} \sin(k\pi x) =$$

$$= -\frac{2}{\pi}\sin(\pi x) - \frac{1}{\pi}\sin(2\pi x) - \frac{2}{3\pi}\sin(3\pi x) - \dots$$

pozn. $\neq 0$ $\exists x$ (např. $x = \sqrt{2}$)

$\boxed{(1\,zp.)}$: $a_0 = \frac{1}{2}\int_{-2}^{2}(x+1)\,dx = \frac{1}{2}\left[\frac{x^2}{2}+x\right]_{-2}^{2} = \frac{1}{2}\left(\cancel{\frac{4}{2}}+2-\cancel{\frac{4}{2}}+2\right) = 2$

$$a_k = \frac{1}{2}\int_{-2}^{2}(x+1)\cos\left(\frac{k\pi}{2}x\right)dx = \left|\begin{array}{ll} u = x+1 & u' = 1 \\ v' = \cos(\;) & v = \frac{2}{k\pi}\sin\left(\frac{k\pi}{2}x\right) \end{array}\right.$$

$$= \underbrace{\cancel{\frac{1}{2}\left[\overset{u}{(x+1)}\overset{v}{\frac{2}{k\pi}}\sin\left(\frac{k\pi}{2}x\right)\right]_{-2}^{2}}}_{=0} - \frac{1}{\cancel{2}}\int_{-2}^{2}\frac{\cancel{2}}{k\pi}\sin\left(\frac{k\pi}{2}x\right)dx =$$

$$= +\frac{2}{k^2\pi^2}\left[\cos\left(\frac{k\pi}{2}x\right)\right]_{-2}^{2} = \frac{2}{k^2\pi^2}\left(\cos k\pi - \overbrace{\cos(-k\pi)}^{=\cos k\pi}\right) = 0$$

$$b_k = \frac{1}{2}\int_{-2}^{2}(x+1)\sin\left(\frac{k\pi}{2}x\right)dx = \left|\begin{array}{ll} u = x+1 & u' = 1 \\ v' = \sin(\;) & v = \frac{-2}{k\pi}\cos\left(\frac{k\pi}{2}x\right) \end{array}\right.$$

$$= \frac{1}{\cancel{2}}\left[\frac{-\cancel{2}\overset{u}{(x+1)}}{k\pi}\overset{v}{\cos}\left(\frac{k\pi}{2}x\right)\right]_{-2}^{2} + \frac{1}{\cancel{2}}\frac{(-\cancel{2})}{k\pi}\int_{-2}^{2}\cos\left(\frac{k\pi}{2}x\right)dx =$$

$$= -\frac{3}{k\pi}\cos k\pi + \frac{(-1)}{k\pi}\cos(-k\pi) - \underbrace{\cancel{\frac{2}{k^2\pi^2}\left[\sin\left(\frac{k\pi}{2}x\right)\right]_{-2}^{2}}}_{=0}$$

$$= \frac{-4}{k\pi}(-1)^k$$

$$F(x) = 1 + \sum_{k=1}^{\infty} -\frac{4}{k\pi}(-1)^k \sin\left(\frac{k\pi}{2}x\right)$$