3.) Taylor

$$T_3(x) = f(x_0) + f'(x_0)(x-x_0) + \frac{f''(x_0)}{2!}(x-x_0)^2 + \frac{f'''(x_0)}{3!}(x-x_0)^3$$

$$\boxed{T_n(x) = f(x_0) + \sum_{k=1}^{n} \frac{f^{(k)}(x_0)}{k!}(x-x_0)^k}$$

13.) $f(x) = e^{-2x}$ $\qquad x_0 = 2$

$f(2) = e^{-4}$ $\qquad f'(2) = -2e^{-2x}\Big|_2 = -2e^{-4}$

$f''(2) = 4e^{-2x}\Big|_2 = 4e^{-4}$ $\qquad f'''(2) = -8e^{-2x}\Big|_2 = -8e^{-4}$

$f^{(k)}(2) = (-2)^k e^{-2x}\Big|_2 = (-2)^k e^{-4}$

$$T_3(x) = e^{-4} - 2e^{-4}(x-2) + \cancel{4}^{\,2}\,\frac{e^{-4}}{\cancel{2}}(x-2)^2 - \cancel{8}^{\,4}\,\frac{e^{-4}}{\cancel{3!}_{\,3}}(x-2)^3$$

$$T_n(x) = e^{-4} + \sum_{k=1}^{n} \frac{(-2)^k e^{-4}}{k!}(x-2)^k$$

pozn.: zkuste si tu sumu rozepsat a přesvědčit se, že sedí i na $T_3(x)$

14.) $\boxed{e^x \approx \sum_{k=0}^{\infty} \frac{x^k}{k!}}$

15.) $f(x) = \ln x \qquad x_0 = 1$

$$f(1) = 0 \qquad f'(1) = \frac{1}{x}\Big|_1 = 1$$

$$f''(1) = -\frac{1}{x^2}\Big|_1 = -1 \qquad f'''(1) = \frac{2}{x^3}\Big|_1 = 2$$

$$f^{IV}(1) = -\frac{6}{x^4}\Big|_1 = -6 \qquad f^{(k)}(1) = (-1)^{k-1}\frac{(k-1)!}{x^k}\Big|_1 = (-1)^{k-1}$$

$$T_3(x) = 0 + 1\cdot(x-1) - \frac{1}{2}(x-2)^2 + \frac{\cancel{2}}{\cancel{3!}_{3}}(x-2)^3$$

$$T_n(x) = 0 + \sum_{k=1}^{n} (-1)^{k-1}\frac{(k-1)!}{k!}(x-1)^k =$$

$$= \sum_{k=1}^{n} (-1)^{k-1}\frac{\cancel{(k-1)}!}{k\cdot\cancel{(k-1)}!}(x-1)^k = \sum_{k=1}^{n} \frac{(-1)^{k-1}}{k}(x-1)$$

nebo

$l = k-1$: $\displaystyle T_n(x) = \sum_{l=0}^{n} (-1)^l \frac{(x-1)^{l+1}}{l+1}$

(posuneme index sumace)

16) $$\boxed{\ln(1+x) \approx \sum_{l=0}^{\infty} (-1)^l \frac{x^{l+1}}{l+1}}$$

$(x \in (-1,1\rangle)$

viz. cv 2 ☺