

$$\dot{X} - AX = \vec{b}$$

3.54b

$$\dot{X} = \underbrace{\begin{pmatrix} -3 & 1 \\ -4 & 2 \end{pmatrix}}_A X + \underbrace{\begin{pmatrix} 1 \\ -2 \end{pmatrix}}_b$$

2. a) o.Ř.

1. b) B.R. + byř

long.

$$|A - \lambda E| = \begin{vmatrix} -3-\lambda & 1 \\ -4 & 2-\lambda \end{vmatrix} = 0$$

$$-(2-\lambda)(3+\lambda) + 4 = 0$$

$$\lambda^2 + \lambda - 2 = 0$$

$$\lambda_{1,2} = \frac{-1 \pm \sqrt{1+8}}{2} = \frac{-1 \pm 3}{2} < \begin{matrix} 1 \\ -2 \end{matrix}$$

rel. v. $\lambda = 1$: $\begin{pmatrix} -4 & 1 & | & 0 \\ -4 & 1 & | & 0 \end{pmatrix}$

$$-4v_1 + v_2 = 0$$

$$v_1 = p \begin{pmatrix} 1 \\ 4 \end{pmatrix}$$

$p \in \mathbb{R} - \{0\}$

rel. v. $\lambda = -2$: $\begin{pmatrix} -1 & 1 & | & 0 \\ -4 & 4 & | & 0 \end{pmatrix}$

$$-w_1 + w_2 = 0$$

$$v_2 = p \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$X_p \dots$ B.R. (přím. v.)

$$X = c_1 e^t \begin{pmatrix} 1 \\ 4 \end{pmatrix} + c_2 e^{-2t} \begin{pmatrix} 1 \\ 1 \end{pmatrix} + \begin{bmatrix} 2 \\ 5 \end{bmatrix}$$

1. b) $\dot{X} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} = \begin{pmatrix} -3 & 1 \\ -4 & 2 \end{pmatrix} \begin{pmatrix} x_{BR} \\ y_{BR} \end{pmatrix} + \begin{pmatrix} 1 \\ -2 \end{pmatrix}$

$$\begin{cases} -3x + y = -1 \\ -4x + 2y = +2 \end{cases}$$

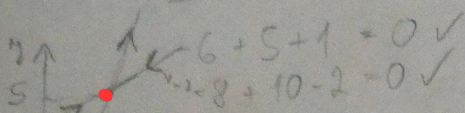
$$-2x = +4$$

$$x = -2$$

$$y = -1 + 6 = 5$$

$$\begin{bmatrix} 2 \\ 5 \end{bmatrix}$$

sedlo (sign $\lambda_1 \neq$ sign λ_2)



$$\vec{x} = \dot{x}(0) = AX + b$$

6.

$$\dot{X} = \begin{pmatrix} 2 & 1 \\ 4 & 2 \end{pmatrix} X$$

$$X_{BR}(t) = \begin{pmatrix} 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 2 & 1 \\ 4 & 2 \end{pmatrix} \begin{pmatrix} x_{BR} \\ y_{BR} \end{pmatrix}$$

$$2x_{BR} + y_{BR} = 0$$

$$y_{BR} = -2x_{BR}$$

$$\begin{vmatrix} 2-\lambda & 1 \\ 4 & 2-\lambda \end{vmatrix} = 0$$

$$(2-\lambda)^2 - 4 = 0$$

$$\lambda^2 - 4\lambda = 0$$

$$\lambda_2 = 4$$



$$v_{II} = \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} \left(\begin{array}{cc|c} -2 & 1 & 0 \\ 4 & 2 & 0 \end{array} \right)$$

$$-2u_1 + u_2 = 0$$

$$v_0 = \begin{pmatrix} w_1 \\ w_2 \end{pmatrix} \left(\begin{array}{cc|c} 2 & 1 & 0 \\ 4 & 2 & 0 \end{array} \right)$$

$$2w_1 + w_2 = 0$$

$$(e^{0t} = 1)$$

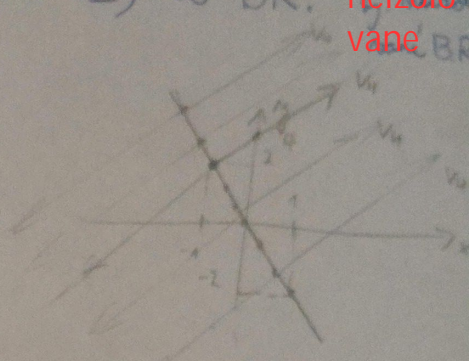
$$\text{O.R. } X(t) = C_1 \begin{pmatrix} -1 \\ 2 \end{pmatrix} + C_2 e^{4t} \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$

$$c) \underline{X(t) = \begin{bmatrix} -1 \\ 2 \end{bmatrix}} \quad (\text{BR}) \quad \text{neizolovane}$$

a) ~~BR~~ BR, λ_{eff}

b) fázorový obr. + O.R.

c) $X(0) = \begin{pmatrix} x(0) \\ y(0) \end{pmatrix} = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$
 A sing. mat. $\Rightarrow \infty$ BR. neizolovane



$p \in \mathbb{R} - \{0\}$

7) A

$$\dot{X} = \begin{pmatrix} 1 & -1 \\ 4 & -3 \end{pmatrix} X$$

~~$$X(t) = \begin{pmatrix} 0 \\ -1 \end{pmatrix}$$~~

$$A - \lambda E = \begin{vmatrix} 1-\lambda & 1 \\ 4 & -3-\lambda \end{vmatrix} = 0$$

$$-(1-\lambda)(3+\lambda) + 4 = 0$$

$$\lambda = -1 \quad \lambda^2 + 2\lambda + 1 = 0 \Rightarrow \lambda_{1,2} = -1$$

$$\begin{array}{l} 2 \ 1 \ | \ 0 \\ 4 \ -2 \ | \ 0 \end{array}$$

$$2u_1 - u_2 = 0$$

$$v_{-1} = P \begin{pmatrix} 1 \\ 2 \end{pmatrix} \quad (\text{jeu 1 ml. os.})$$

$$\text{F.S.} = \{ e^{-t} (v_1); \cancel{te^t (v_1)} \}$$

$$\text{ale } te^{-t} (v_1) + e^{-t} v_2$$

NE!

1.zp.:

$$(A - \lambda E) v_2 = v_1$$

3! rís. (převládá) (V. 2 LA)

$$\begin{pmatrix} 2 & -1 \\ 4 & -2 \end{pmatrix} \begin{pmatrix} w_1 \\ w_2 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \end{pmatrix} \Leftrightarrow \begin{pmatrix} 2 & -1 & | & 1 \\ 4 & -2 & | & 2 \end{pmatrix}$$

$$2w_1 - w_2 = 1$$

$$v_2 = \begin{pmatrix} 0 \\ -1 \end{pmatrix}$$

W partikulární řešení part. !!

$$\text{F.S.} = \{ e^{-t} \begin{pmatrix} 1 \\ 2 \end{pmatrix}; e^{-t} \begin{pmatrix} t \\ 2t-1 \end{pmatrix} \}$$

2.zp.:

(eliminace met.) → převedem na ODR 2.ř.

$$\begin{aligned} \dot{x}(t) &= x - y \\ \dot{y}(t) &= 4x - 3y \end{aligned}$$

$$\Rightarrow y = x - \dot{x} \quad \left| \frac{d}{dt} \right.$$

$$\dot{x} - \ddot{x} = \dot{y} = 4x - 3(x - \dot{x})$$

$$\ddot{x} + 2\dot{x} + x = 0$$

$$\lambda^2 + 2\lambda + 1 = 0 \quad \lambda_{1,2} = -1$$

$$\text{F.S.} = \{ e^{-t}, te^{-t} \}$$

$$y = x - \dot{x}$$

$$\text{F.S.} = \{ 2e^{-t}, 2te^{-t} - e^{-t} \}$$



pro y: