Understanding large-scale atmospheric and oceanic flows with layered rotating shallow water models

V. Zeitlin

¹Laboratory of Dynamical Meteorology, ENS, Paris, France

Non-homogeneous Fluids and Flows, Prague, August 2012

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Importance of moisture in the atmosphere: obvious. Influences large-scale dynamics via the latent heat release, due to condensation and precipitation.

Atmospheric circulation modeling: equation of state of the moist air extremely complex. Discretization/averaging: problematic.

Current parametrizations of precipitations and latent heat release:

relaxation to the equilibrium (saturation) profile of humidity ⇒ threshold effect ⇒ essential nonlinearity Consequences: no linear limit; linear thinking: modal decomposition, linear stability analysis, etc impossible ⇒ problems in quantifying predictability of moist - convective dynamical systems.

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Aims and method I

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Aim:

Understanding the influence of condensation and latent heat release upon large-scale dynamical processes

Reminder:

- Simplest model for large-scale motions: rotating shallow water.
- Link with primitive equations: vertical averaging
- Baroclinic effects: 2 (or more) layers.

Problem with this approach for moist air: averaging of essentially nonlinear equation of state.

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Aims and method II

Our approach

- Combine (standard) vertical averaging of primitive equations between the isobaric surfaces with that of Lagrangian conservation of moist enthalpy
- Allow for convective fluxes (extra vertical velocity) across the isobars
- Link these fluxes to condensation
- Use relaxation parametrization in terms of bulk moisture in the layer for the condensation/precipitation

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Aims and method III

Advantages:

- Simplicity, qualitative analysis of basic phenomena straightforward
- Fully nonlinear in the hydrodynamic sector
- Well-adapted for studying discontinuities, in particular precipitation fronts
- Efficient numerical tools available (finite-volume codes for shallow water)
- Various limits giving known models
- Inclusion of topography (gentle or steep) straightforward

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Primitive equations in pseudo-height coordinates

$$\frac{d}{dt}\mathbf{v} + f\mathbf{k} \times \mathbf{v} = -\nabla \phi$$

$$\frac{d}{dt}\theta = 0$$

$$\nabla \cdot \mathbf{v} + \partial_z \mathbf{w} = 0$$

$$\partial_z \phi = g \frac{\theta}{\theta_0}$$

 $\mathbf{v} = (u, v)$ and w - horizontal and vertical velocities, $\frac{d}{dt} = \partial_t + \mathbf{v} \cdot \nabla + w \partial_z$, f - Coriolis parameter, θ - potential temperature, ϕ - geopotential.

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Moisture and moist enthalpy

Condensation turned off: conservation of specific humidity of the air parcel:

$$\frac{d}{dt}q=0.$$

Condensation turned on: θ and q equations acquire source and sink. Yet the moist enthalpy $\theta + \frac{L}{c_p}q$, where L-latent heat release, c_p -specific heat, is conserved for any air parcel on isobaric surfaces:

$$\frac{d}{dt}\left(\theta + \frac{L}{c_{p}}q\right) = 0,$$

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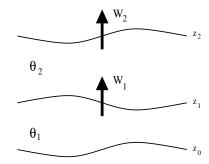
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Vertical averaging with convective fluxes

3 material surfaces:

$$w_0 = \frac{dz_0}{dt}, \quad w_1 = \frac{dz_1}{dt} + W_1, \quad w_2 = \frac{dz_2}{dt} + W_2.$$



Mean-field + constant mean $\theta \rightarrow$

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Averaged momentum and mass conservation equations:

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 $\begin{cases} \partial_t \mathbf{v}_1 + (\mathbf{v}_1 \cdot \nabla) \mathbf{v}_1 + f \mathbf{k} \times \mathbf{v}_1 = -\nabla \phi(\mathbf{z}_1) + g \frac{\theta_1}{\theta_0} \nabla \mathbf{z}_1, & \text{enorm models} \\ \partial_t \mathbf{v}_2 + (\mathbf{v}_2 \cdot \nabla) \mathbf{v}_2 + f \mathbf{k} \times \mathbf{v}_2 = -\nabla \phi(\mathbf{z}_2) + g \frac{\theta_2}{\theta_0} \nabla \mathbf{z}_2 + \frac{\mathbf{v}_1 - \mathbf{v}_2}{h_2} W_1^{\text{ral properties}} \end{cases}$

$$\begin{cases}
\partial_t h_1 + \nabla \cdot (h_1 \mathbf{v}_1) = -W_1, \\
\partial_t h_2 + \nabla \cdot (h_2 \mathbf{v}_2) = +W_1 - W_2,
\end{cases}$$

Linking convective fluxes to precipitation I

Bulk humidity: $Q_i = \int_{z_{i-1}}^{z_i} q dz$. Precipitation sink:

$$\partial_t Q_i + \nabla \cdot (Q_i \mathbf{v}_i) = -P_i.$$

In precipitating regions ($P_i > 0$), moisture is saturated $q(z_i) = q^s(z_i)$ and the temperature of the air-mass $W_i dt dx dy$ convected due to the latent heat release $\theta(z_i) + \frac{L}{c_p} q^s(z_i)$, is the one of the upper layer: θ_{i+1} . We assume "dry" stable background stratification:

$$\theta_{i+1} = \theta(z_i) + \frac{L}{c_p}q(z_i) \approx \theta_i + \frac{L}{c_p}q(z_i) > \theta_i,$$

with constant $\theta(z_i)$ and $q(z_i)$.

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Integrating the moist enthalpy we get

$$W_i = \beta_i P_i$$

with a positive-definite coefficient

$$eta_i = rac{L}{c_p(heta_{i+1} - heta_i)} pprox rac{1}{q(z_i)} > 0.$$

Last step: relaxation formula with relaxation time τ .

$$P_i = \frac{Q_i - Q_i^s}{\tau} H(Q_i - Q_i^s)$$

where H(.) is the Heaviside (step) function.

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2-layer model with a dry upper layer

Vertical boundary conditions: upper surface isobaric $z_2 = \text{const}$, geopotential at the bottom constant (ground) $\phi(z_0) = \text{const}, Q_2 = 0, Q_1 = Q$:

$$\begin{cases} \begin{array}{l} \partial_t \textbf{\textit{v}}_1 + (\textbf{\textit{v}}_1 \cdot \nabla) \textbf{\textit{v}}_1 + f \textbf{\textit{k}} \times \textbf{\textit{v}}_1 = -g \nabla (h_1 + h_2), \\ \partial_t \textbf{\textit{v}}_2 + (\textbf{\textit{v}}_2 \cdot \nabla) \textbf{\textit{v}}_2 + f \textbf{\textit{k}} \times \textbf{\textit{v}}_2 = -g \nabla (h_1 + \alpha h_2) + \frac{\textbf{\textit{v}}_1 - \textbf{\textit{v}}_2}{h_2} \beta P \\ \partial_t h_1 + \nabla \cdot (h_1 \textbf{\textit{v}}_1) = -\beta P, \\ \partial_t h_2 + \nabla \cdot (h_2 \textbf{\textit{v}}_2) = +\beta P, \\ \partial_t Q + \nabla \cdot (Q \textbf{\textit{v}}_1) = -P, \end{array} \end{cases}$$
 Ceneral properties of the model conservation law Characteritics and fronts Example: scattering of a simple wave on a moisture front introducing evaporation of the properties of the model conservation and relation to the known models of the model conservation and relation to the known models of the model conservation and relation to the known models of the model conservation and relation to the known models of the model conservation and relation to the known models of the model conservation and relation to the known models of the model conservation to the known models of the model conservation and relation to the known models of the model conservation to the known models of the known models of

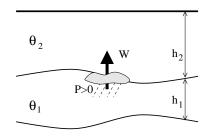
 $\alpha = \frac{\theta_2}{\theta_1}$ - stratification parameter.

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Constructing the model



Sketch of the model



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Immediate relaxation limit

$$au o 0, \; \Rightarrow \; P = -Q^s \nabla \cdot \mathbf{v}_1 \; ext{(Gill, 1982), and}$$
 $\partial_t \mathbf{v}_1 + (\mathbf{v}_1 \cdot \nabla) \mathbf{v}_1 + f \mathbf{k} \times \mathbf{v}_1 = -g \nabla (h_1 + h_2),$ $\partial_t \mathbf{v}_2 + (\mathbf{v}_2 \cdot \nabla) \mathbf{v}_2 + f \mathbf{k} \times \mathbf{v}_2 = -g \nabla (h_1 + \alpha h_2)$ $-\frac{\mathbf{v}_1 - \mathbf{v}_2}{h_2} \beta Q^s \nabla \cdot \mathbf{v}_1,$ $\partial_t h_1 + \nabla \cdot (h_1 \mathbf{v}_1) = +\beta Q^s \nabla \cdot \mathbf{v}_1,$ $\partial_t h_2 + \nabla \cdot (h_2 \mathbf{v}_2) = -\beta Q^s \nabla \cdot \mathbf{v}_1,$

humidity staying at the saturation value: $Q = Q^s$.

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$$m{v}^{bt} = rac{h_1 \, m{v}_1 + h_2 \, m{v}_2}{h_1 + h_2}, \,\, m{v}^{bc} = m{v}_1 - m{v}_2,$$

and linearizing in the hydrodynamic sector gives:

$$\left\{ \begin{array}{l} \partial_t \mathbf{v}^{bc} + f \mathbf{k} \times \mathbf{v}^{bc} = -g_{\mathbf{e}} \nabla \eta, \\ \partial_t \eta + H_{\mathbf{e}} \nabla \cdot \mathbf{v}^{bc} = -\beta P, \\ \partial_t Q + Q_{\mathbf{e}} \nabla \cdot \mathbf{v}^{bc} = -P, \end{array} \right. ,$$

where $g_e = g(\alpha - 1)$, $Q_e = \frac{H_e}{H_1}Q^s$, η - perturbation of the interface, H_e - equivalent height.

Model first proposed by Gill (1982) and studied by Majda et al (2004, 2006, 2008).

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Quasigeostrophic limit

In the small Rossby number limit on the β -plane Lapeyre & Held (2004) model follows:

$$\begin{split} \frac{d_1^{(0)}}{dt} (\nabla^2 \psi_1 + y - \frac{\eta_1}{D_1}) &= \frac{\beta P}{D_1}, \\ \frac{d_2^{(0)}}{dt} (\nabla^2 \psi_2 + y - \frac{\eta_2}{D_2}) &= -\frac{\beta P}{D_2}, \end{split}$$

Here $\frac{d_i^{(0)}}{dt} = \partial_t + (\boldsymbol{v}_i^{(0)} \cdot \nabla)$, $\boldsymbol{k} \times \boldsymbol{v}_i^{(0)} = -\nabla \psi_i$, $D_i = \frac{H_i}{H_0}$, and $\psi_{1,2}$ (geostrophic streamfunctions) are related to the free-surface (η_2) and interface (η_1) perturbations as:

$$\psi_1 = \eta_1 + \eta_2, \quad \psi_2 = \eta_1 + \alpha \eta_2.$$

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1-layer moist-convective RSW

In the limit $H_1/(H_1+H_2) \rightarrow 0$ the reduced-gravity one-layer moist-convective shallow water follows (Bouchut, Lambaerts, Lapeyre & Zeitlin, 2009):

$$\begin{cases} \partial_t \mathbf{v}_1 + (\mathbf{v}_1 \cdot \nabla) \mathbf{v}_1 + f \mathbf{k} \times \mathbf{v}_1 = -\nabla \eta, \\ \partial_t \eta + \nabla \cdot \{\mathbf{v}_1 (1 + \eta)\} = -\beta \mathbf{P}, \\ \partial_t Q + \nabla \cdot (Q \mathbf{v}_1) = -\mathbf{P}, \end{cases}$$

(Nondimensional equations, η - free-surface perturbation)

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Horizontal momentum

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$$(\partial_t + \mathbf{v}_1 \cdot \nabla)(\mathbf{v}_1 h_1) + \mathbf{v}_1 h_1 \nabla \cdot \mathbf{v}_1 + f \mathbf{k} \times (\mathbf{v}_1 h_1)$$

$$= -g \nabla \frac{h_1^2}{2} - g h_1 \nabla h_2 - \mathbf{v}_1 \beta \mathbf{P},$$

$$(\partial_t + \mathbf{v}_2 \cdot \nabla)(\mathbf{v}_2 h_2) + \mathbf{v}_2 h_2 \nabla \cdot \mathbf{v}_2 + f \mathbf{k} \times (\mathbf{v}_2 h_2)$$

$$= -\alpha g \nabla \frac{h_2^2}{2} - g h_2 \nabla h_1 + \mathbf{v}_1 \beta \mathbf{P},$$

Red: moist convection drag. Total momentum: $\mathbf{v}_1 h_1 + \mathbf{v}_2 h_2$ is not affected by convection.

Energy

Energy densities of the layers:

$$\begin{cases} e_1 = h_1 \frac{\mathbf{v}_1^2}{2} + g \frac{h_1^2}{2}, \\ e_2 = h_2 \frac{\mathbf{v}_2^2}{2} + g h_1 h_2 + \alpha g \frac{h_2^2}{2}, \end{cases}$$

For the total energy $E = \int dx dy (e_1 + e_2)$ we get:

$$\partial_t \mathbf{E} = -\int d\mathbf{x} \, \beta P\left(gh_2(1-lpha) + rac{(\mathbf{v}_1 - \mathbf{v}_2)^2}{2}
ight).$$

1st term: production of PE (for stable stratification); 2nd term destruction of KE.

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Potential vorticity

$$(\partial_t + \mathbf{v}_1 \cdot \nabla) \frac{\zeta_1 + f}{h_1} = \frac{\zeta_1 + f}{h_1^2} \beta P,$$

$$(\partial_t + \mathbf{v}_2 \cdot \nabla) \frac{\zeta_2 + f}{h_2} = -\frac{\zeta_2 + f}{h_2^2} \beta P + \frac{\mathbf{k}}{h_2} \cdot \left\{ \nabla \times \left(\frac{\mathbf{v}_1 - \mathbf{v}_2}{h_2} \beta P \right) \right\}$$

where $\zeta_i = \mathbf{k} \cdot (\nabla \times \mathbf{v}_i) = \partial_x \mathbf{v}_i - \partial_y \mathbf{u}_i$ (i = 1, 2)- relative vorticity.

PV in each layer is not a Lagrangian invariant in precipitating regions.

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Conservation laws



Moist enthalpy and moist PV

Moist enthalpy in the lower layer: $m_1 = h_1 - \beta Q$ and is always locally conserved:

$$\partial_t m_1 + \nabla \cdot (m_1 \mathbf{v}_1) = 0.$$

Conservation of the moist enthalpy in the lower layer allows to derive a new Lagrangian invariant, the moist PV:

$$(\partial_t + \mathbf{v}_1 \cdot \nabla) \frac{\zeta_1 + f}{m_1} = 0.$$

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Quasilinear form and characteristic equations

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1-d reduction: $\partial_{\nu}(...) = 0$, \Rightarrow quasilinear system:

$$\partial_t \mathbf{f} + \mathbf{A}(\mathbf{f})\partial_{\mathbf{X}}\mathbf{f} = \mathbf{b}(\mathbf{f}).$$

Characteristic equation: $det(\mathbf{A} - c\mathbf{I}) = 0$

"Dry" characteristic equation

$$\mathcal{F}(c) = \left\{ (u_1-c)^2 - gh_1
ight\} \left\{ (u_2-c)^2 - lpha gh_2
ight\} - gh_1 gh_2$$

▶ "Moist" characteristic equation ($\tau \rightarrow 0$)

$$\mathcal{F}^{m}(c) = \mathcal{F}(c) + ((u_1 - u_2)^2 - (\alpha - 1)gh_2)g\beta Q^s = 0.$$



Characteristic velocities about the rest state

"Dry" characteristics:

$$C_{\pm}=g(H_1+\alpha H_2)\frac{1\pm\sqrt{\Delta}}{2},$$

"Moist" characteristics:

$$C_{\pm}^{m}=g(H_{1}+\alpha H_{2})\frac{1\pm\sqrt{\Delta^{m}}}{2}.$$

Here $C = c^2$ and

$$\Delta = 1 - \frac{4H_1H_2(\alpha - 1)}{(H_1 + \alpha H_2)^2} = \frac{(H_1 - \alpha H_2)^2 + 4H_1H_2}{(H_1 + \alpha H_2)^2}.$$

$$\Delta^m = \Delta + \frac{4(\alpha - 1)\beta Q^s H_2}{(H_1 + \alpha H_2)^2}.$$

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Moist vs dry characteristic velocities

 c^m is real for positive moist enthalpy of the lower layer in the state of rest : $M_1 = H_1 - \beta Q^s > 0$, and

$$C_{-}^{m} < C_{-} < \frac{g(H_{1} + \alpha H_{2})}{2} < C_{+} < C_{+}^{m},$$

for $0 < M_1 < H_1 \Rightarrow$ moist internal (mainly baroclinic) mode propagates slower than the dry one, consistent with observations.

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Discontinuities in dependent variables (no rotation)

Rankine-Hugoniot (RH) conditions (immediate relaxation):

$$\begin{cases} -s[v_1h_1 + v_2h_2] + [u_1v_1h_1 + u_2v_2h_2] = 0, \\ -s[m_1] + [m_1u_1] = 0, \\ -s[h_2] + [h_2u_2 + \beta Q^su_1] = 0. \end{cases}$$

s - propagation speed of the discontinuity. Remark: mass conservation \rightarrow moist enthalpy conservation in the lower layer.

Due to $\lim_{x_s \to a} \lim_{b \to x_s} \int_a^b P = 0$, P does not enter RH conditions for u, v, h.

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Discontinuities in derivatives

RH conditions linearized about the rest state:

$$\begin{cases} (s^2 - C_+)(s^2 - C_-)[\partial_x u_1] = -(\alpha - 1)gH_2g\beta[P], \\ (s^2 - C_+^m)(s^2 - C_-^m)[\partial_x u_1] = -s(\alpha - 1)gH_2g\beta[\partial_x Q]. \end{cases}$$

For a configuration where it rains at the right side of the discontinuity, $P_-=0$ and $P_+=-Q^s\partial_x u_{1+}>0$, there exist five types of precipitation fronts:

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Precipitation fronts

- 1. the dry external fronts, $\sqrt{C_+} < s < \sqrt{C_+^m}$,
- 2. the dry internal subsonic fronts, $\sqrt{C_{-}^{m}} < s < \sqrt{C_{-}}$,
- 3. the moist internal subsonic fronts, $-\sqrt{C_{-}^{m}} < s < 0$,
- 4. the moist internal supersonic fronts, \sqrt{C}

$$-\sqrt{C_+} < s < -\sqrt{C_-},$$

5. the moist external fronts, $s < -\sqrt{C_+^m}$.

This result confirms previous studies within a linear baroclinic model (Frierson *et al*, 2004).

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Wave scattering on a moisture front: setting

Localized internal simple wave centred at $x_P = 2$ and moving eastward:

$$u_1(x,0) = \begin{cases} \sigma(x - x_P)^2 + U_0 & \text{if } -\sqrt{\frac{U_0}{\sigma}} \le x - x_P \le \sqrt{\frac{U_0}{\sigma}}, \\ 0 & \text{otherwise, } U_0 = 0.01, \sigma = -1 \end{cases}$$
(1)

Stationary moisture front at $x_M = 5$, saturated air at the east, unsaturated at the west:

$$Q(x,0) = Q^{s}\{1 + q_{0} \tanh(x - x_{M})H(-x + x_{M})\}, q_{0} = 0.05.$$
(2)

Strong downflow convergence in the lower layer $\rightarrow P > 0$ near the moisture front.

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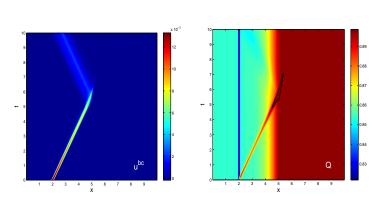
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Wave scattering on a moisture front: baroclinic velocity and moisture



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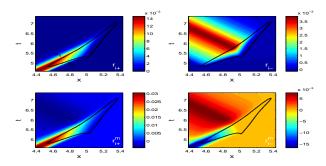
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Wave scattering on a moisture front: condensation zone I



Dry and moist internal Riemann invariants. $s_{1,2}$ -precipitation fronts (dry subsonic and moist supersonic).

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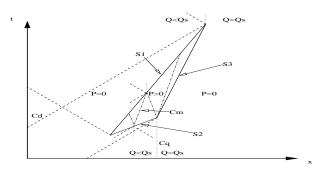
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Characteristics and fronts in the condensation zone



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Evaporation and its parametrizations

In the presence of evaporation source *E*

$$\partial_t Q + \nabla \cdot (Q \mathbf{v}_1) = E - P$$

Hence:

$$\partial_t m_1 + \nabla \cdot (m_1 \mathbf{v}_1) = -\beta \mathbf{E}$$

Simple parametrizations of E (may be combined):

- ► Relaxational: $E = \frac{\hat{Q} Q}{\tau_E} H(m_1)$, where \hat{Q} equilibrium value.
- ▶ Dynamic: $E = \alpha_E |\mathbf{v}_1| H(m_1)$

 m_1 should stay positive (plays a role of static stability)

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Baroclinic Bickley jet

Geostrophically balanced upper-layer jet on the *f*-plane. non-dimensional profiles of velocity and thikness perturbations:

$$ar{u}_1=0,\quad ar{\eta}_1=rac{1}{lpha-1} anh(y), \ ar{u}_2=\mathrm{sech}^2(y),\quad ar{\eta}_2=rac{-1}{lpha-1} anh(y).$$

No deviation of the free surface: $\bar{\eta}_1 + \bar{\eta}_2 = 0$. Parameters: Ro = 0.1, Bu = 10 - typical for atmospheric jets. Lecture 4:
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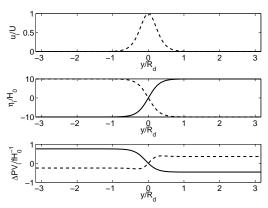
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Bickley jet: zonal velocity \bar{u}_i , thickness deviation $\bar{\eta}_i$ and PV anomaly. Lower (upper) layer: solid black (dashed gray).

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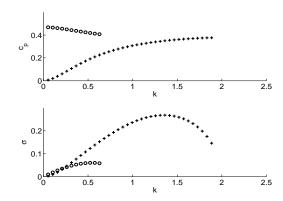
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Linear stability diagram



Phase velocity (top) and growth rate (bottom)

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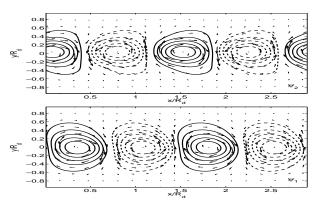
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The most unstable mode



Most unstable mode of the upper-layer Bickley jet. Upper(top) and lower (bottom) layer- geostrophic streamfunctions and velocity (arrows) fields. Lecture 4:
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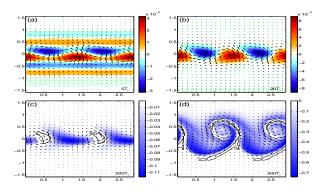
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Early stages: evolution of moisture



Evolution of the moisture anomaly $Q-Q_0$ with superimposed lower-layer velocity. Black contour: condensation zones.

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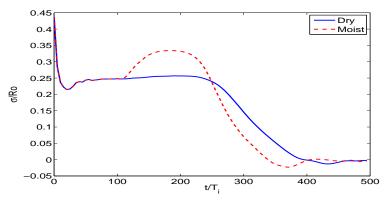
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Early stages: growth rates



Red: moist, blue: dry simulations. \Rightarrow Transient increase in the growth rate due to condensation.

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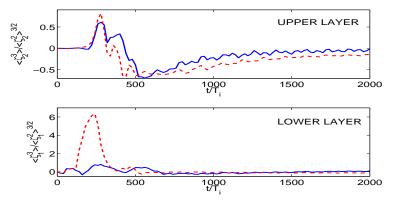
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Cyclone-anticyclone asymmetry



Skewness of relative vorticity. Red: moist, blue: dry simulations.

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How condensation enhances cyclones: 1-layer model

For $Ro \rightarrow 0$ and $Bu \sim O(1)$, close to saturation $\psi \sim \tilde{q} << 1$:

$$(\partial_t + \mathbf{v}^{(0)} \cdot \nabla) \left[\nabla^2 \psi - \psi \right] = \beta P, \tag{3}$$

$$(\partial_t + \mathbf{v}^{(0)} \cdot \nabla) \left[\tilde{q} - Q_s \nabla^2 \psi \right] = -P, \tag{4}$$

 $\mathbf{v}^{(0)} = (-\partial_y \psi, \partial_x \psi)$ - geostrophic velocity, $\psi = \bar{\eta} + \eta$, and \tilde{q} is moisture anomaly with respect to Q_s .

 \Rightarrow PV of the fluid columns which pass through the precipitating regions increases. For $au \to 0$ $\tilde{q} \approx 0$, and:

$$Q_s(\partial_t + \mathbf{v}^{(0)} \cdot \nabla) \left[\nabla^2 \psi \right] \approx P_{\tau \to 0} > 0,$$
 (5)

⇒ increase of geostrophic vorticity in the precipitation regions.

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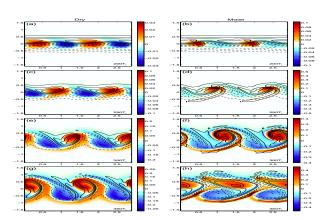
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Dry vs moist simulations: evolution of relative vorticity



Lower layer: colors, upper layer: contours. Condensation: solid black.

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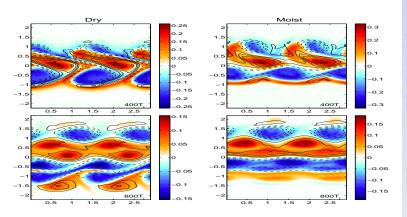
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Dry vs moist simulations: formation of secondary zonal jets at late stages



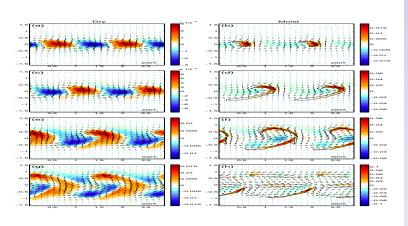


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Unbalanced (aheostrophic) motions: baroclinic divergence





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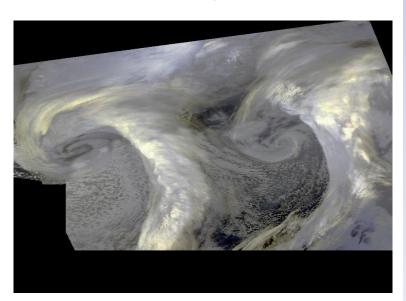
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Moist baroclinic instability in Nature



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The model

- Physically and mathematically consistent
- Simple, physics transparent
- Efficient high-resolution numerical schemes available
- Benchmarks: good

Moist vs dry baroclinic instability

- local enhancement of the growth rate of the moist-convective instability at the precipitation onset,
- significant increase in intensity of ageostrophic motions during the evolution of the moist instability,
- substantial cyclone anticyclone asymmetry, which develops due to the moist convection effects.
- substantial differences in the structure of zonal jets resulting at the late stage of saturation.

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Presentation based on:

- Lambaerts J.; Lapeyre G.; Zeitlin V. and Bouchut F. "Simplified two-layer models of precipitating atmosphere and their properties" *Phys. Fluids* 23, 046603, 2011.
- Lambaerts J.; Lapeyre G. and Zeitlin V. "Moist versus Dry Baroclinic Instability in a Simplified Two-Layer Atmospheric Model with Condensation and Latent Heat Release" J. Atmos. Sci. 69, 1405-1426, 2012.

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