Stratified turbulent flows

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Summary

- Introduction
- Basic concepts for stratified flow
- Governing equations for stratified flows: The Boussinesq approximation
- Turbulence in stratified flows
- Unbounded stable stratified flow
- Wall bounded stable stratified flows
- Particle dispersion in stable stratified flows

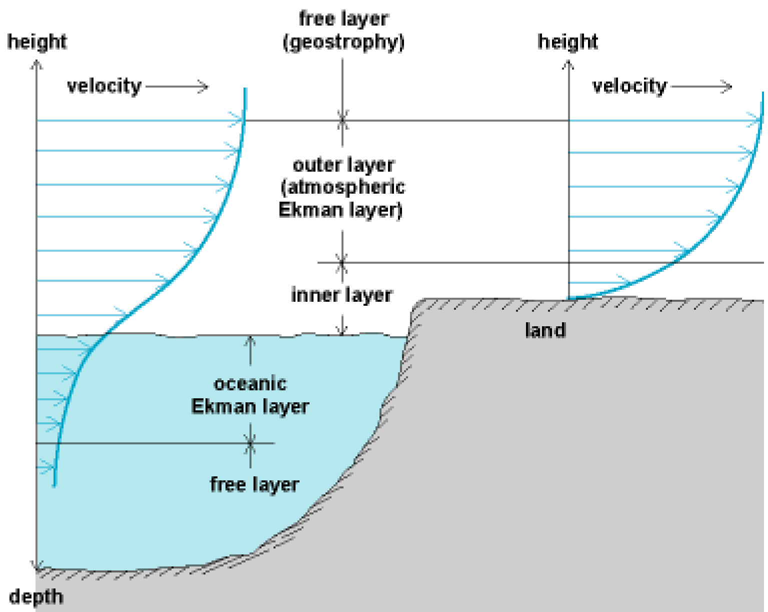
Solution of Environmental problems often requires the knowledge of fluid-mechanics complex processes

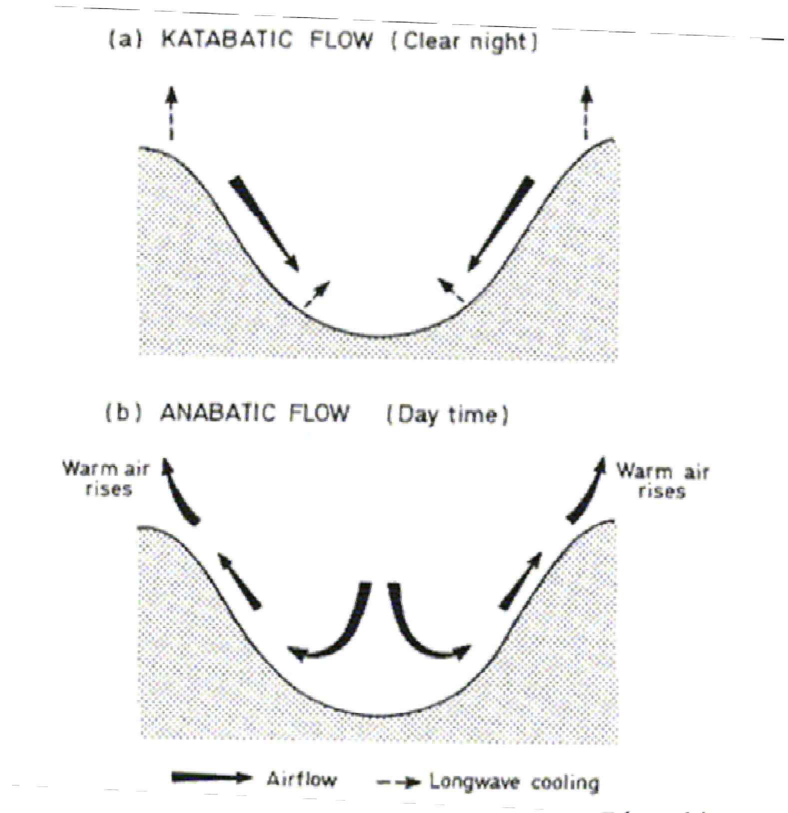
Although quite different from case to case environmental fluid mechanics problems are often characterized by:

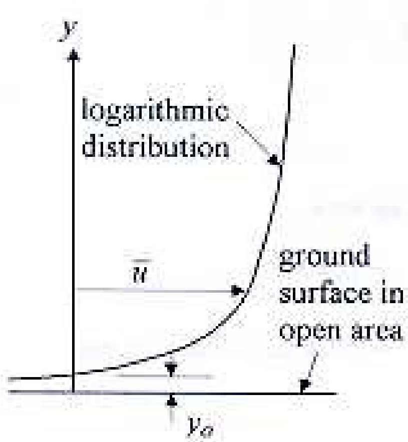
- Vertical variation of temperature and concentration of some active/passive scalars
- Rotation effects at scales of the order of some kilometers
- Presence of plumes, jets, thermals….
- Presence of complex topography ruling mass, momentum and concentration transport and mixing

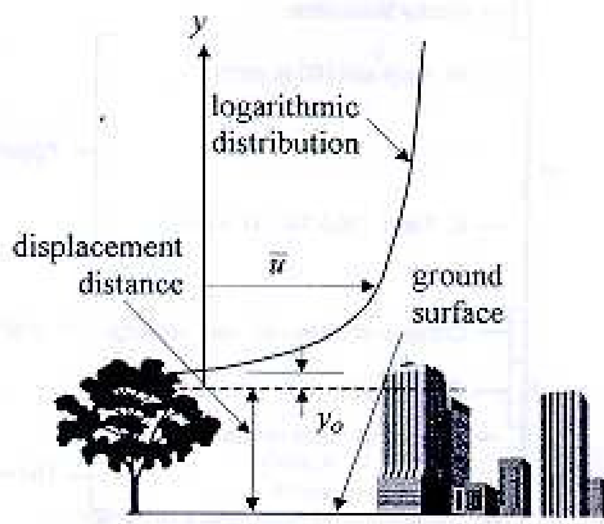
In the present lectures we focus on the effect of stratification, which, on environmental scales, is always present

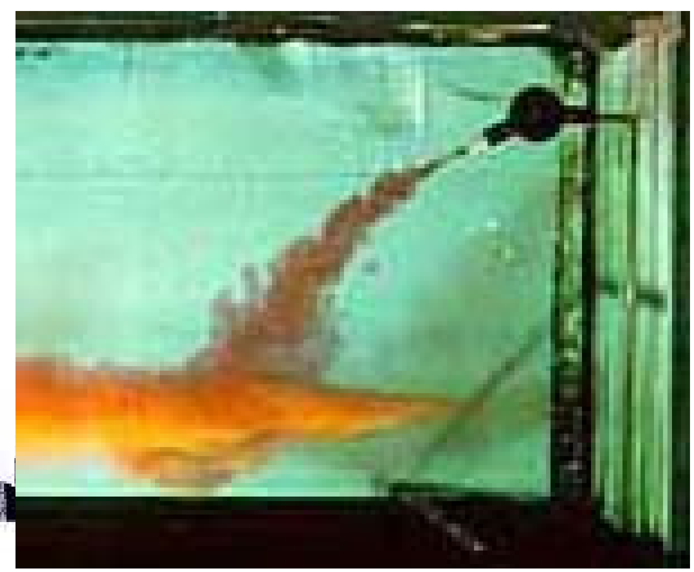
Some nice examples











Some basic concepts

In the general cases fluid density is a function of time and space:

$$\rho = \rho(\vec{x}, t)$$

In complex systems composed of more than one component, fluid density is related to thermodynamic quantities and concentration of dissolved phases as :

$$\rho = \rho(p, T, C_i)$$

p is pressure

T is temperature

C_i is concentration of the *i-th* dissolved specie

Static equilibrium of a fluid column

A fluid is defined as ‘incompressible’ is density is not function of pressure

An incompressible fluid is in a stable equilibrium condition if the density decreases upward

An incompressible fluid is in an unstable equilibrium condition if the density increases upward

An incompressible fluid is in neutral equilibrium condition if the density is constant

In a compressible medium the neutral equilibrium condition occurs if *entropy is constant* in the fluid column

Static equilibrium of a compressible fluid column

In a fluid, pressure decreases with height as

$$\boxed{\frac{\partial p}{\partial z} = -\rho g}$$

In a compressible medium density decreases with height because of the change of pressure

If a fluid parcel is moved upward it expands and thus reduces its density.

The fluid column is in neutral equilibrium if, in the new position, the fluid parcel finds the same density as that gained during its own expansion.

This happens if the displacement is done isoentropically.

As a consequence a fluid column is in neutral equilibrium if entropy is constant with the height

The lapse rate

Consider the case of a monophase fluid, obeying the perfect gas law

It can be shown that the adiabatic change of temperature with height is :

$$\boxed{\frac{dT_a}{dz} \equiv \Gamma_a = -\frac{g}{C_p}}$$

This, the *lapse rate*, is the largest rate at which temperature can decrease without causing instability in the fluid column

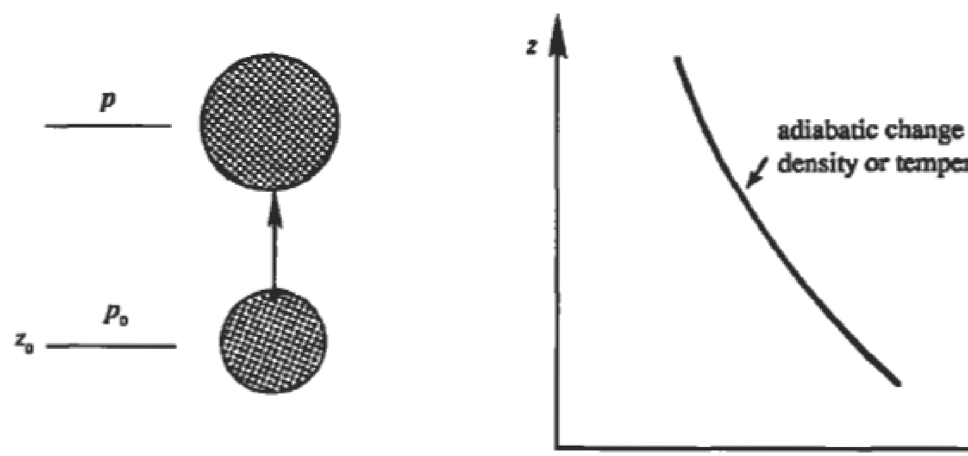


Figure 1.8 Adiabatic expansion of a fluid particle displaced upward in a compressible medium.

For atmosphere at normal temperature and pressure $\Gamma_a = 10°C / Km$

The potential temperature and density

Suppose pressure and temperature at a certain height are *T* and *p*

The potential temperature is the temperature attained by the fluid parcel if taken adiabatically at a reference pressure p_s

The relationship between the actual temperature and the potential one is

$$T = \theta \left(\frac{p}{p_s} \right)^{\frac{\gamma - 1}{\gamma}}$$

θ	Potential temperature
$\gamma = C_p / C_v$	Ratio between specific heats

The potential density is the density attained by the fluid parcel if taken adiabatically at a reference pressure p_s

The relationship between the potential density and temperature is

$$-\frac{1}{\rho_\theta} \frac{d\rho_\theta}{dz} = \frac{1}{\theta} \frac{d\theta}{dz}$$

Stability of a compressible fluid column

Considering the hydrostatic relationship and the perfect gas law, after some math:

$$\frac{T}{\theta}\frac{d\theta}{dz} = \frac{dT}{dz} + \frac{g}{C_p} = \Gamma - \Gamma_a$$

The fluid column is in neutral conditions if the temperature gradient is equal to the lapse rate. It follows:

$$\frac{d\theta}{dz} > 0 \quad \text{stable} \quad \frac{d\rho_\theta}{dz} < 0$$

$$\frac{d\theta}{dz} = 0 \quad \text{neutral} \quad \frac{d\rho_\theta}{dz} = 0$$

$$\frac{d\theta}{dz} < 0 \quad \text{unstable} \quad \frac{d\rho_\theta}{dz} > 0$$

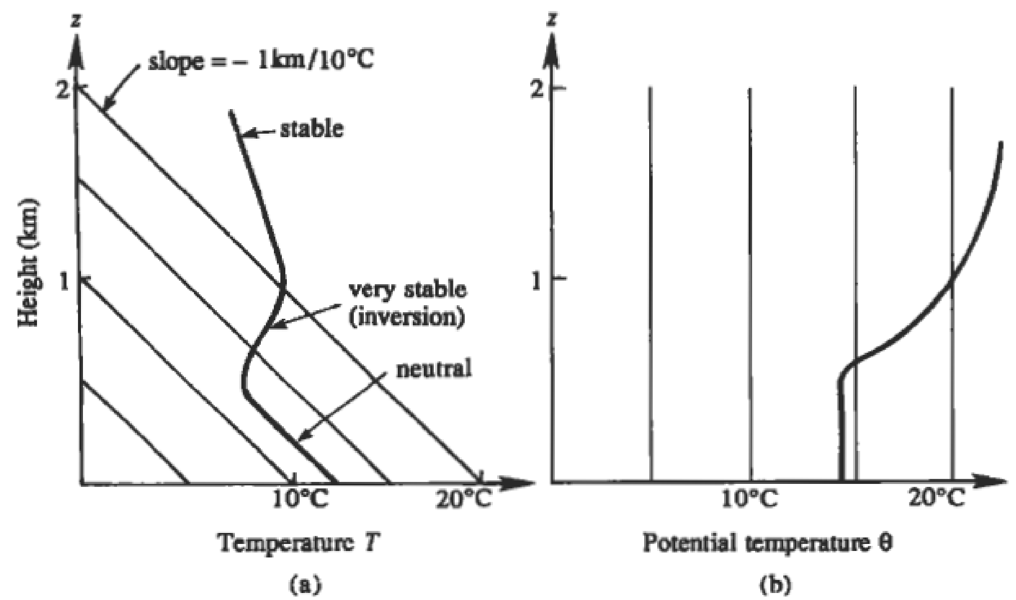


Figure 1.9 Vertical variation of the (a) actual and (b) potential temperature in the atmosphere. Thin straight lines represent temperatures for a neutral atmosphere.

Stability in the ocean

In the ocean is more convenient to express the stability using potential density

It is defined as the density attained by a fluid parcel if taken at a reference pressure isoentropically and at a constant salinity

Take advantage of the concept of speed of sound $c^2 = \partial p/\partial\rho$ defined at constant entropy and salinity

A uniform state is such that $dp_a = c^2 d\rho_a$

The density in a neutral state changes according to

$$\frac{d\rho_a}{dz} = \frac{1}{c^2}\frac{dp_a}{dz} = \frac{1}{c^2}(-\rho_a g) = -\frac{\rho g}{c^2}$$

The stability in the ocean is determined by the sign of the potential density

$$\frac{d\rho_{pot}}{dz} = \frac{d\rho}{dz} - \frac{d\rho_a}{dz} = \frac{d\rho}{dz} + \frac{\rho g}{c^2}$$

Note that $\rho g / c^2 \approx 1000\left[Kg/m^3\right]10\left[m/s^2\right] \Big/ 10^6\left[m^2/s^2\right] = 10^{-2}\left[Kg/m^3\,\frac{1}{m}\right]$

How important are compressibility effects?

There are three main situations where compressibility effects must be considered:

- Steady high speed flows (*Ma=U/c >0.3*). In this case pressure changes cause strong density changes in the flow. Since *c=350 m/s* in air and *c=1470 m/s* in water, in EFM usually *Ma<<0.3*
- Unsteady flows, for rapidly variable phenomena (see for example the water hammer problem)
- When the vertical scale of the phenomenon is so large that hydrostatic pressure changes the fluid density. For air, density variations can be neglected when the vertical scale *L<<* $c^2/g \sim 10\,\mathrm{km}$

In all other cases, compressibility effects can be neglected and the governing equations can be simplified accordingly, keeping in mind that we must consider the potential density to account for the stability of the fluid column.

Governing equations for fluid motion with weak density variations: The Boussinesq approximation (1)

Continuity equation for a compressible medium:

$$\frac{1}{\rho}\frac{D\rho}{Dt} + \nabla \cdot \mathbf{u} = 0,$$

When $\frac{\partial \rho}{\partial t} << u\frac{\partial \rho}{\partial x}$, if *U, L*, and $\delta\rho$ are respectively typical velocity

length and density scales of the process, we can write

$$\frac{(1/\rho)(D\rho/Dt)}{\nabla \cdot \mathbf{u}} \sim \frac{(1/\rho)u(\partial\rho/\partial x)}{\partial u/\partial x} \sim \frac{(U/\rho)(\delta\rho/L)}{U/L} = \frac{\delta\rho}{\rho} = \alpha\delta T \ll 1,$$

It follows that continuity equation becomes $\nabla \cdot \mathbf{u} = 0.$

The density variations practically do not change the fluid volume

Governing equations for fluid motion with weak density variations: The Boussinesq approximation (2)

$\nabla \cdot \mathbf{u} = 0.$ implies simplifications in the momentum equation:

$$\rho \frac{D\mathbf{u}}{Dt} = -\nabla p + \rho \mathbf{g} + \mu \nabla^2 \mathbf{u}.$$

Consider an hypothetical density state ρ_0 and an associated static pressure field p_0, such that $\frac{\partial p_0}{\partial z} = -g\rho_0$, if $p=p_0+p'$ and $\rho=\rho_0+\rho'$

$$\left(1 + \frac{\rho'}{\rho_0}\right) \frac{D\mathbf{u}}{Dt} = -\frac{1}{\rho}\nabla p' + \frac{\rho'}{\rho_0}\mathbf{g} + \nu \nabla^2 \mathbf{u},$$

If ρ'/ρ_0 << 1 and inertial accelerations are much smaller than gravity, the density perturbation ρ' appears only in the gravitational term, and it cannot be neglected.

Governing equations for fluid motion with weak density variations: The Boussinesq approximation (3)

The thermal energy equation is:

$$\rho \frac{De}{Dt} = -\nabla \cdot \mathbf{q} - p(\nabla \cdot \mathbf{u}) + \phi.$$

with $e = C_v T$, $q = -k\nabla T$ and ϕ viscous heating

Although the velocity divergence is negligible the term $p(\nabla \cdot \mathbf{u})$ may be comparable to the other terms of the energy equation:

$$-p\nabla \cdot \mathbf{u} = \frac{p}{\rho}\frac{D\rho}{Dt} \simeq \frac{p}{\rho}\left(\frac{\partial \rho}{\partial T}\right)_{\mathrm{p}} \frac{DT}{Dt} = -p\alpha \frac{DT}{Dt}$$

Using the perfect gas law, after some math the energy equation is

$$\rho C_{\mathrm{p}} \frac{DT}{Dt} = -\nabla \cdot \mathbf{q} + \phi,$$

Under the B-approximation ϕ is negligible and the equation assumes the form of an advection-diffusion equation of the temperature field

$$\frac{DT}{Dt} = \kappa \nabla^2 T,$$ with $\kappa \equiv k/\rho C_{\mathrm{p}}$ the thermal diffusivity

The Boussinesq approximation: *summary*

When the density variations in the fluid column are small, under the following circumstances:

- small velocity scale
- slow processes
- vertical scale small compared to c^2/g

The Navier-Stokes equations assume the following form:

$$\nabla \cdot \vec{u} = 0$$

$$\frac{D\vec{u}}{Dt} = -\frac{1}{\rho_0}\nabla p + \nu \nabla^2 \vec{u} - g\frac{\rho}{\rho_0}$$

$$\frac{D\rho}{Dt} = k\nabla^2 \rho$$

Note that since, $\dfrac{\Delta\rho}{\rho_0} = -\alpha \Delta T$, the energy equations can be written for the (potential) density

The Boussinesq approximation: more active scalars

In many applications we have more than one stratifying agent:

In the ocean we usually have (potential) temperature and salinity

In the atmosphere we usually have (potential) temperature and humidity

In these cases we need evolution equations for both scalars.

Continuity and momentum equations remain unchanged.

Additionally we have:

$$\frac{DT}{Dt} = k_T \nabla^2 T$$

$$\frac{DC}{Dt} = k_C \nabla^2 C$$

$$\frac{\Delta\rho}{\rho_0} = -\alpha \Delta T + \beta \Delta C$$

With β the coefficient of cubic expansion for the scalar considered

Turbulence in stratified flows mean and fluctuating field

The Ba-NS equations rule the motion of a laminar as well as a turbulent fluid column, provided that all the turbulence scales are resolved

Let's operate the Reynolds decomposition:

$$u_i = U_i + u'_i$$
$$p = P + p'$$
$$\rho = \Upsilon + \rho'$$

In flow fields where density is homogeneous in the horizontal planes, we can further decompose the mean pressure, as the sum of a mean hydrodynamic pressure and an hydrostatic pressure field in equilibrium with the mean density field

$$P = P_H + P_\rho$$

$$-\nabla P_\rho - g\Upsilon = 0$$

The Reynolds Average NS Equations of turbulent stratified flows: one active scalar

$$\frac{\partial U_j}{\partial x_j} = 0$$

$$\frac{\partial U_i}{\partial t} + \frac{\partial U_i U_j}{\partial x_j} = -\frac{1}{\rho}\frac{\partial P}{\partial x_i} + \nu \frac{\partial^2 U_i}{\partial x_j \partial x_j} - \frac{\partial \overline{u'_i u'_j}}{\partial x_j}$$

$$\frac{\partial \mathrm{Y}}{\partial t} + \frac{\partial \mathrm{Y} U_j}{\partial x_j} = \kappa \frac{\partial^2 \mathrm{Y}}{\partial x_j \partial x_j} - \frac{\partial \overline{\rho' u'_j}}{\partial x_j}$$

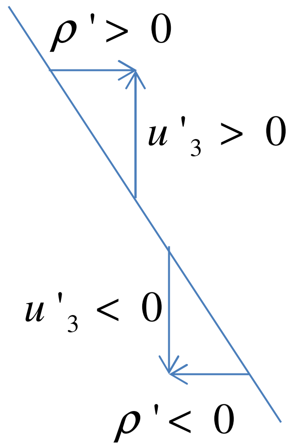
The buoyancy fluxes $\overline{\rho' u_i}$ $(i = 1,2,3)$ affect the mean density field

Apparently there is not feedback effect of potential density (apart the modification of the mean pressure field) on momentum.

The scalar affects the turbulent mean field through modification of the Reynolds stresses

The vertical buoyancy flux

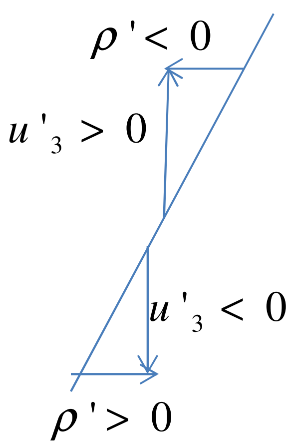
It has to be interpreted as vertical flux of mass associated to fluctuations of temperature or concentration



$$\frac{\partial \rho}{\partial z} < 0 \quad , \quad \frac{\partial T}{\partial z} > 0$$

$$\overline{u'_3 \rho'} > 0 \quad , \quad \overline{u'_3 T'} < 0$$

Stable stratified case



$$\frac{\partial \rho}{\partial z} > 0 \quad , \quad \frac{\partial T}{\partial z} < 0$$

$$\overline{u'_3 \rho'} < 0 \quad , \quad \overline{u'_3 T'} > 0$$

Unstable stratified case

The TKE equation for stratified flows

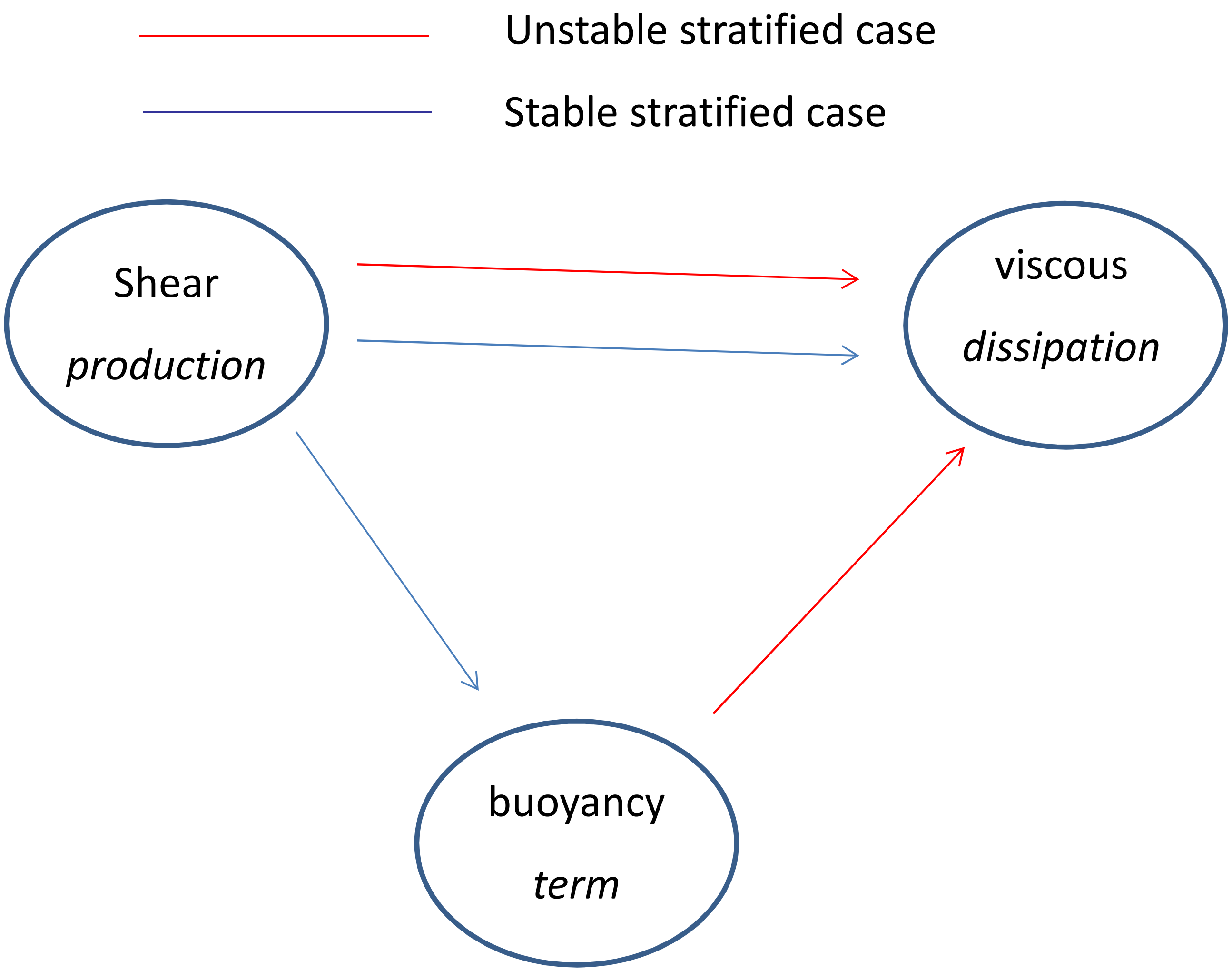
$$\frac{\partial}{\partial t}K+\underbrace{\frac{\partial}{\partial x_k}U_k K}_{advection}=-\underbrace{\left(\overline{u'_i u'_j}\frac{\partial U_i}{\partial x_j}\right)}_{production}-\underbrace{\frac{\partial}{\partial x_k}\overline{u'_i u'_i u'_k}}_{triple\ \ correlation}$$

$$-\underbrace{\frac{\partial}{\partial x_k}\left(\overline{p'u'_k}\right)}_{pressure\ \ diffusion}+\underbrace{\nu\frac{\partial}{\partial x_k}\frac{\partial}{\partial x_k}K}_{\substack{viscous\\ diffusion}}-\underbrace{2\nu\overline{\frac{\partial u'_i}{\partial x_k}\frac{\partial u'_i}{\partial x_k}}}_{\substack{viscous\\ dissipation}}-\underbrace{\delta_{i3}\frac{g}{\rho_0}\overline{\rho' u'_3}}_{buoyancy\ \ \ prod./destr.}$$

In stable stratified case $\overline{\rho' u_3} > 0$, buoyancy acts as destruction term

In unstable stratified case $\overline{\rho' u_3} < 0$, buoyancy produces TKE

Source/Sink terms in the TKE equation



Unstable stratification: *quick summary*

Both shear stress and buoyancy term contribute to produce turbulent kinetic energy

Strong mixing characterizes the fluid column, with the presence of convective cells spanning the whole height of the fluid column

Unstable conditions produce convective motion able to carry biological and chemical matter from the bottom surface toward the top layers of the fluid

This has strong implications, depending on the environmental process investigated.

Unstable stratification: *typical cases in the low atmosphere*

The atmospheric boundary layer under typical diurnal conditions. In this case heating of the ground produces instability of the fluid column and rapid mixing in the low atmosphere.

- From one side this condition allows the transport of pollutants in the higher levels of the atmosphere and reduces their concentration at the ground level

- From the other side, pollutants released at a certain height through chimneys are pushed down by the convective cells at the ground level increasing the concentration of pollutants

Unstable stratification: *typical cases in water basins*

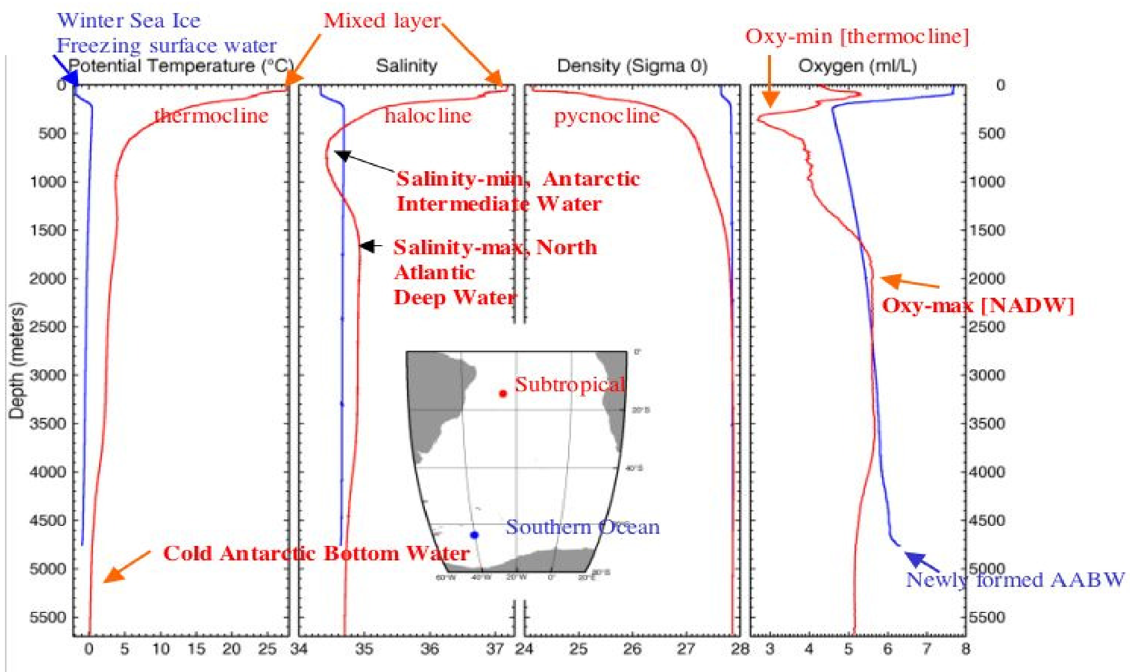
Rapid cooling of the upper surface of the ocean or of a lake, producing unstable conditions and the formation of descending plumes of cold water, from one side, and ascending plumes of warm water from the other side

- In the oceans this produces rapid vertical mixing salinity, heat, nutrients and chemical species

- In lakes, the uprising of nutrients, typically relegated in the lower layer, may cause the over-production of algae and rapid consume of oxygen in the water column

Stable stratification: *an overview*

In the oceans stable stratification inhibits vertical transport of:
-Chemical species (Oxygen)
-Nutrients

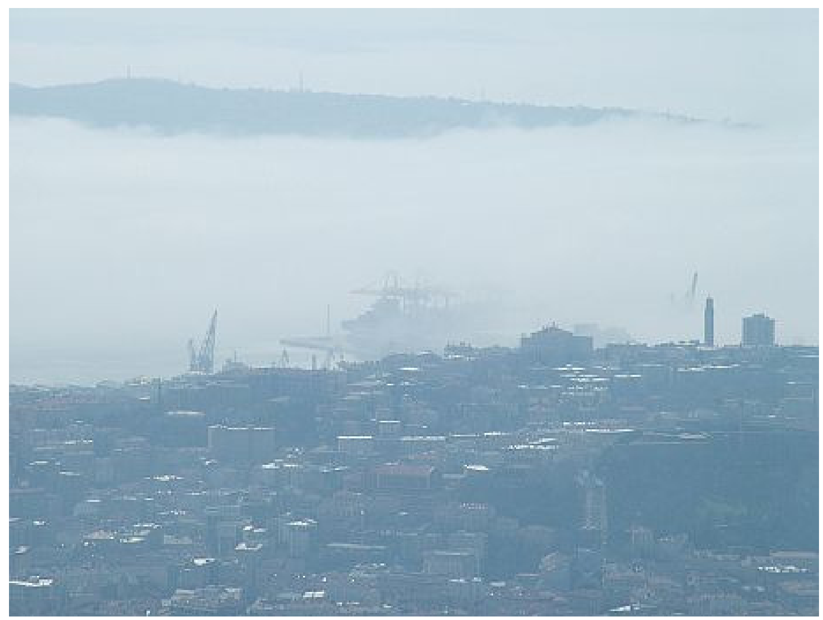


Stable stratification: *an overview*

In atmosphere applications, stable stratification (thermal inversion) suppresses vertical transport of polluting particulate released at the ground level

Thermal inversion during late Autumn:

The Val Badia (Trentino) and the City of Trieste



The Brunt-Vaisala frequency

Consider a fluid parcel with density ρ_0 in an environment with a vertical distribution of density $\rho(z)$.
Let's move the fluid parcel by a small quantity z'. The 2nd Newton law says:

$$\rho_0 \frac{d^2 z'}{dt^2} = -g\left[\rho_0 - \rho(z')\right] \approx g \frac{d\rho}{dz} z'$$

We obtain the equation of the harmonic oscillator:

$$\frac{d^2 z'}{dt^2} + N^2 z' = 0 \qquad (1)$$

With:

$$N^2 = -\frac{g}{\rho_0}\frac{d\rho}{dz}$$

The Brunt-Vaisala frequency

Unstable stratification $\qquad \dfrac{d\rho}{dz} > 0 \qquad N^2 = i\left|N^2\right|$

Neutral stratification $\qquad \dfrac{d\rho}{dz} = 0 \qquad N^2 = 0$

Stable stratification $\qquad \dfrac{d\rho}{dz} < 0 \qquad N^2 > 0$

- Under unstable stratification the solution of Eq.1 gives an exponential solution (the fluid parcel, once moved from its initial solution definitively moves far from the initial position)

- Under neutral stratification the fluid parcel remains in the new position

- Under stable stratification the fluid parcel oscillates around the initial position with a frequency smaller than or equal to N.

Stable stratification: *theoretical background*

The RICHARDSON NUMBER *Ri* rules the response of a turbulent field to the effect of stratification.

$$Ri = \frac{Potential \quad energy}{Kinetic \quad energy}$$

Different forms of the Richardson number are often in use:

$$Ri_g = \frac{N^2}{S^2} = \frac{-\dfrac{g}{\rho_0}\dfrac{d\rho}{dz}}{\left(\dfrac{du}{dz}\right)^2}$$

$$Ri_\tau = \frac{gh\Delta\rho}{\rho_0 u_\tau^2}$$

$$Ri_f = \frac{B}{P} = \frac{\dfrac{g}{\rho_0} <\rho' w'>}{<u' w'> \dfrac{du}{dz}} = \frac{Ri_g}{\mathbf{Pr}_T}$$

Stable stratification: *theoretical background*

The theory of Miles (J. Fluid Mech., 1961) gives

$$Ri_g > 0.25$$

For the linear stability of a system with the mean shear S aligned with the mean density gradient N_z

Observations and numerical results give a value of the mixing efficiency:

$$\eta = \frac{B}{\varepsilon} \approx \frac{B}{P} = Ri_f = \mathbf{0.15 - 0.20}$$

Stable stratification: theoretical background

The turbulent Prandtl number rules the amount of turbulent mixing of the potential density compared to the turbulent mixing of momentum

$$\mathrm{Pr}_T = \frac{\nu_T}{\kappa_T}$$

For a passive scalar (*i.e.* a concentration of a dispersed pollutant) Reynold's analogy suggests:

$$\mathrm{Pr}_T \approx 1$$

This holds in stable stratified flows approximately for

When

$$Ri_g < 0.15 \div 0.20$$

$$Ri_g > 0.20 \qquad \rightarrow \qquad Pr > 1$$

Momentum mixing larger than scalar mixing in strongly stratified regimes

The turbulent Prandtl number

When

$$Ri_g > 0.20 \qquad \rightarrow \qquad Pr > 1$$

In the buoyancy dominated regime, *Ri* beyond the critical value *0.20*, the Prandtl number has a superlinear behavior

This is due to the presence of internal waves, able to transfer momentum in the vertical direction but not able to transfer mass and heat.

Studies of stable stratified turbulence

In stable stratified flows there is the competing effect between the mechanical production of turbulent kinetic energy and buoyancy destruction

Here after we describe laboratory-scale numerical experiments aimed at understanding the dynamics of turbulent stable stratified flows

Among the variety of investigations we focus on studies where the numerical simulations are performed using:

- Direct numerical simulation (DNS)
- Large eddy simulation (LES)

In DNS all the turbulence scale are solved directly, through numerical integration of the unsteady 3-dimensional Navier-Stokes Equations

In LES the large energy carrying scales of motion are resolved through numerical integration of the unsteady 3-dimensional Navier-Stokes Equations, whereas the more isotropic, universal and small scales of motion are parameterized by means of a subgrid-scale (SGS) model

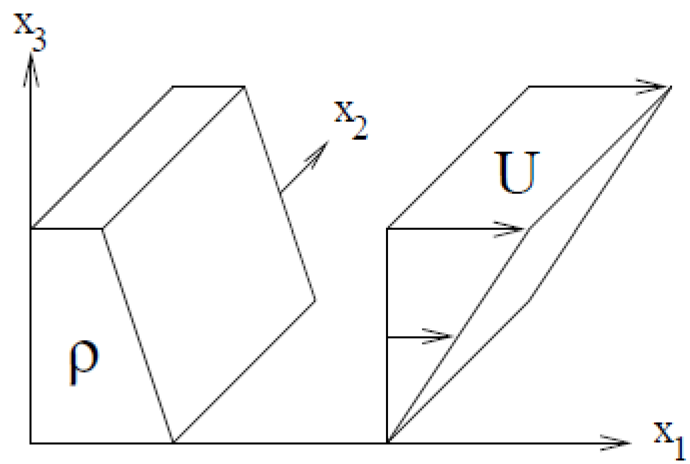
Studies of stable stratified turbulence

We mention numerical studies with increasing complexity:

- Homogeneous turbulence with vertical shear and vertical density gradient
- Plane channel flow with horizontal isothermal walls
- Free surface channel flow with imposed heat fluxes
- Homogeneous turbulence with inclined shear and vertical density gradient
- Plane channel flow with vertical isothermal walls
- Particle dispersion in a stratified wind driven Ekman layer
- Particle dispersion in a stratified bottom Ekman layer

Free shear stratified flows

The simplest case where we can study the effect of competition between mechanical production and buoyancy destruction is the problem depicted below



The gradient Richardson number

$$Ri_g = \frac{g/\rho_0 \, \dfrac{d\rho}{dz}}{\dfrac{dU}{dz}} = \frac{N^2}{S^2}$$

is constant in space and time

Free shear stratified flows

DNS performed by
Gerz et al., J.Fluid Mech., 1989;
Holt et al., J. Fluid Mech., 1992;
Jacobitz et al., 1997
Jacobitz and Sarkar, 1999
Diamessis and Nomura, 2000

LES performed by
Kaltenback et al., 1994

Laboratory experiments carried out by
Rohr et al. 1988, J.Fluid Mech., salty stratified water in a laboratory flume,
Piccirillo and van Atta, J. Fluid Mech., 1995 thermally stratified water flume

Hereafter we discuss results obtained by Sarkar and co-workers using DNS

Homogeneous stratified turbulence: *numerical features*

Using the Rogallo transformation, the Boussinesq form of the NS-equations, can be written for the fluctuating velocity, density and pressure fields

Taking advantage of homogeneity in the three directions for the fluctuating field, simulations have been carried out over a triple periodic box

Fractional step algorithms employed for the integration of the unsteady NS-equations

Different space-discretization methods have been used (finite differences, spectral collocation method)

Homogeneous stratified turbulence: *main features*

This flow is inherently unsteady

In the neutral case, turbulent kinetic energy grows exponentially with time

$$K = K_0 \exp\{\gamma S t\}$$

with

$S = \dfrac{dU}{dz}$ the background shear and

γ the growth rate of K

Homogeneous stratified turbulence: *parameters*

The gradient Richardson number $$Ri_g = \frac{N^2}{S^2}$$

The Prandtl number $$\Pr = \frac{\nu}{k}$$

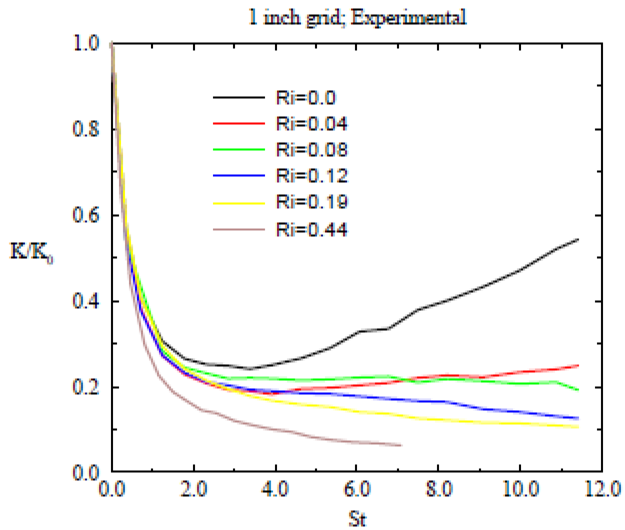
The Reynolds number $$\mathrm{Re}_\lambda = \frac{u\lambda}{\nu}$$

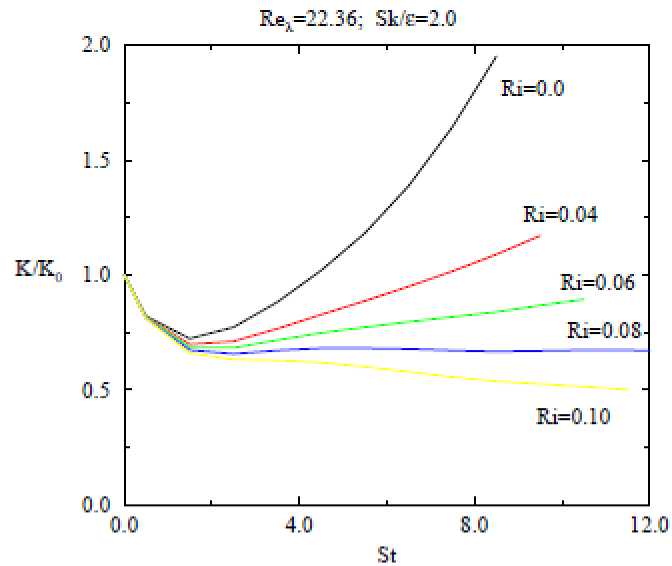
The Froude numbers $$Fr_h = u / Nl \qquad Fr_v = \frac{w}{Nl}$$

The shear number $$\frac{SK}{\varepsilon} \text{ or } \frac{Sl}{u}$$

u is a velocity scale, i.e. $\sqrt{K}$, *l* is an integral length scale, λ is the Taylor microscale

Homogeneous stratified turbulence: *K(t)*





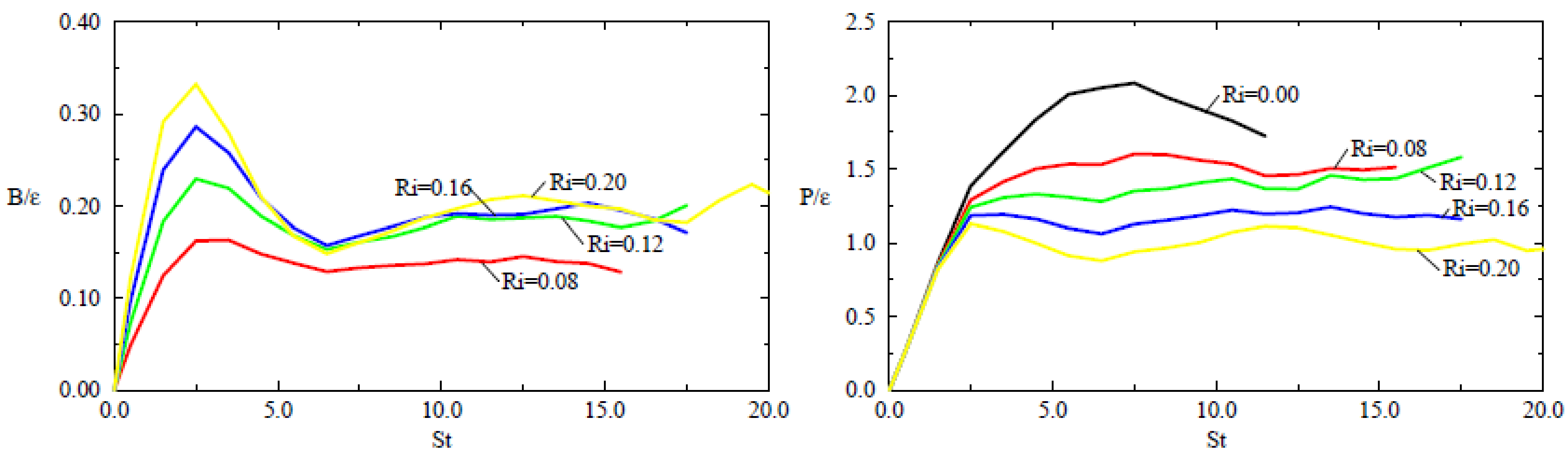
Laboratory experiments (van Atta and co-workers) and DNS studies (Jacobitz, Sarkar and van Atta, 1997) show that stratification inhibits the growth of *K*

K grows or decays in time, depending on the value of Ri_g

There is a critical value such of Ri_g such that *K* remains constant in time

Ri_g is function of Re_λ and SK/ε

Homogeneous stratified turbulence: *K(t)* budget



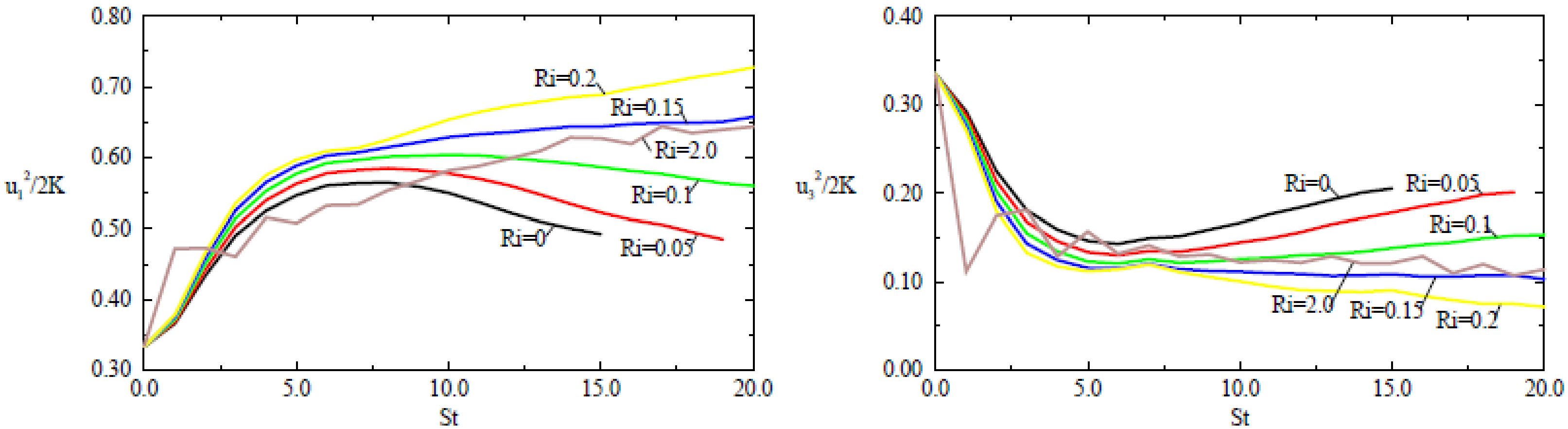
Beyond $Ri_{g,crit}$ the B/ε ratio saturates around 0.20

P/ε decreases with increasing level of stratification, quantified by Ri_g .

Production decreases due to suppression of the Reynolds shear stress $\overline{u'w'}$

Part of energy goes into generation of internal waves (reversible process)

Homogeneous stratified turbulence: *anisotropy*



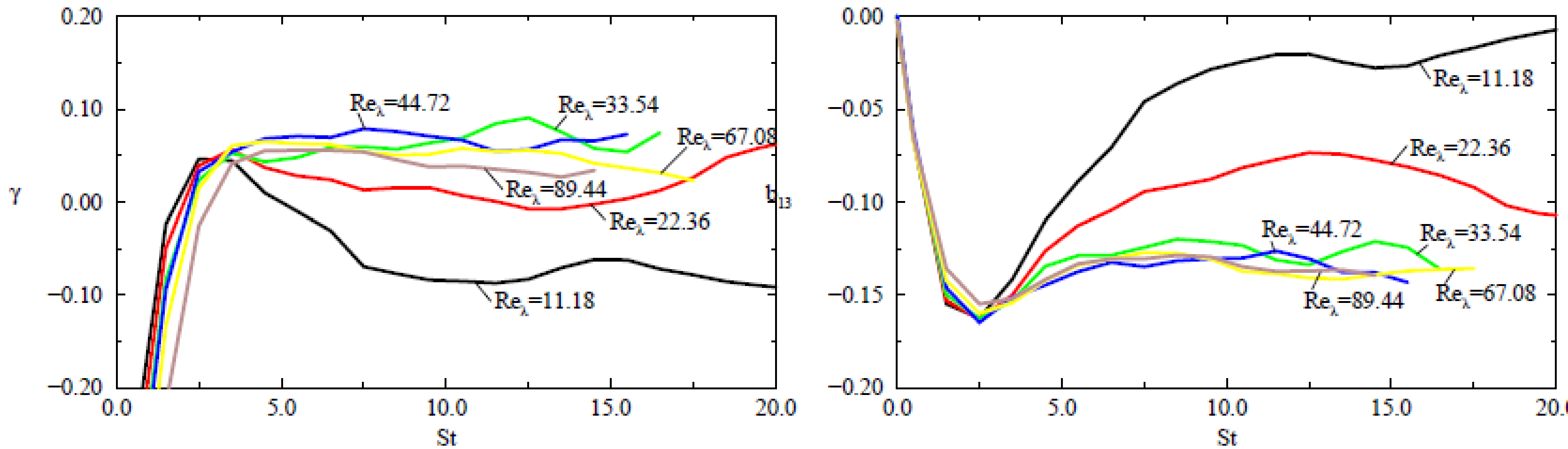
With increasing stratification the turbulent kinetic energy is mainly composed of horizontal fluctuations

This is due to the fact that stratification destroys pressure-strain correlation $\overline{p's'_{ii}}$ implying that turbulent kinetic energy produced along the streamwise direction is less transferred over the vertical direction, where buoyancy destruction acts

Dimensional analysis (see Sarkar and co-workers 1997,1999) suggests that 2D-turbulence asymptotic behavior is not reached . The authors showed that

$$\frac{u'_3}{u'_1} \approx Fr_w \quad \text{and } Fr_w \approx O(1) \text{ for } Ri_g \to \infty$$

Homogeneous stratified turbulence: *Re-number effects*



Sarkar and co-workers (1997,1999) observed that:

The growth rate of *K* $$\gamma = \frac{1}{SK}\frac{dK}{dt}$$

The Reynolds stress anisotropy $$b_{13} = \frac{\overline{u'_1 u'_3}}{2K}$$

are nearly independent on Re_λ provided that its value is large enough

Wall-Bounded Stratified Turbulence (WBST)

Homogeneous stratified turbulence is representative of physical situations where turbulence develops in the core of the fluid column

In most situations turbulence develops close to solid walls and inhomogeneity is present in the wall normal direction

In this case turbulent statistics vary along the vertical.

$$S=S(z),\ Ri_g = Ri_g(z)\ ,\ N=N(z)\ ....$$

An additional complication derives from the boundary conditions on the thermal field. They vary from case to case and affect the fate of stratified turbulence

Here we discuss two simple, yet representative, cases:

- Plane channel flow with horizontal walls, vertical stratification and imposed temperature at the solid walls (Armenio and Sarkar, J. Fluid Mech., 2002)

- Free-surface horizontal channel, vertical stratification, imposed heat flux at the free surface and adiabatic bottom wall (Taylor et al., Phys. Fluids, 2005)

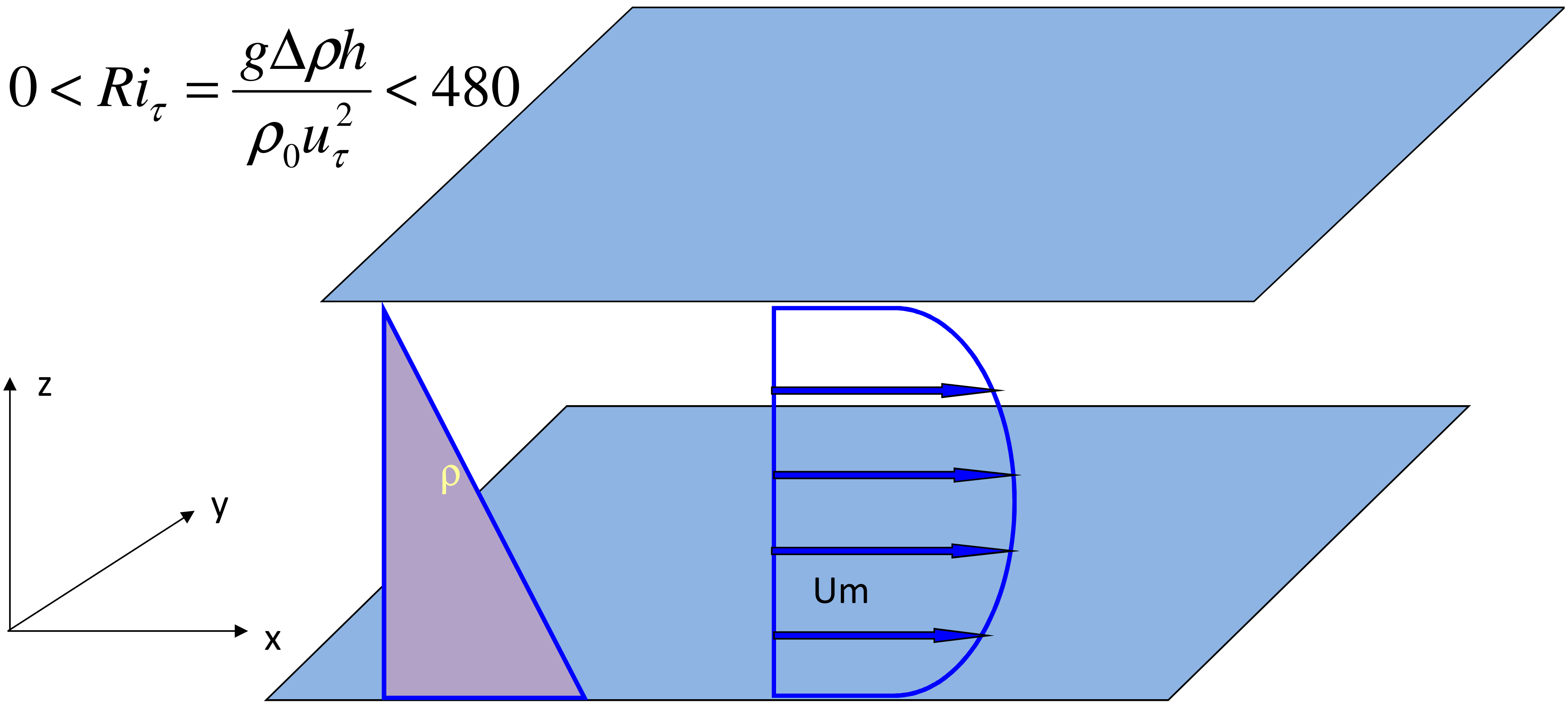
Horizontal stable stratified plane channel flow (SSPCF): *boundary conditions*

This case may be archetypal of the atmospheric boundary layer, under thermal inversion

$$\mathrm{Re}_\tau = 180$$

$$\mathrm{Pr} = 0.71$$

$$0 < Ri_\tau = \frac{g\Delta\rho h}{\rho_0 u_\tau^2} < 480$$



HSSPCF: *literature survey*

Laboratory experiments carried out by

Komori et al., J. Fluid Mech., 1983. Stable stratified channel flow in an horizontal water flume.

LES performed by

Garg et al., Phys. Fluids, 2000. The authors analyzed:
the response of the system under weak stratification conditions,
the transient under strong stratification

Armenio and Sarkar, J. Fluid Mech., 2002 (AS02). The authors analyzed the final response of the system, under a variety of conditions.

DNS performed by

Garcia-Villalba and del Ãlamo Phys. Fluids, 2011: The authors used larger domain size larger Re number and larger stratification levels compared to AS02

HSSPCF: *purposes of the AS02 study*

To understand the state of turbulent motion under strong stable stratification

How do profiles of relevant quantities adjust under stable stratification?

What is the more significant form of the Richardson number for parameterization purposes among the following ones?

Friction *Ri* number

$$Ri_\tau = \frac{g\Delta\rho h}{\rho_0 u_\tau^2}$$

Bulk *Ri* number

$$Ri_b = \frac{g\Delta\rho h}{\rho_0 u_b^2}$$

Gradient *Ri* number

$$Ri_g(z) = \frac{N^2(z)}{S^2(z)}$$

HSSPCF: *AS02 numerical model*

-Boussinesq approximation of the Navier-Stokes equations employed under the assumption that the density variations are small when compared to the bulk density of the flow

-Filtered Navier-Stokes equations solved using the second-order accurate, finite difference algorithm of Zang et al.,J. Comp. Phys., 1994

-Dynamic mixed model employed for the parameterization of the SGS momentum fluxes (Armenio and Piomelli, FTC, 2000)

-Dynamic eddy viscosity model employed for the parameterization of the SGS density fluxes

-The model constant is averaged over the planes of homogeneity

-The wall layer is directly resolved (no wall-layer modeling)

HSSPCF: *theoretical background*

Main features:

-Due to the presence of imposed wall temperature, the buoyancy flux $<\rho' w'>$ (and consequently) the destruction term $-g<\rho' w'>/\rho_0$ is large where the maximum production of turbulent kinetic energy occurs;

-Behavior of the gradient Richardson number along the fluid column:

$$Ri_g\,|_{wall} = Ri_\tau \frac{Nu}{2\mathrm{Re}_\tau^2} << 1$$

$$Ri_g(z) = Ri_\tau \frac{\kappa^2 Nu}{2\,\mathrm{Pr}\,\mathrm{Re}_\tau\,\kappa_\rho} \frac{z}{h} \quad \text{(within the log layer)}$$

$$Ri_g\,|_{centerline} = \infty$$

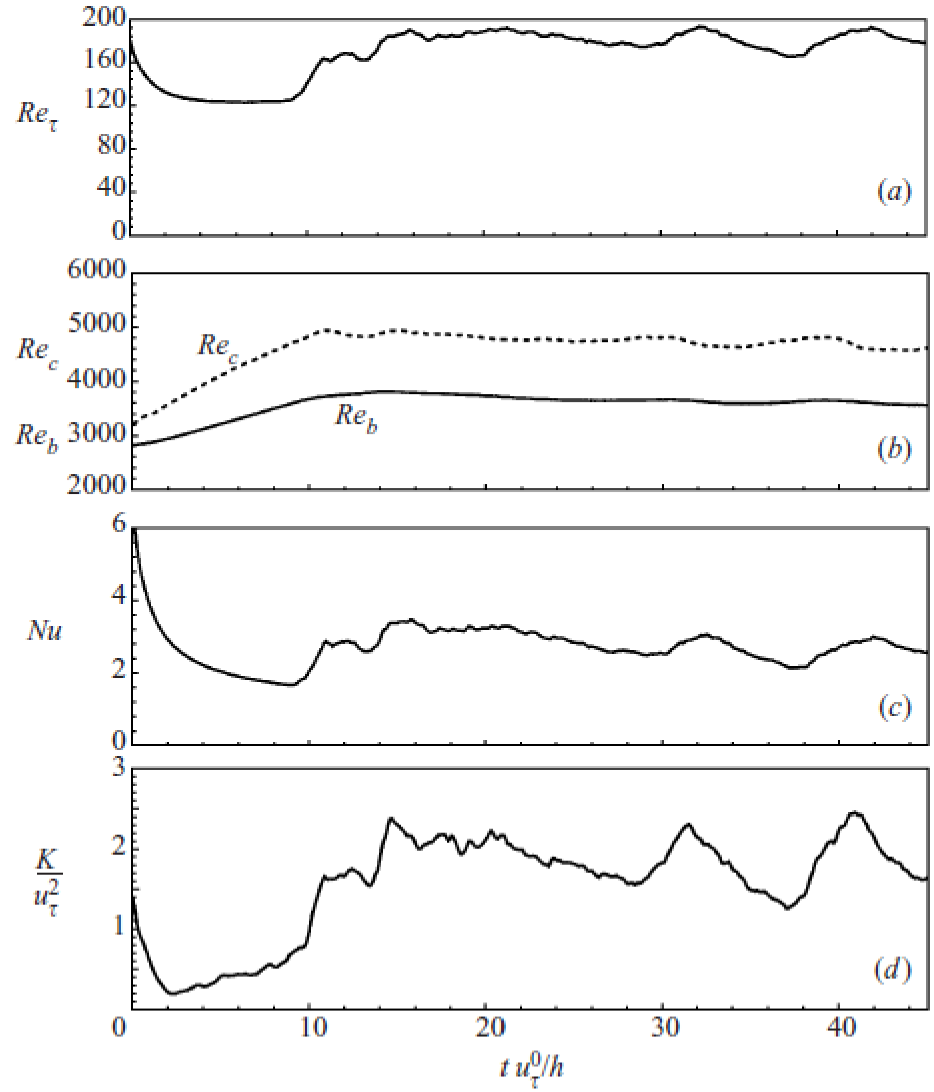
Practically, it looks like $0 < Ri_g \le \infty$!!!

HSSPCF: *transient toward a steady state*

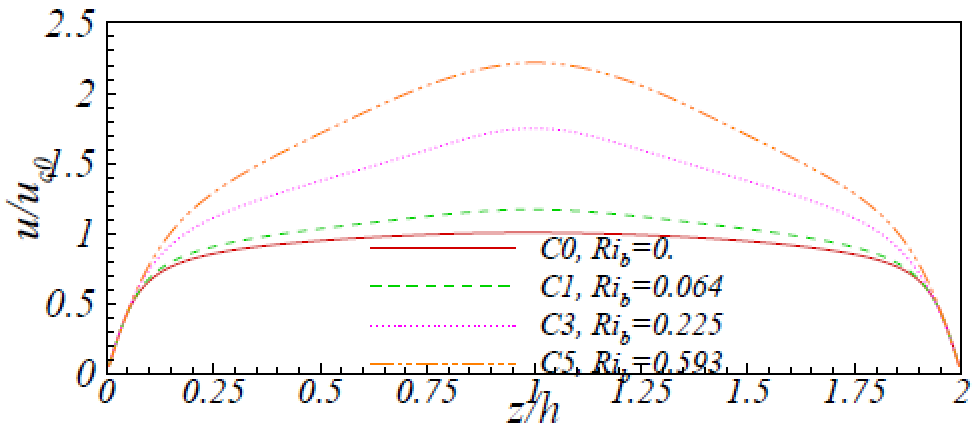
The simulations were initialized from a steady case at smaller Ri_b number and imposing a larger value of Ri_b

The figure depicts the behavior of some bulk quantities for the case Ri_b =0.0685

Partial re-laminarization is observed during the early stages, followed by a sudden re-transition to turbulence



HSSPCF: *the integral balance*



The simulations were run imposing a constant mean driving pressure gradient

The integral balance gives:

$$2h\Pi = 2\tau_w \quad \Rightarrow \quad \tau_w = h\Pi$$

The wall shear stress does not change from case to case, since it depends on the imposed driving pressure gradient.

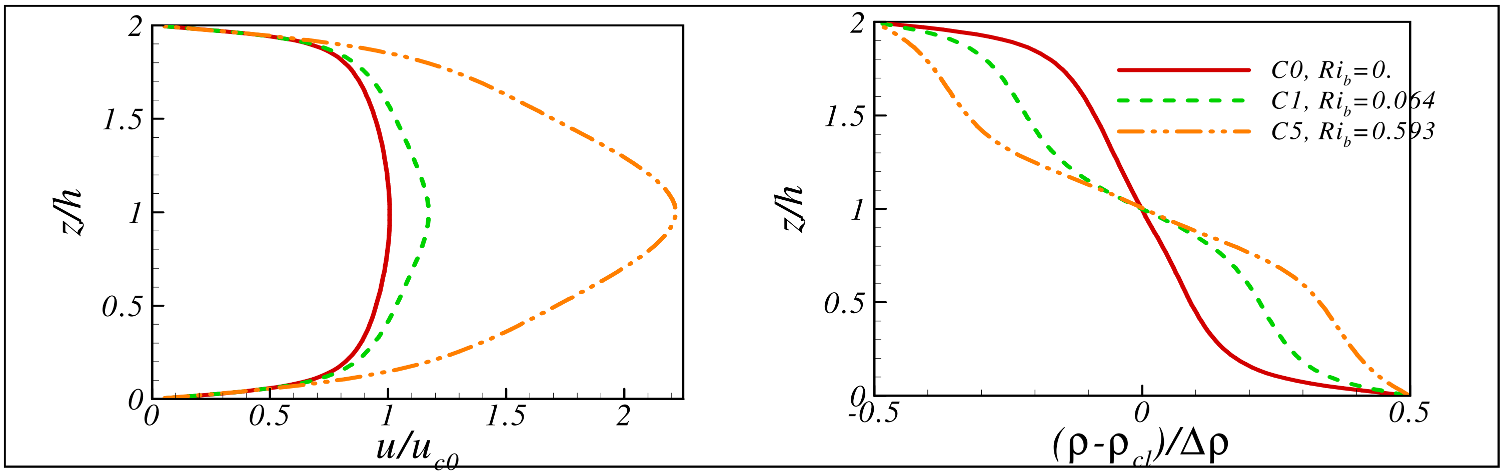
The increase of stratification increases the flow rate and thus decreases the friction coefficient

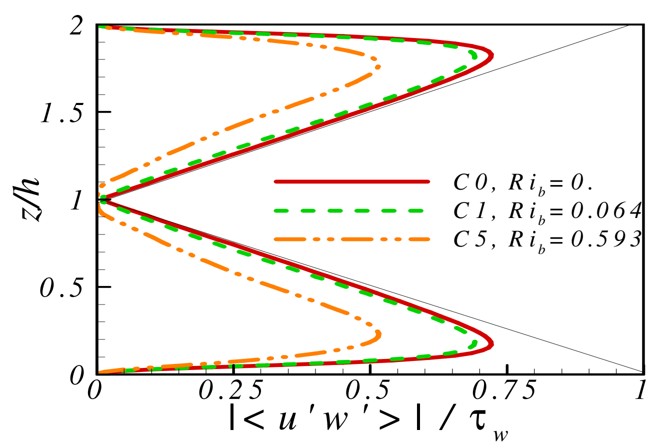
$$C_f = 2\left({u_\tau}/{u_b} \right)^2$$

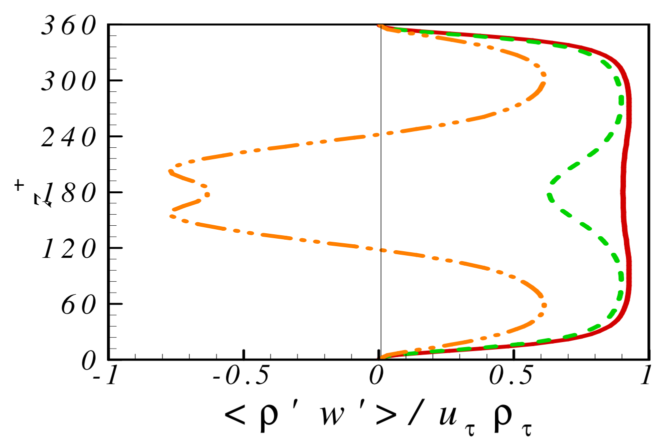
HSSPCF: *the mean field*

$$\left| -\langle u'w' \rangle + \nu \frac{\partial u}{\partial z} \right| = \tau_w * |z-1|$$

$$-\langle \rho' w' \rangle + \kappa \frac{\partial \rho}{\partial z} = \cos t$$



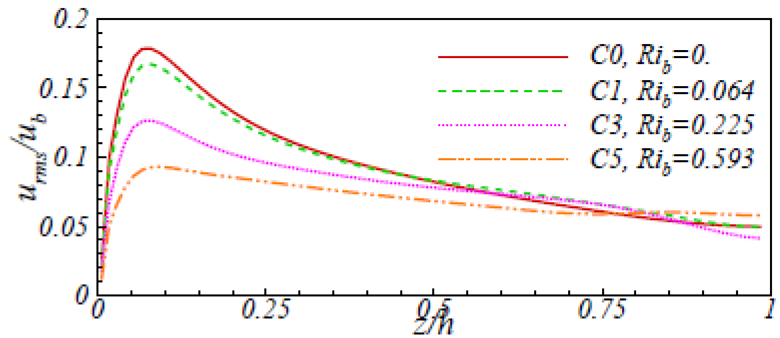


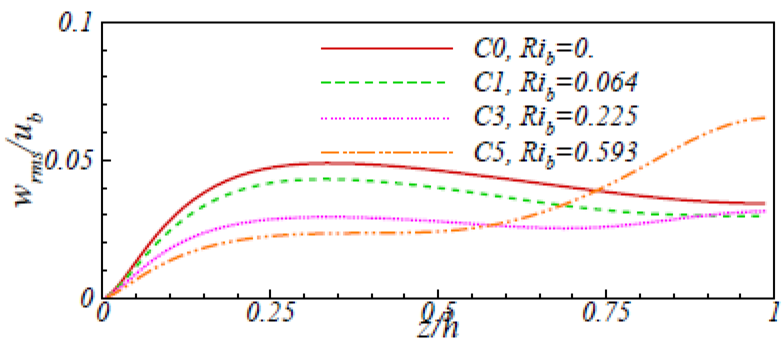


HSSPCF: *the normal Reynolds stresses*

Increasing stratification reduces the level of fluctuations in the three directions in the near wall region

In strongly stratified cases increased vertical fluctuations in the core of the channel indicate the presence of internal waves

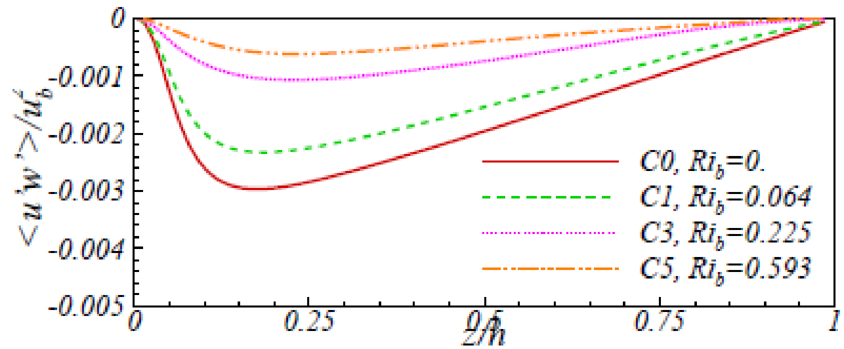




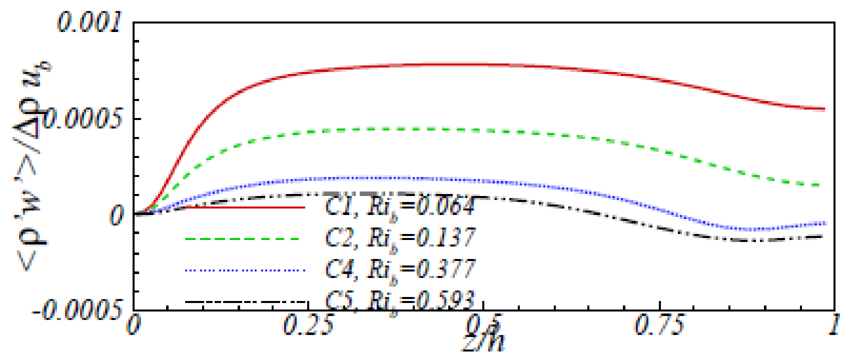
HSSPCF: *the turbulent vertical fluxes*

Increasing stratification reduces:

- cross-correlation between horizontal and vertical velocity fluctuations
- Buoyancy fluxes in the vertical direction



In strongly stratified cases counter-gradient buoyancy fluxes present in the core region (in agreement with Komori et al. experiments)

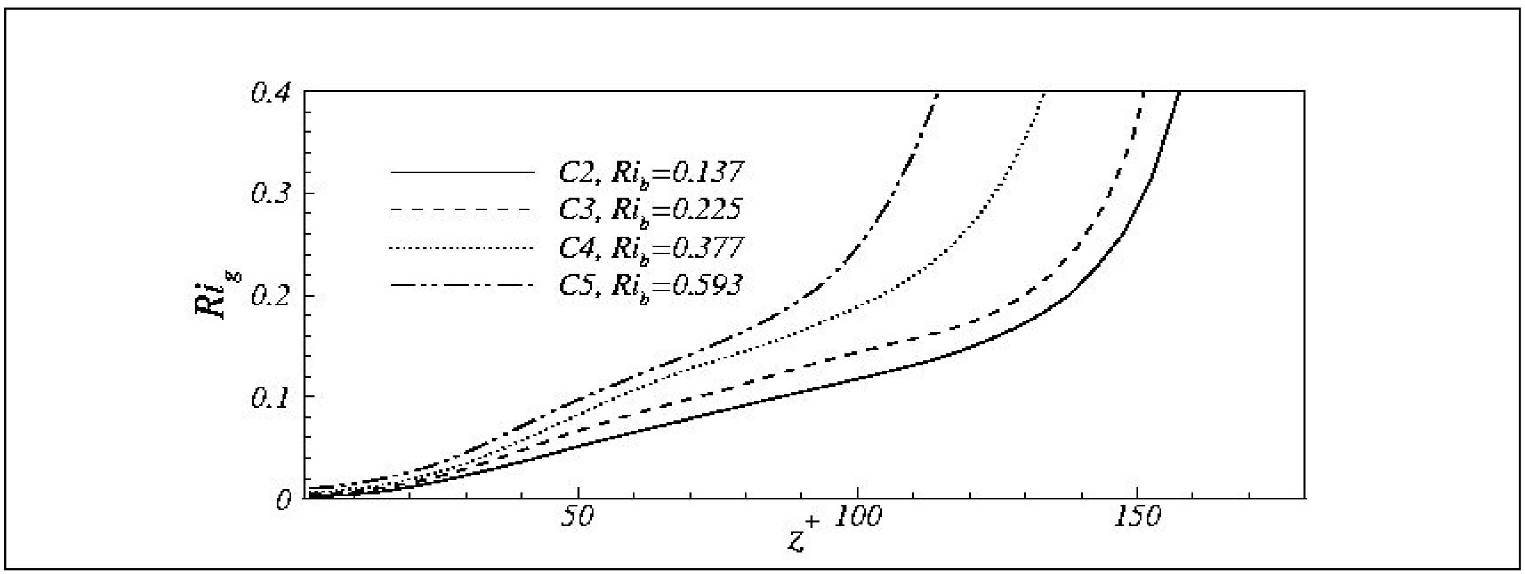


HSSPCF: *the gradient Richardson number*

Ri_g was found to vary from values close to *0* (at the wall) to infinite (centerline)
It was found to increase linearly in the core region

The slope of $Ri_g(z)$ changes when its value is *0.20-0.25*

The vertical location where $Ri_g(z)=0.20\text{-}0.25$ moves closer to the wall with increased stratification



HSSPCF: *…versus Ri_g*

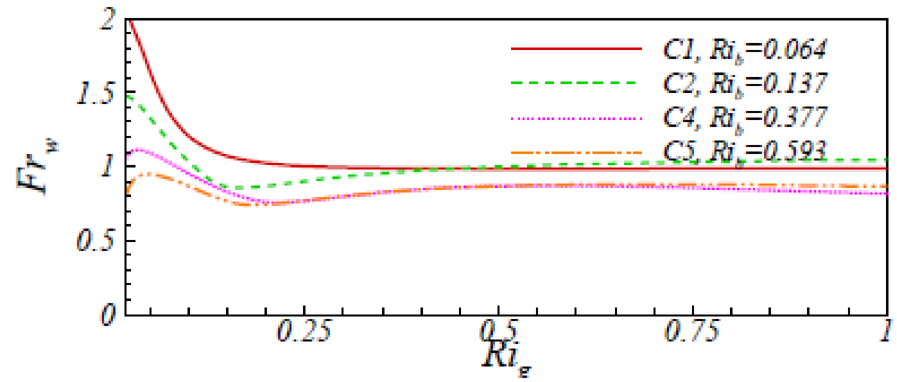
We now plot relevant quantities as a function of Ri_g

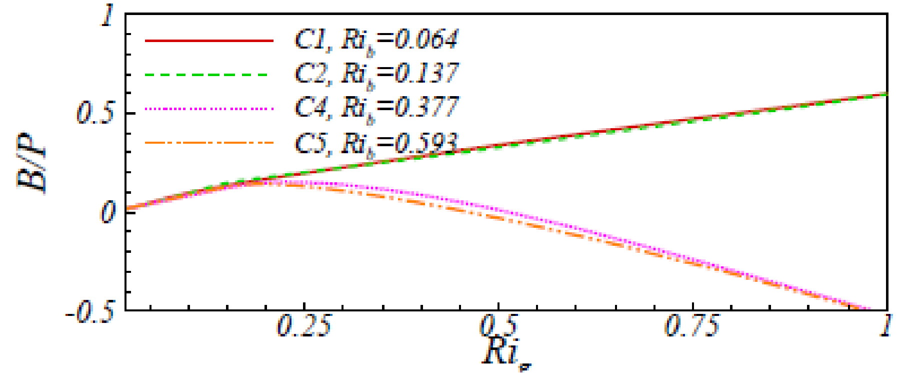
The vertical Froude number collapses to values *O(1)* in the strongly stratified Regions, similarly to the HT case

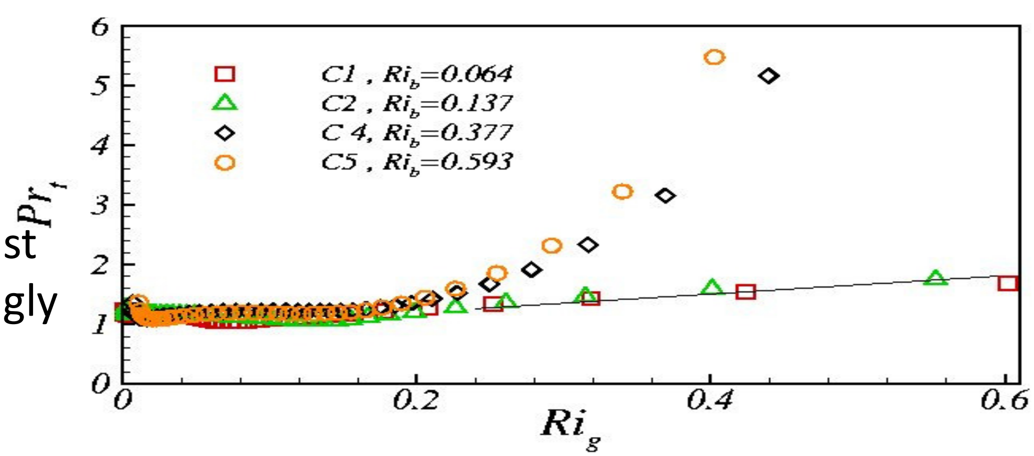
In the strongly stratified cases, the ratio *B/P*, a surrogate of the mixing efficiency, peaks to *0.2* for $Ri_g=0.20\text{-}0.25$

The turbulent Prandtl number is *O(1)* for $Ri_g=0.20\text{-}0.25$, according to the Reynolds analogy.
It has a superlinear behavior for $Ri_g>0.25$

The SGS dynamic model is able to adjust to the flow conditions also in the strongly stratified regions



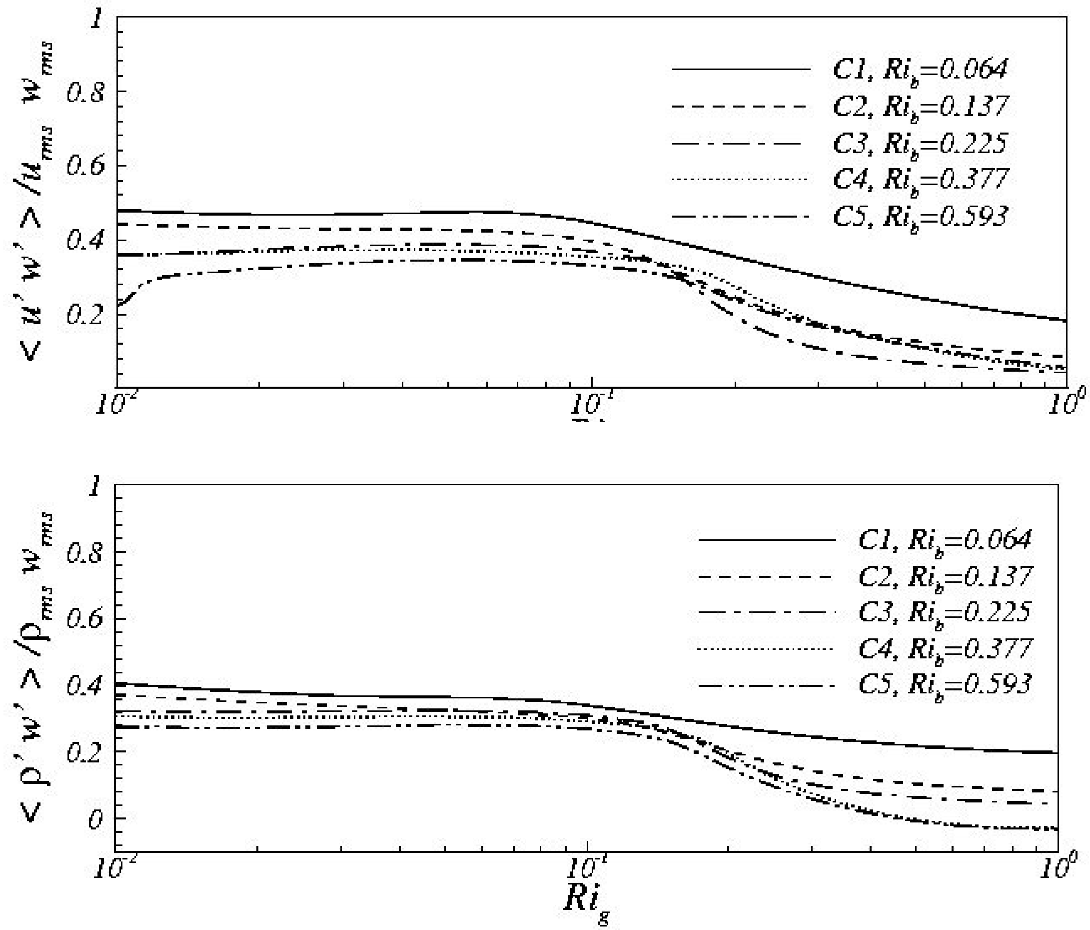




HSSPCF: *correlations versus* Ri_g

The correlation coefficients
of momentum and mass abruptly decay for $Ri_g > 0.2$

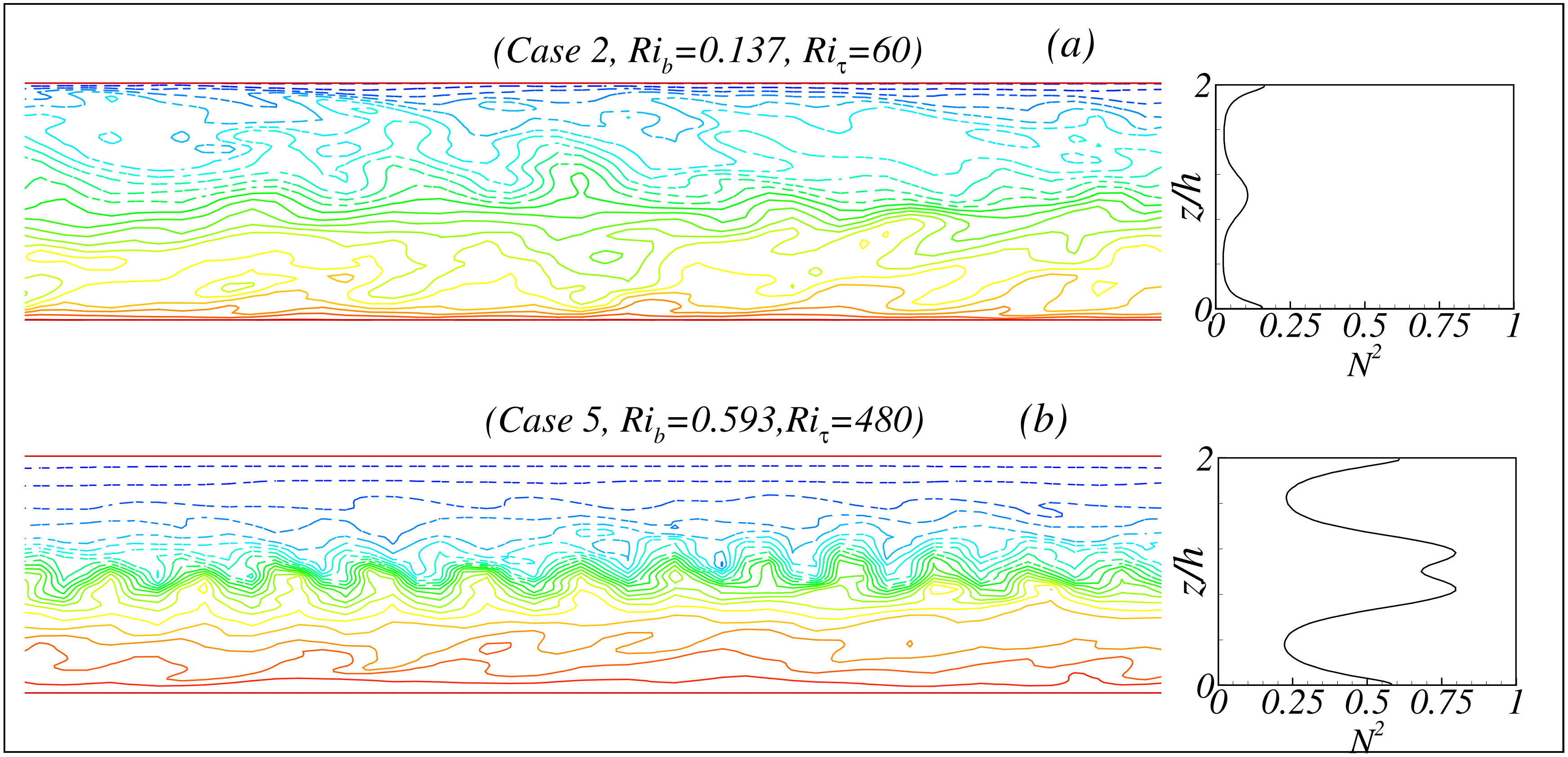
They also depend on the overall level of stratification (Ri_τ)



HSSPCF: *identification of flow regions versus Ri_g*

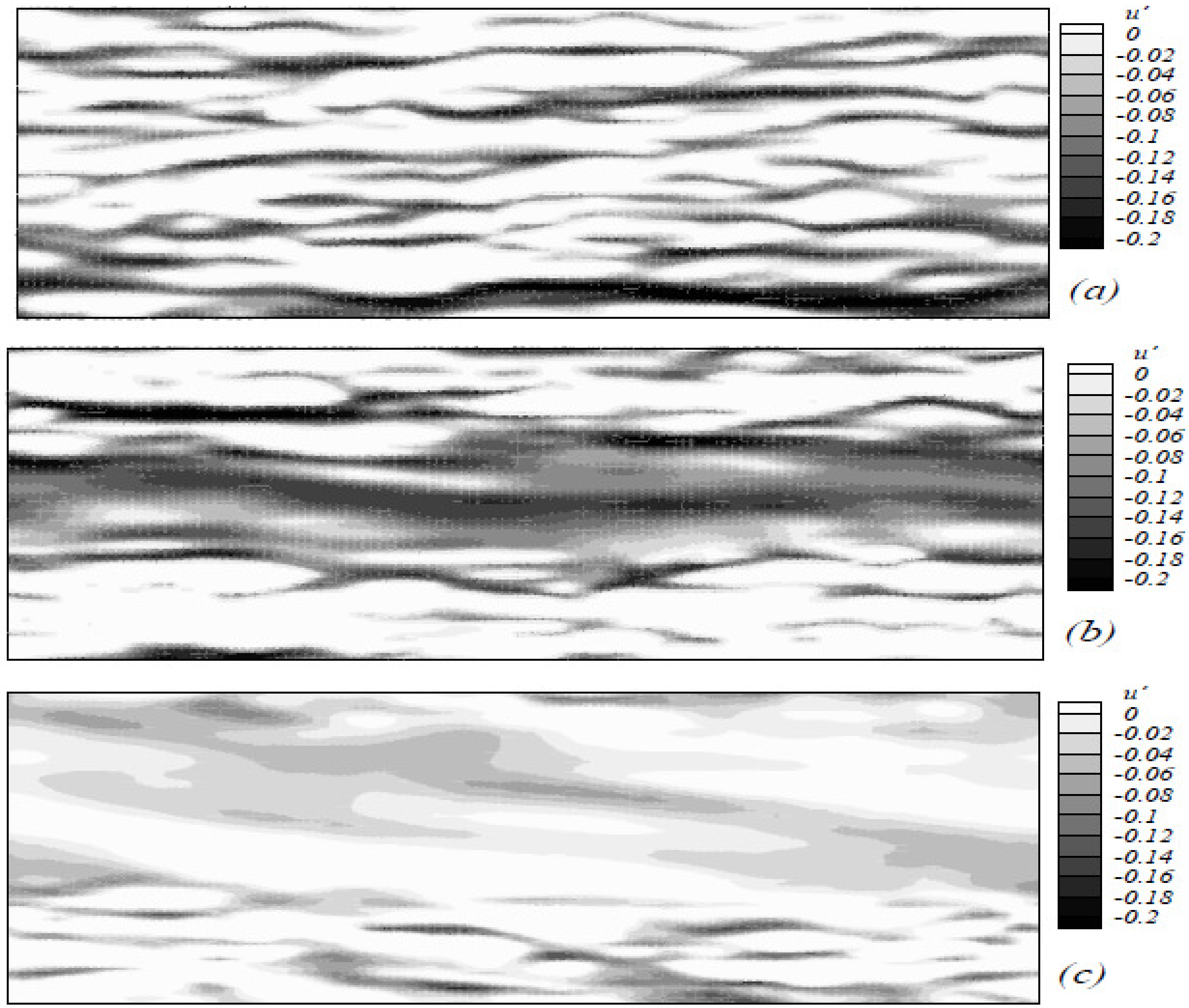
Two separate regions detected:

- A near wall *buoyancy-affected* region where active turbulence is present (Ri_g <0.2)
- An outer *buoyancy-dominated* region characterized by internal waves and counter gradient buoyancy fluxes



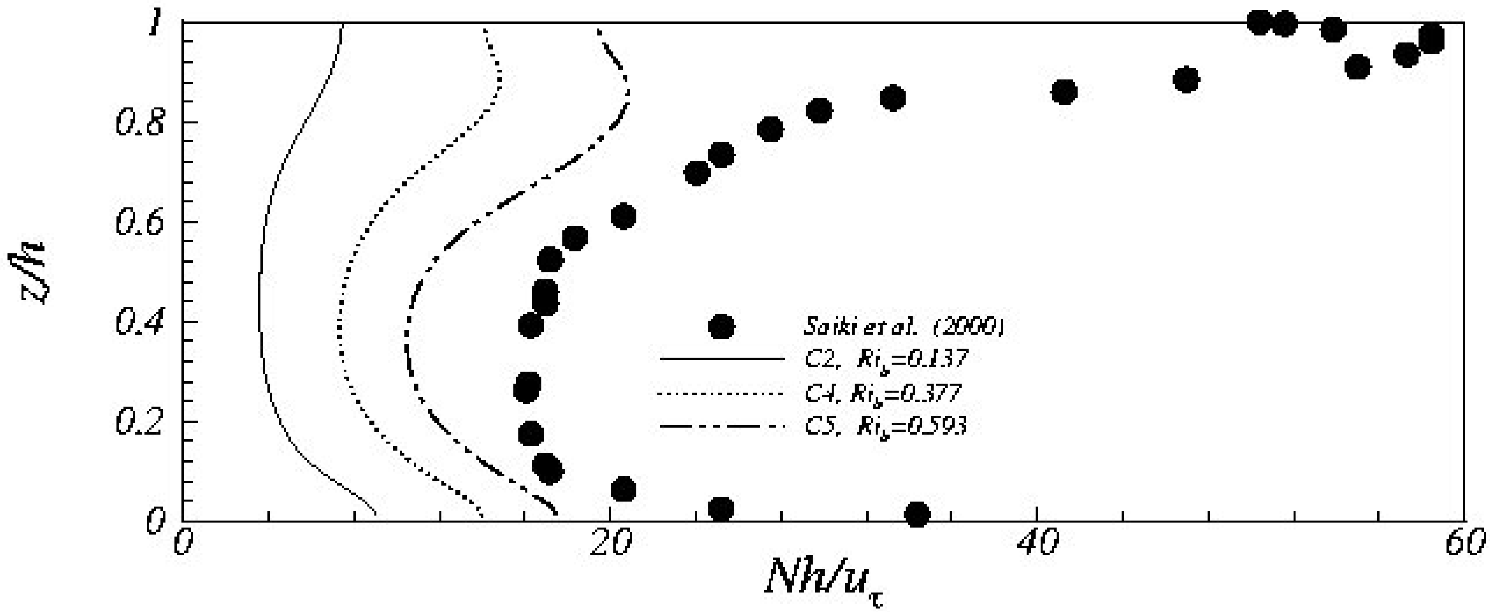
HSSPCF: near wall *turbulent structures*

Near-wall streaks weakened by increased stratification



HSSPCF: *Similarity with the atmospheric stably stratified boundary layer*

The vertical distribution of the Brunt-Vaisala frequency obtained in this very simple geometric and physical configuration is similar to that obtained in numerical studies of the stable planetary boundary layer (Saiki et al., Boundary-layer Met., 2000)



HORIZONTAL FREE-SURFACE CHANNEL WITH ADIABATIC BOTTOM WALL AND IMPOSED HEAT FLUX FROM THE TOP (HFSC)

This problem is archetypal of the turbulence developing in a shallow water basin:
We do not consider complications coming from:
-Rotation
-Free surface effects (waves, Langmuir circulation)
-Wind shear

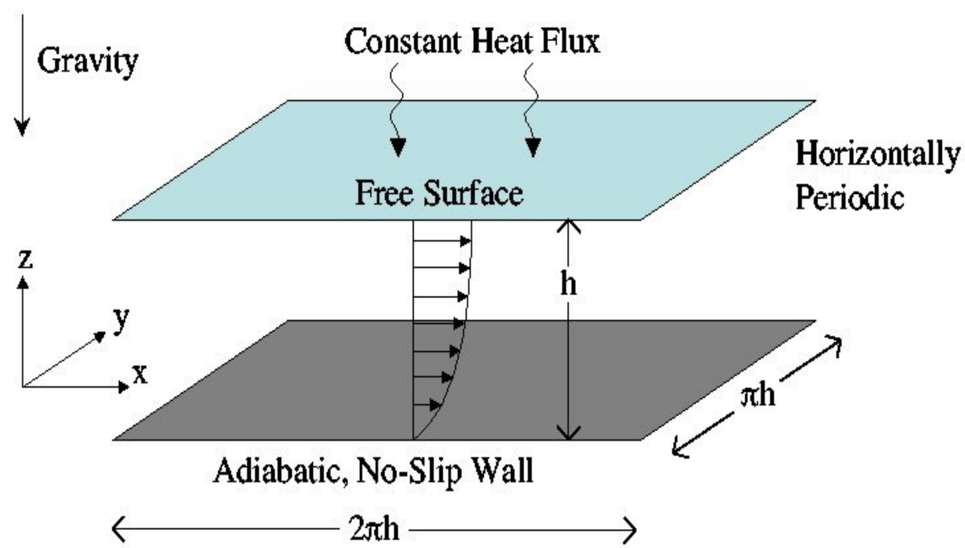
What is the effect of just changing the boundary conditions with respect to the case previously discussed?

Numerical set-up:

$$\mathbf{Re}_\tau = \mathbf{400}$$

$$\mathbf{Pr} = \mathbf{5.0}$$

$$\mathbf{0} < Ri_\tau < \mathbf{500}$$



HFSC: *main features*

Due to the presence of adiabatic bottom wall and imposed net incoming heat flux at the free surface, the region where turbulence production occurs, is at the opposite side with respect to that where buoyancy destruction occurs.

-Behavior of the gradient Richardson number:

$$Ri_g \,|_{wall} = \mathbf{0}$$

$$Ri_g(z) = f(z) \; \textit{not known}$$

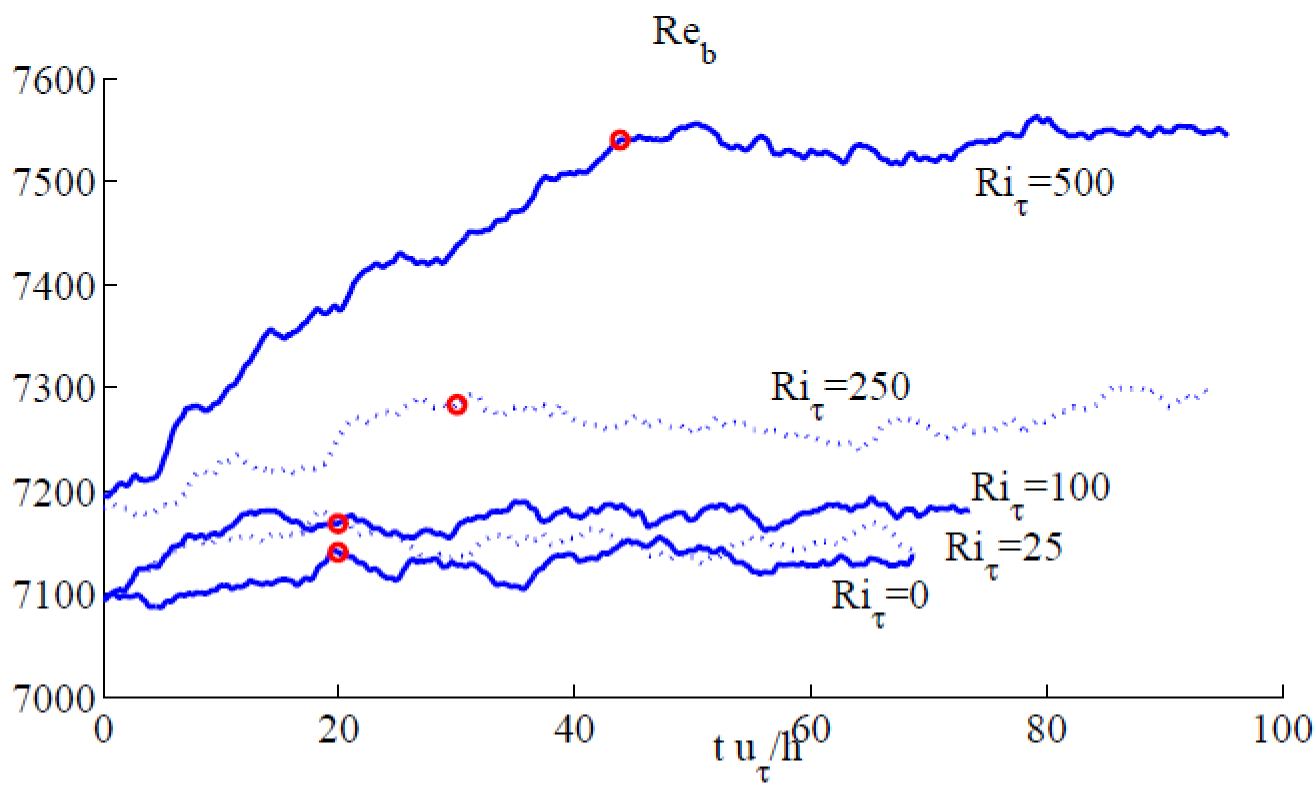
$$Ri_g \,|_{free\ surface} = \infty$$

The friction Richardson number is defined as:

$$Ri_\tau = \frac{gh^2}{\rho_0 u_\tau^2} \left.\frac{d\rho}{dz}\right|_{fs}$$

HFSC: *time evolution of Re_b*

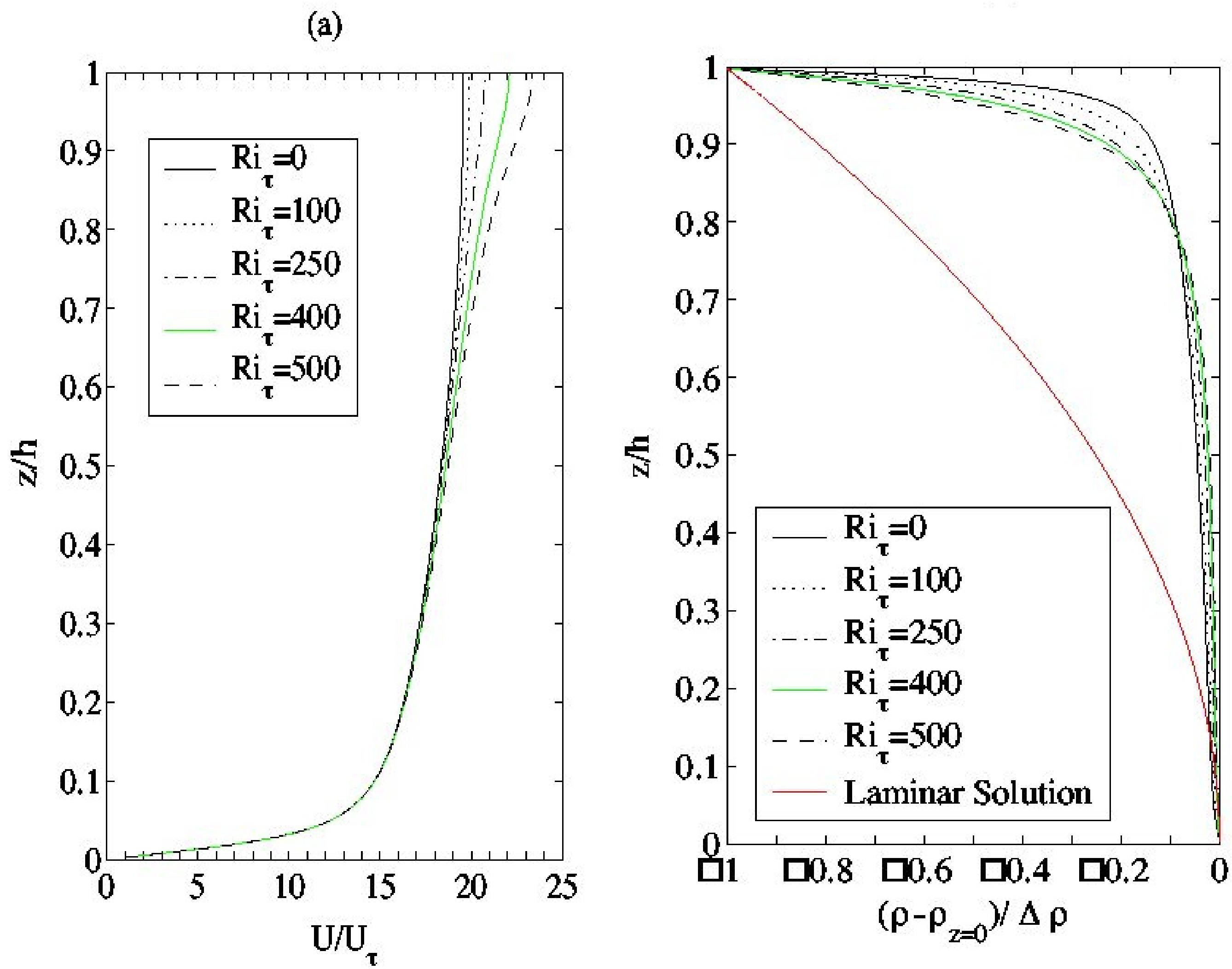
Statistics accumulated after the initial transient, beyond the red spot



HFSC: *the mean field*

The mean velocity increases with Ri_τ in the free surface region

The layer where a strong thermocline is present thickens with Ri_τ

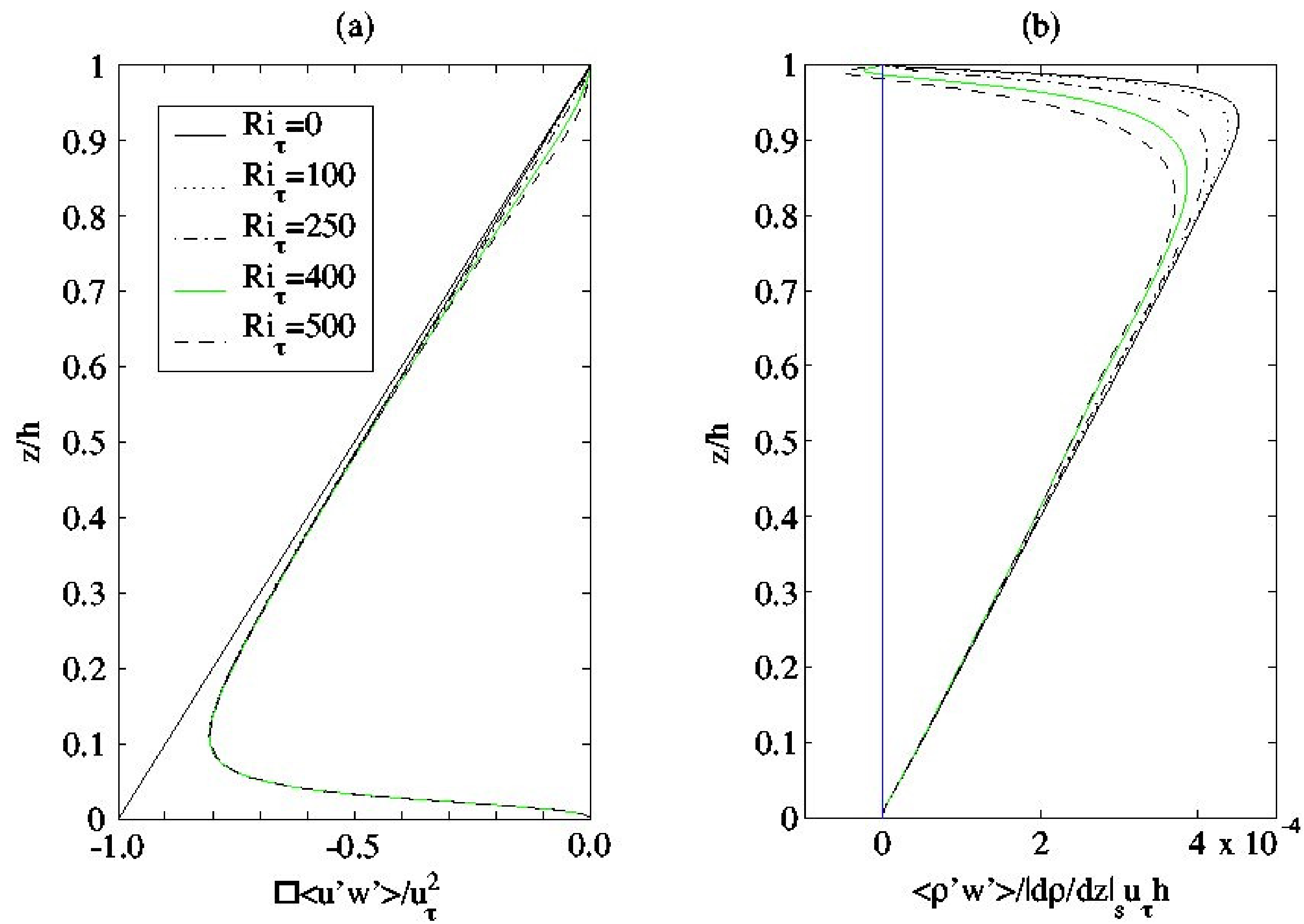


HFSC: *the turbulent fluxes*

The Reynolds shear stress weakly decreases with Ri_τ in the free surface region

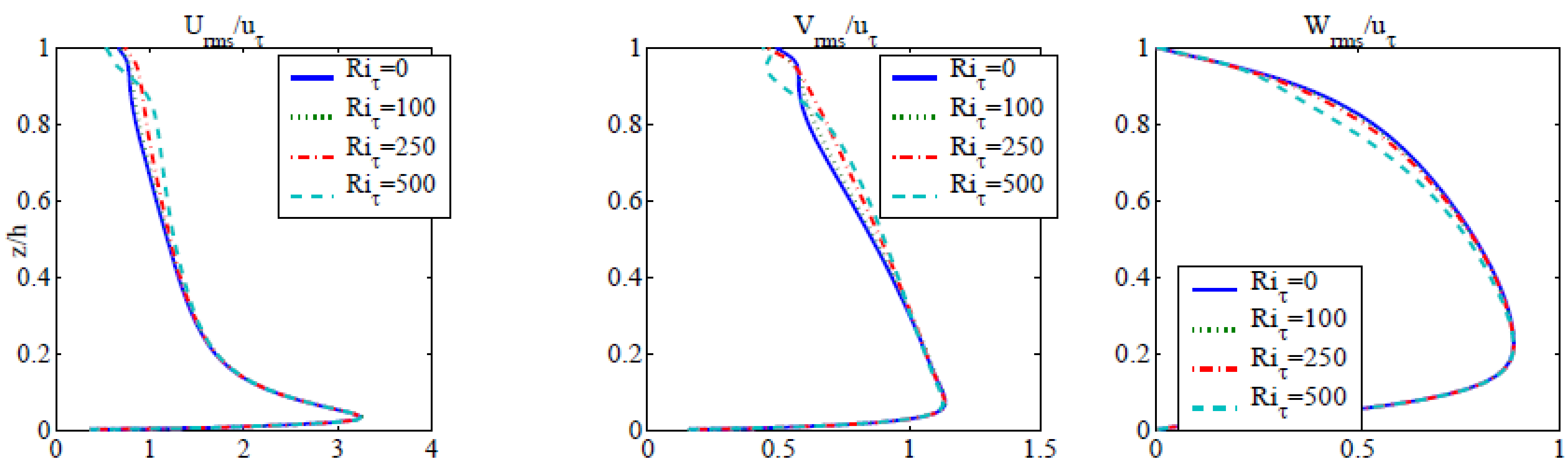
The buoyancy flux strongly decreases in the free-surface layer

Counter-gradient buoyancy fluxes present in the free-surface region under strong stratification



HFSC: *the normal Reynolds stresses*

The normal Reynolds stresses is weakly affected by stratification, only in the upper region of the fluid column

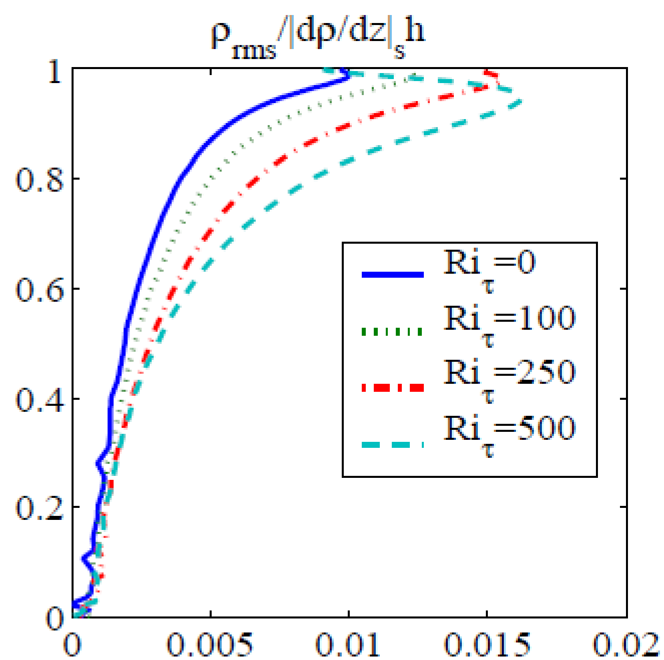


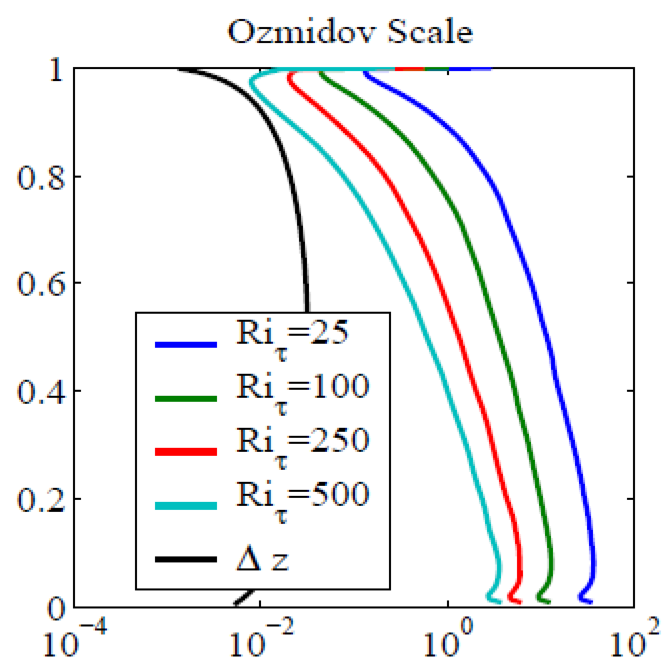
HFSC: *additional statistics*

The level of density fluctuations strongly increases with stratification, associated to the presence of internal waves

The smallest scale directly affected by stratification $L_0 = \left(\frac{\varepsilon}{N^3} \right)^{1/2}$

decreases with stratification. However it remains larger than the smallest scale directly resolved in the simulation

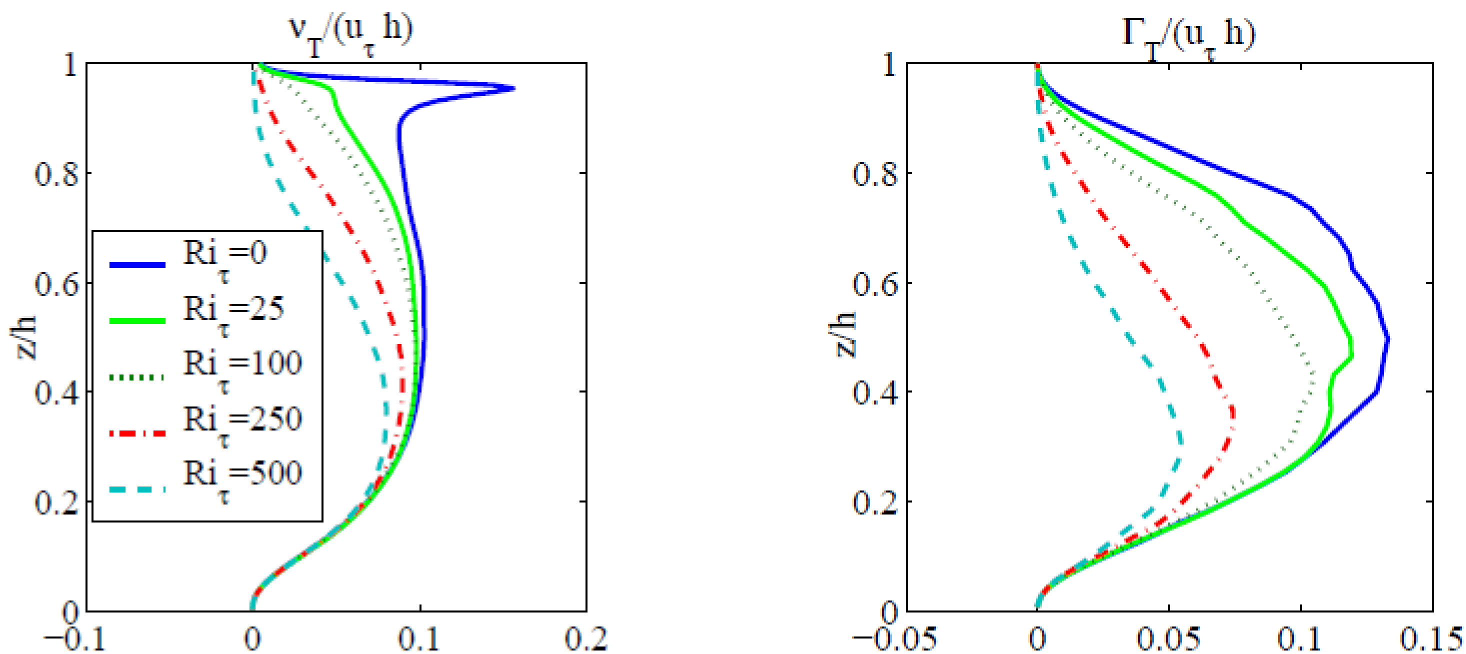




HFSC: *turbulent diffusivities*

The diffusivity of the scalar is much more affected by stratification than the momentum diffusivity (eddy viscosity)

In this case it clearly appears that the Reynolds analogy as well as the constant turbulent Prandtl number assumption are no longer valid

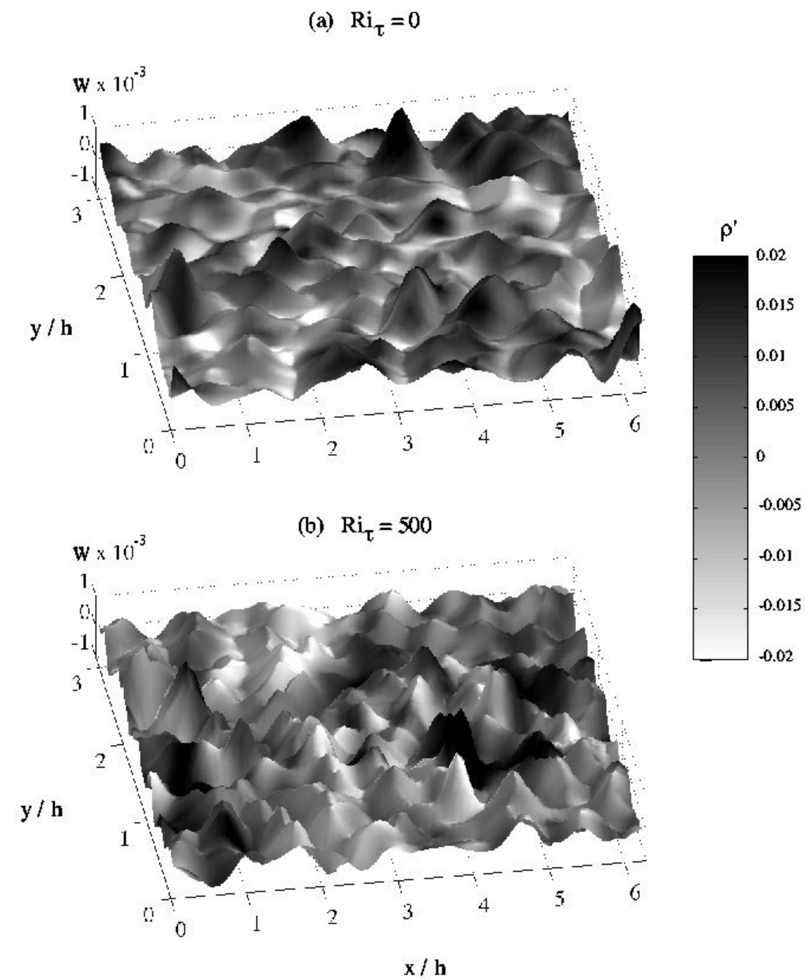


HFSC: *turbulent structures*

The analysis of the instantaneous field and of the coherent structures shows that:

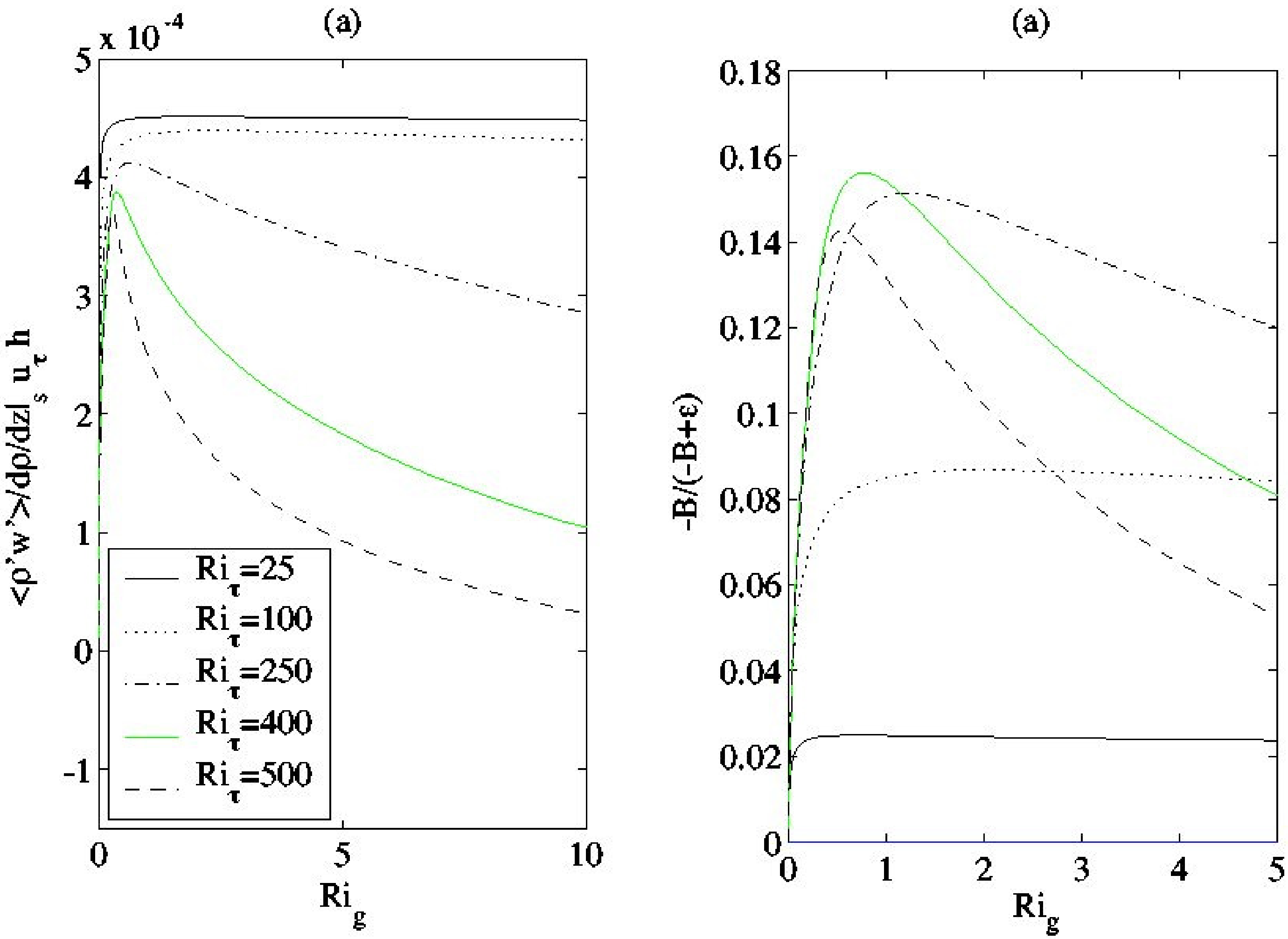
-The free-surface potential energy barrier inhibits upwelling of turbulent patches from the bottom region

-The free-surface vertical spiral structures are weakly affected by stratification



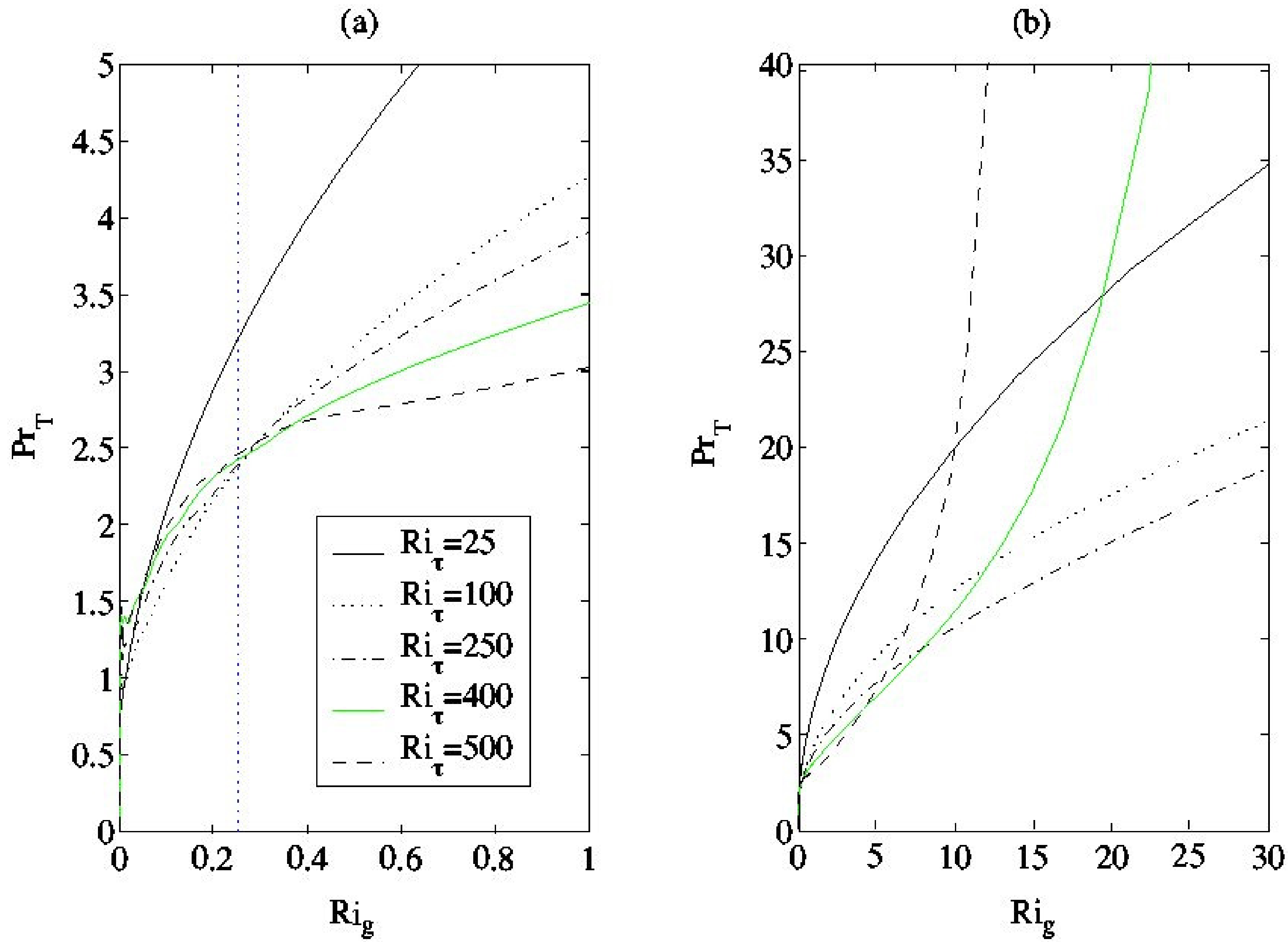
HFSC: *the role of Ri_g*

The gradient Richardson number Ri_g does not appear to be a universal parameter for collapsing turbulent quantities



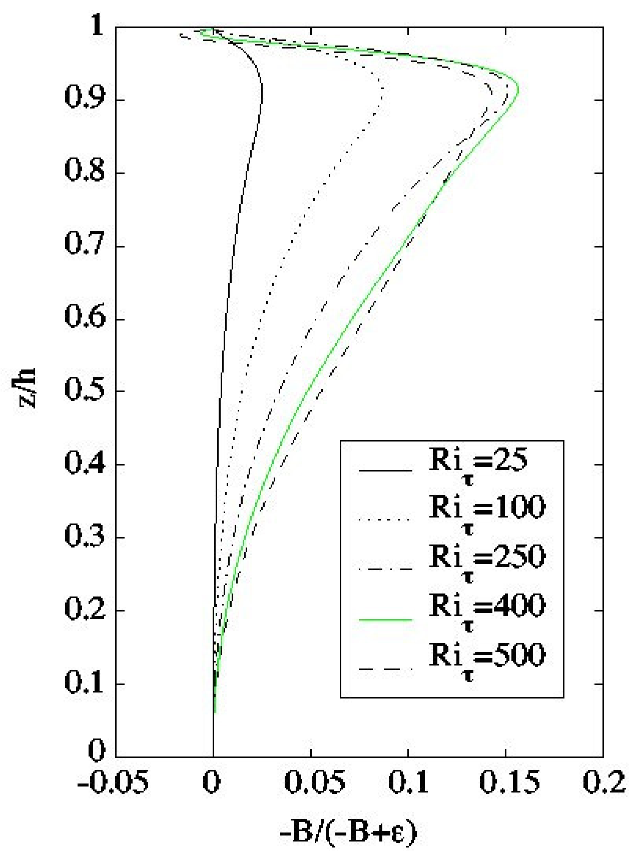
HFSC: *the role of* Ri_g

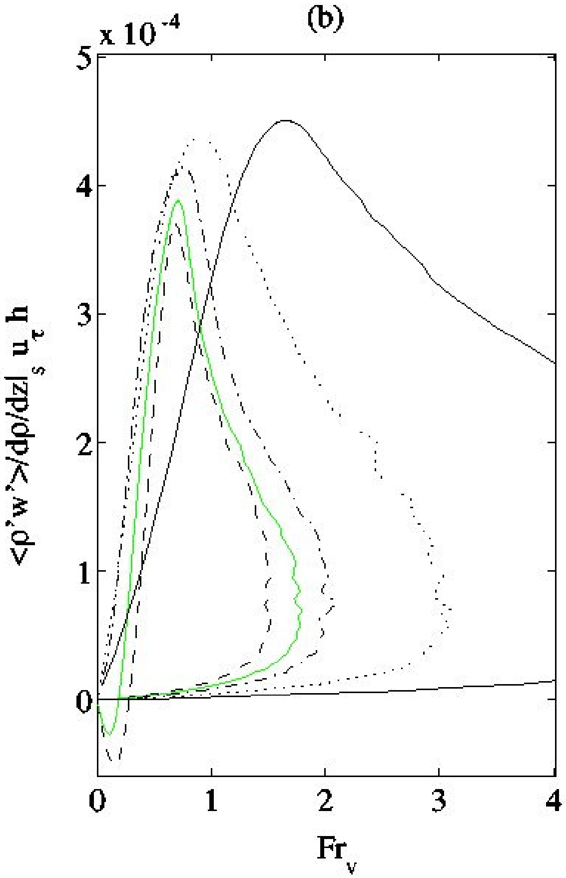
For example, the turbulent Prandtl number does not exhibit a universal behavior when plotted against the gradient Richardson number Ri_g

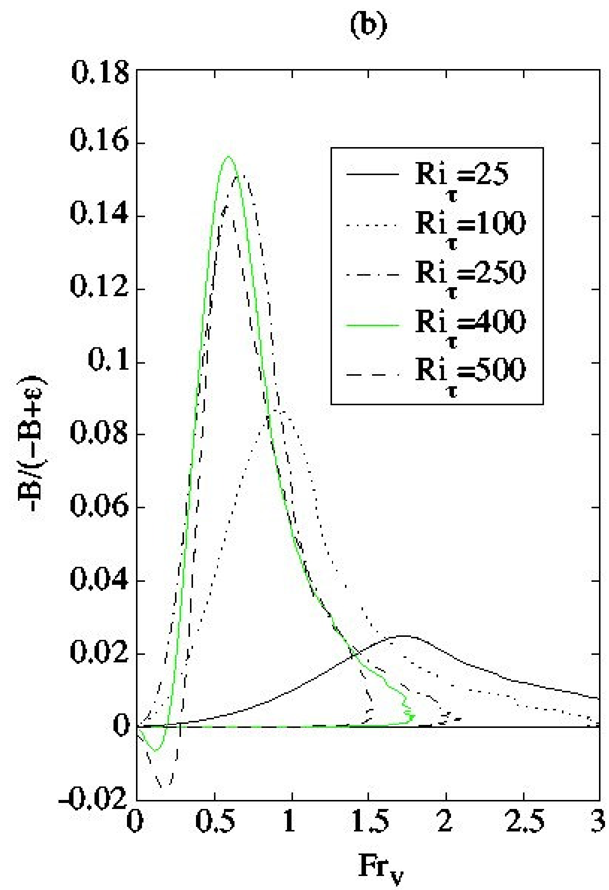


HFSC: and…what about *Fr*$_w$?

The vertical Froude number $Fr_w = \dfrac{w_{rms}}{L_E N}$ with $L_E = \dfrac{\rho_{rms}}{d\overline{\rho}/dz}$ the Ellison scale appears to work much better than Ri_g







Vertical shear and stratification: *some remarks*

Stable Stratification:

- reduces the level of TKE in the fluid column

- induces anisotropy in the flow field

Ri_g is a meaningful parameter when mechanical production and buoyancy destruction are in *real* competition, meaning that density gradient and shear are present in the same region.
In these cases the value Ri_g=0.20-0.25delimitates two separate regions:

- a *buoyancy affected* region where turbulence dominates and is only affected by stratification
- a *buoyancy dominated* region where internal waves and counter-gradient b-fluxes are present. In this case classical turbulent scaling doesn’t hold

In other situations, other scaling parameters must be used, i.e. the vertical Froude number

Inclined shear and vertical stratification (ISVS)

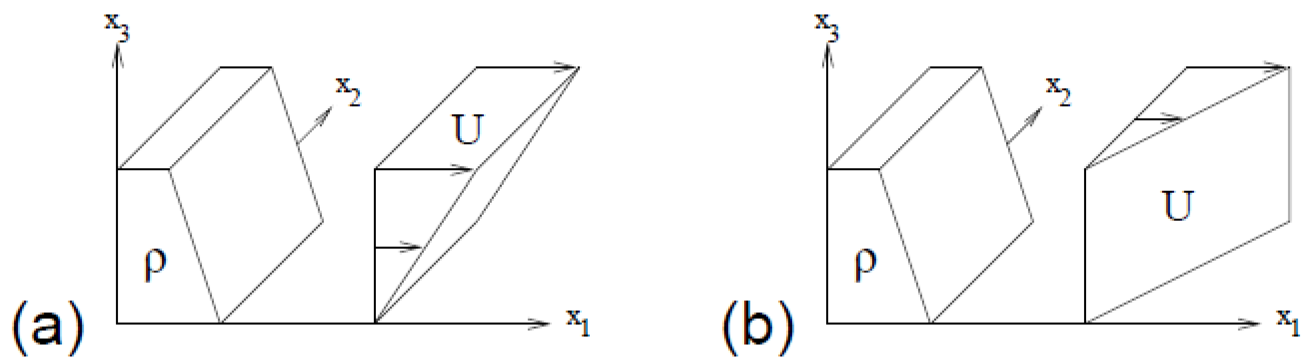
There are several practical situations where the shear is not vertical. Among the others:

- Seamounts where high dissipation rate has been observed (Lueck and Mudge, 1997)
- Straits and channels. Horizontal shear $\frac{dU}{dy} \approx 0.2 \div 0.1 s^{-1} \approx \frac{dU}{dz}$ has been observed in the Haro strait by Farmer et al.
- Anthropogenic or natural discharges in the ocean

In these cases, what found for the vertical shear case, may be not valid. There is not a general criterion for the linear stability analysis as for the vertical shear case (Miles criterion)

Here we discuss some studies of the effect of stratification on flows characterized by the presence of not vertical shear in wall bounded turbulence (Armenio and Sarkar, TCFD, 2004)

ISVS: schematic of the problem for the HT case (Jacobitz and Sarkar, Phys. Fluids, 1999)



The figure shows the limiting cases:

- (a) pure vertical shear (inclination angle $\theta = 0$)
- (b) pure horizontal shear (inclination angle $\theta = \pi/2$)

The results of the study showed that:

- in (b) the critical value of the gradient Richardson number increases by one order of magnitude compared to case (a)

- In (b) the limiting value of mixing efficiency B/e=0.4, twice the value 0.2 of case (a)

Vertical stable stratified plane channel flow (VSSPCF)

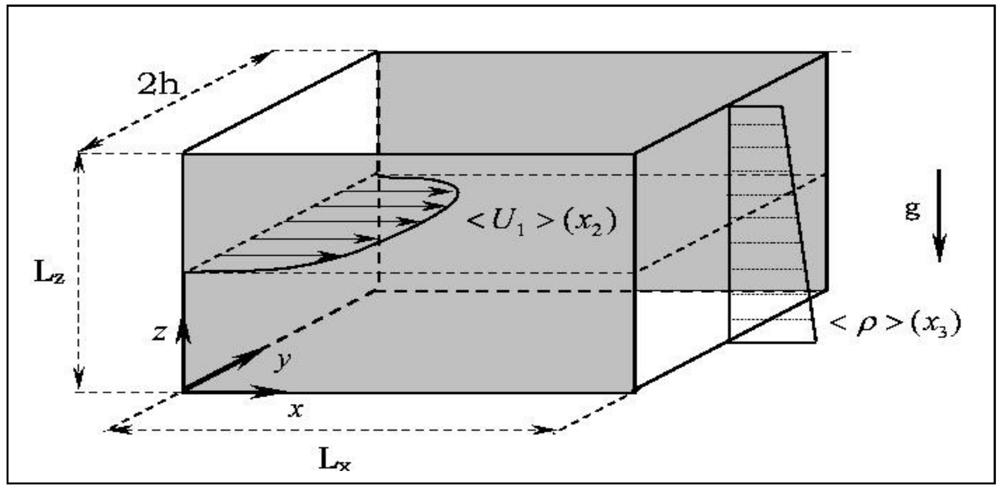
This case is archetypal of the turbulent stratified flow evolving within a canyon-like geometry.
The purpose was to study the fate of the turbulent field, when the stable stratification is not aligned with the forcing mean shear, also in light of the Jacobitz and Sarkar study (1999)

$$\mathbf{Re}_\tau = \mathbf{390}$$

$$\mathbf{Pr} = \mathbf{5.0}$$

$$\mathbf{0} < Ri_{\tau,h} < \mathbf{500}$$

$$Ri_{\tau,v} = \mathbf{100{,}200}$$

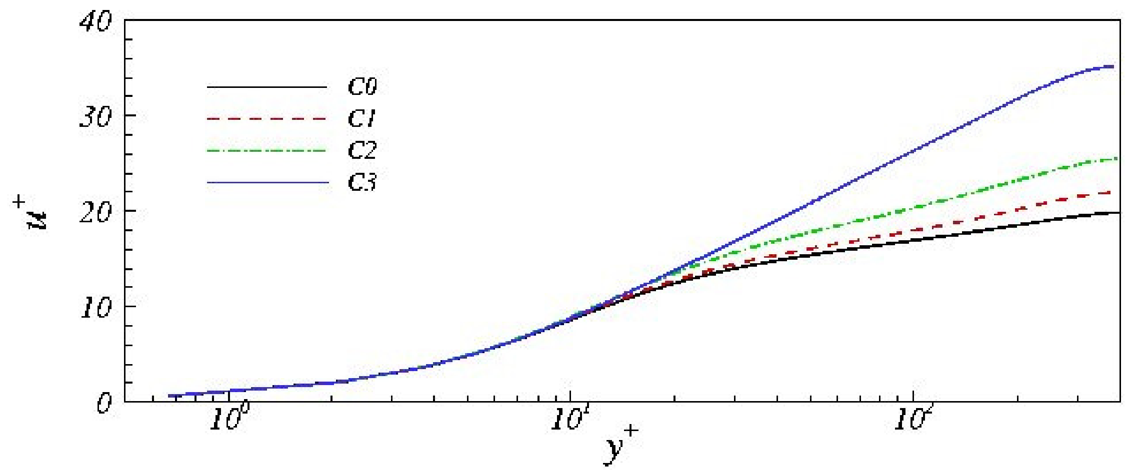


Three cases with HS and two cases with VS, at the same friction Reynolds number were considered

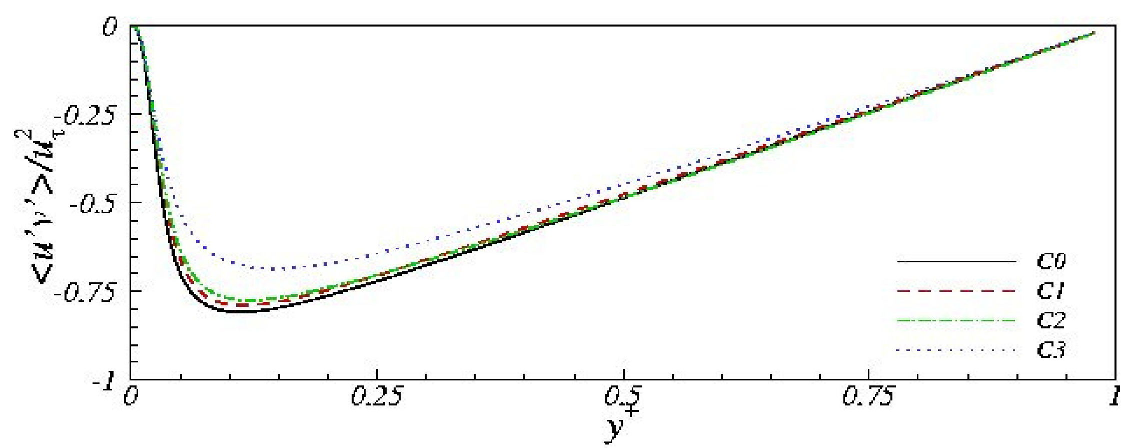
$Case$	Re_τ	Re_b	Ri_τ	Ri_b	$c_f \times 10^2$
$C0$	390	7320	0	0	0.60
$C1$	390	7350	7	0.021	0.59
$C2$	390	7500	15	0.044	0.57
$C3$	390	9025	100	0.20	0.39
$CV1$	390	8700	200	0.21	0.42
$CV2$	390	9380	400	0.36	0.36

VSSPCF: *mean field and Reynolds shear stress*

The mean velocity increases with stratification.

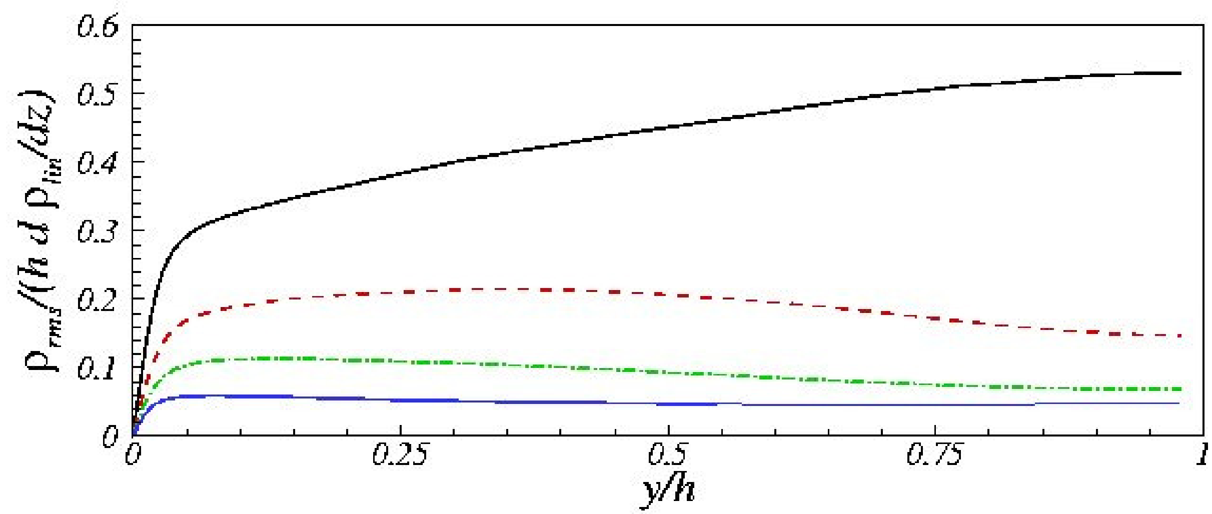


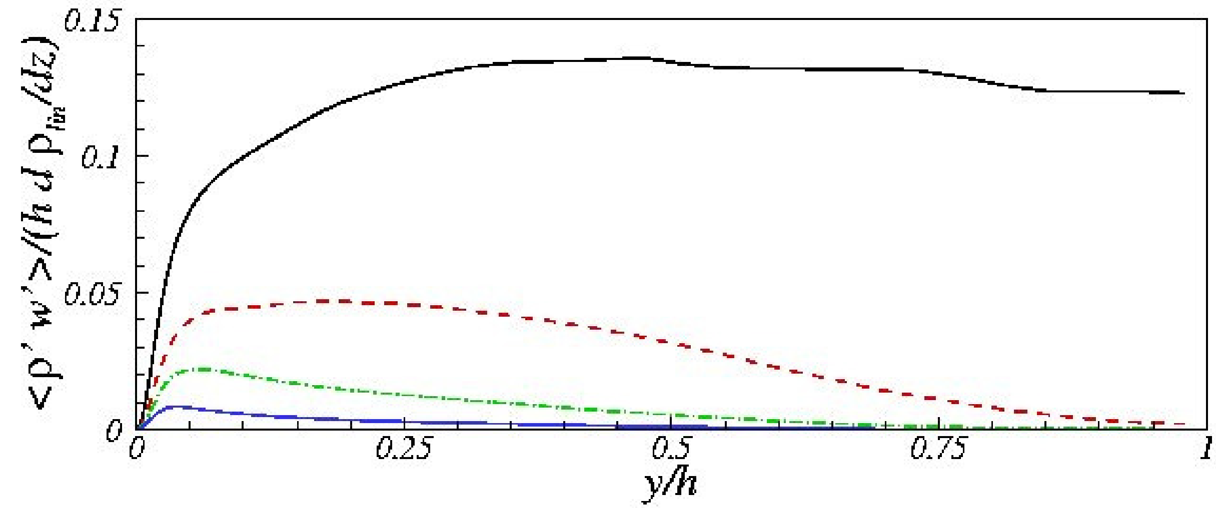
The wall-normal Reynolds shear stress decreases with Ri_τ, at a minor extent than in the vertical shear case



VSSPCF: *the density field*

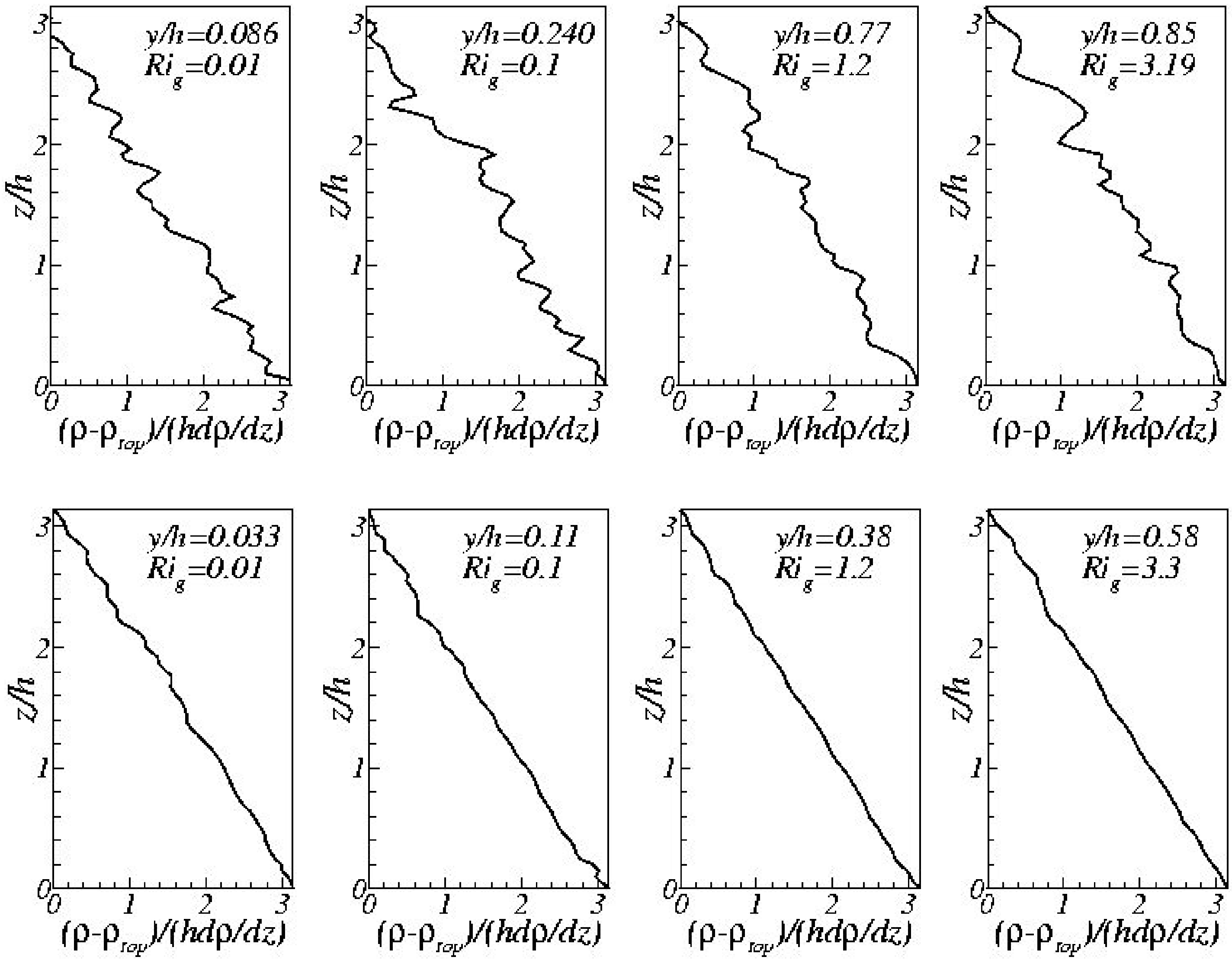
The density fluctuations and the vertical buoyancy fluxes are strongly inhibited by stratification.





VSSPCF: *the instantaneous density field*

C1, Ri_b=0.04



Stratification inhibits overturning of density and associated vertical mixing.

C3, Ri_b=0.59

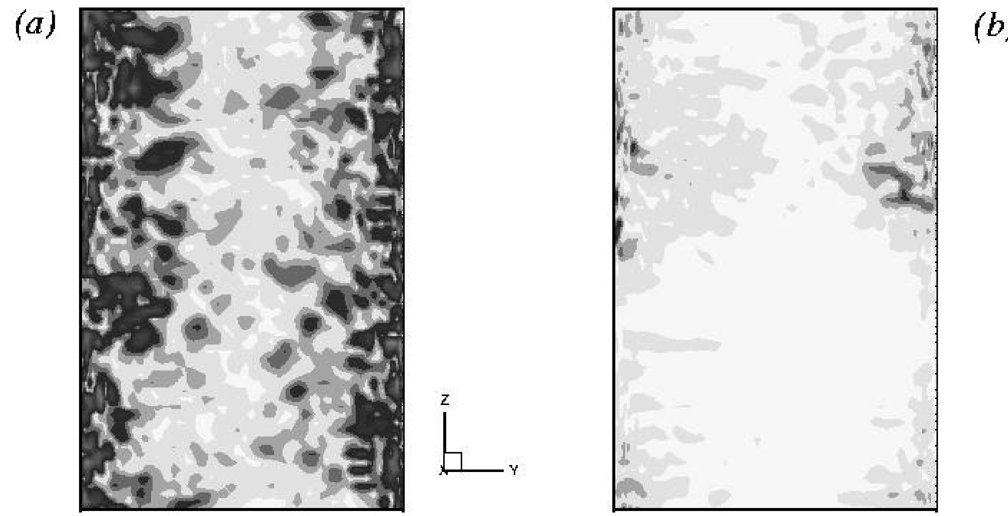
VSSPCF: *who mixes?*

Who is responsible of vertical mixing in the present case?

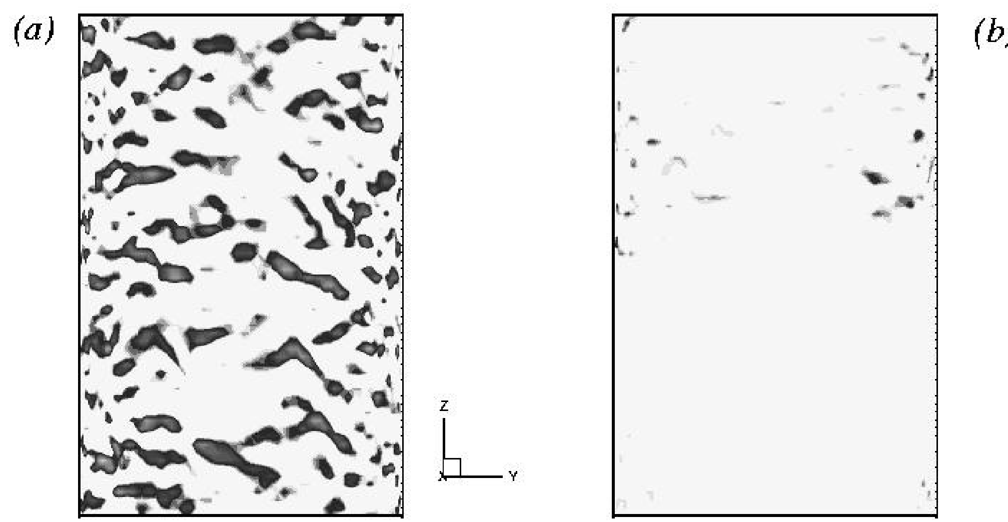
Instantaneous horizontal vorticity enhances rotational motion of the fluid and associated density overturning.

When stratification is strong, the vortical motion is not able to overcome the barrier of potential energy present in the flow.

Inst. horizontal vorticity: (a) C1; (b) C3



Inst. Vert. density gradient: (a) C1; (b) C3

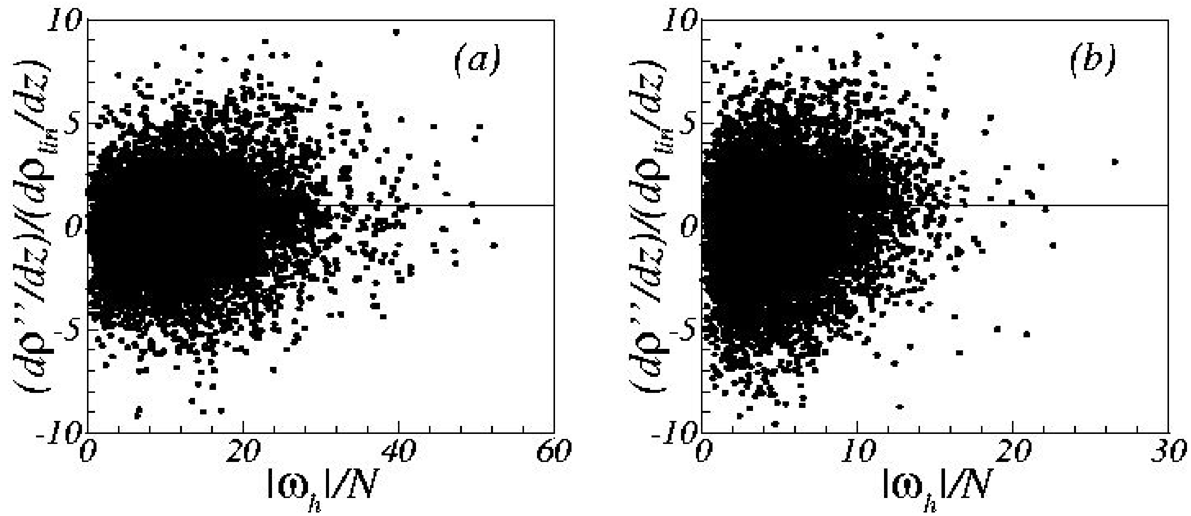


VSSPCF: *scatter plots*

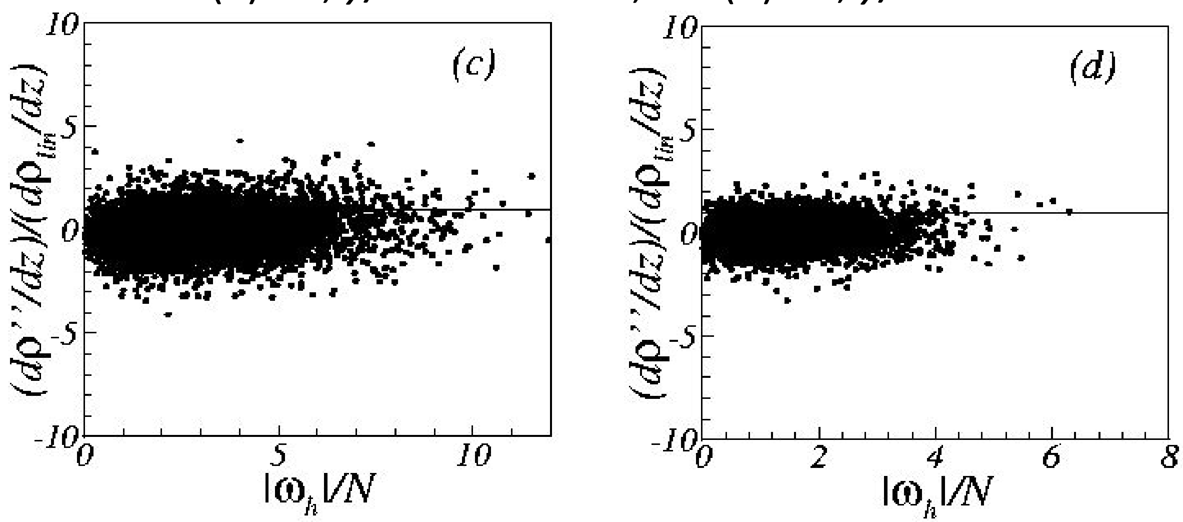
The scatter plot of instantaneous vertical density gradient against the horizontal vorticity shows a strong correlation between each other when stratification is weak.

Overturning events above the grey line

(a) C1, *y/h=0.046* ; (b) C1, *y/h=0.34*



(a) C3, *y/h=0.046* ; (b) C3, *y/h=0.34*

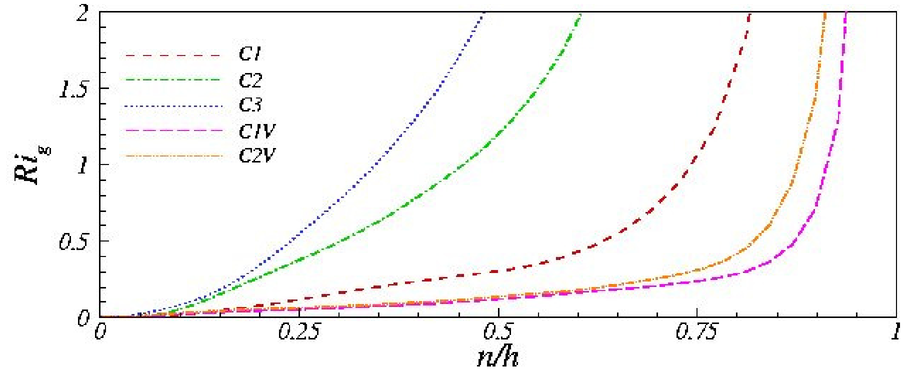


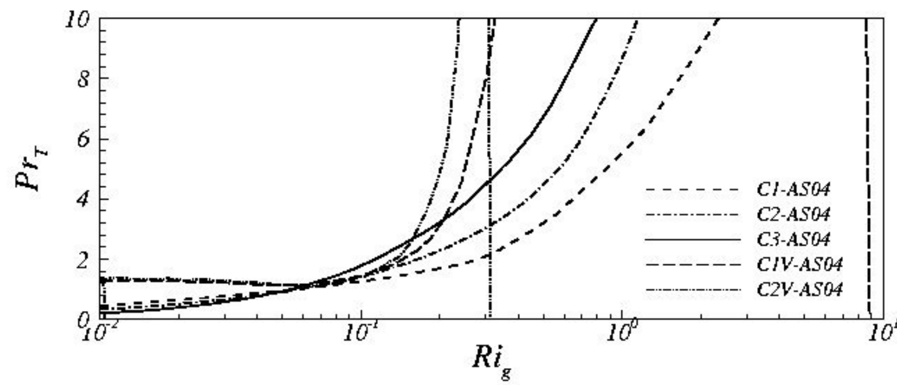
VSSPCF: *HS versus VS*

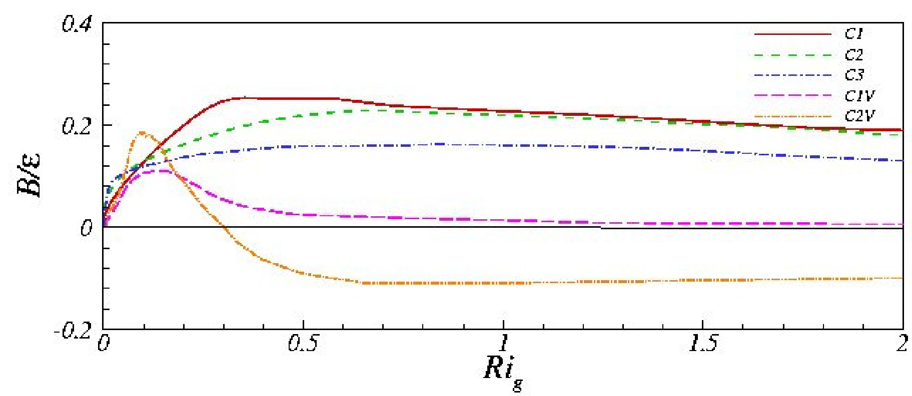
•The gradient Ri does not exhibit the knee at a value around 0.20-0.25;

•The turbulent Prandtl number is not well behaved as in the cases of VS;

•The mixing efficiency remains relatively large even in case of strong stratification.



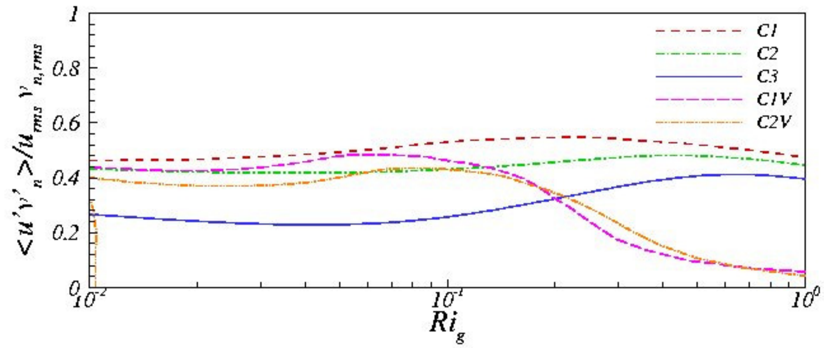


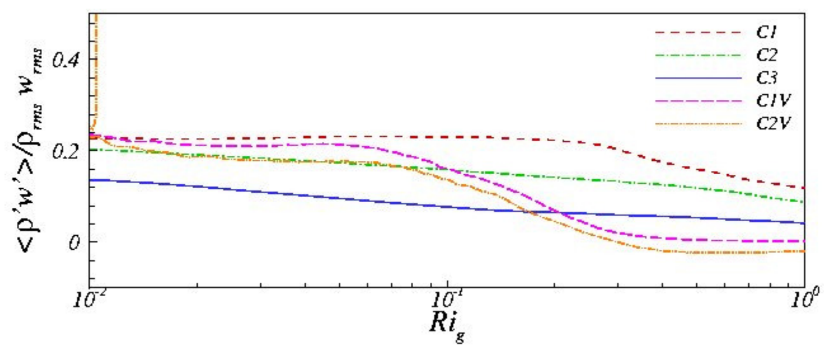


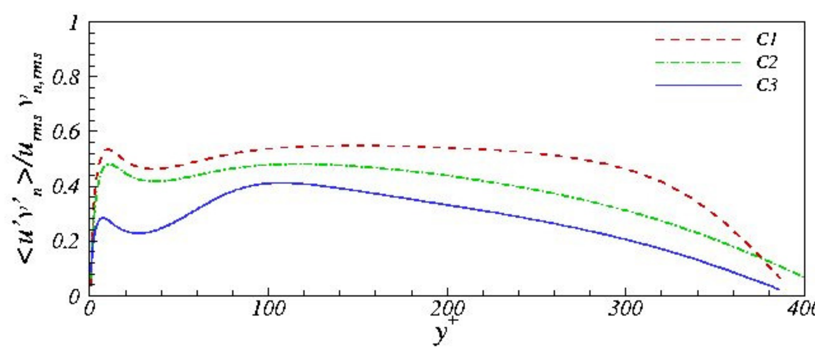
VSSPCF: *HS versus VS*

•Consistently with Jacobitz and Sarkar, 1999, the correlation coefficients decay at a value of Ri_g of the order O(1)

•The correlation coefficients are also strongly dependent on the friction Richardson number







VSSPCF: *remarks*

• When the mean shear is orthogonal to the density gradient:

• The mean shear is able to promote turbulence in a stably stratified environment more than the mean vertical shear;

• A single buoyancy affected region is observable in the wall-normal direction. Internal waves and countergradient buoyancy fluxes are not observed even in case of strong stratification;

• The decay of some turbulent quantities is observed for values of Ri_g - *O(1)*, one order of magnitude larger that the values observed in the case of vertical shear.

Analysis of environmental problems with particulate matter:
The Lagrangian-Eulerian Approach

The dispersed phase

(Particulate)

Ambient Fluid

(air or water)

Lagrangian treatment:

Equation of motion for each particle of the swarm

Eulerian treatment:

Governing equations solved via LES/DNS

Interpolation of the Eulerian field at the particle position

The particle motion equation

The complete form of the particle motion equation was formulated by Maxey and Riley, Phys. Fluids A (1983). In this formulation the force acting over a single point particle is composed of several contributions: Stokes drag, gravity term, added mass, pressure drag, Basset term.

Armenio and Fiorotto, Phys. Fluids 1993, showed that under most circumstances Stokes drag is the leading term , and a simplified form of the MR equation gives accurate results. The dimensional simplified MR equation, in a rotating frame of reference is:

$$\frac{\mathrm{d}V_{\mathrm{p},i}}{\mathrm{d}t_{\mathrm{d}}} = \left(\frac{\rho_{\mathrm{p}} - \rho_{\mathrm{tot,d}}}{\rho_{\mathrm{p}}}\right) g\delta_{i3} + \frac{3}{4D_{\mathrm{p}}} C_{\mathrm{D}} |U_{\mathrm{p},i} - V_{\mathrm{p},i}| (U_{\mathrm{p},i} - V_{\mathrm{p},i}) - 2\epsilon_{ijk}\Omega_j V_{p,k}$$

$V_{p,i}$ *i*-comp. of the particle velocity $U_{p,i}$ *i*-comp. of fluid velocity at particle location
ρ_p particle density
ρ_{tot} total fluid density
D_p particle diameter
C_D drag coefficient
Ω_i background vorticity

'd' denotes dimensional quantities

The particle motion equation in a stratified flow

Now, consider the total density as the sum of a constant reference one ρ_0 and a perturbation density $\rho_d(x_{di}, t_d)$.

If we make the equation non-dimensional with a length scale δ, ρ_0, a velocity scale u_τ, a time scale u_τ/δ we obtain:

$$\frac{\mathrm{d}v_{\mathrm{p},i}}{\mathrm{d}t} = \left(1 - \frac{1}{\Delta\rho}\right)\frac{1}{Fr^2}\delta_{i3} - \frac{Ri}{\Delta\rho}\rho\delta_{i3} + \frac{3}{4d_\mathrm{p}}C_\mathrm{D}|u_{\mathrm{p},i} - v_{\mathrm{p},i}|(u_{\mathrm{p},i} - v_{\mathrm{p},i}) - \epsilon_{ijk}\frac{\Omega_j}{\Omega_3}v_{p,k}$$

With

$$\Delta\rho = \rho_\mathrm{p}/\rho_0, \quad v_{\mathrm{p},i} = V_{\mathrm{p},i}/u_\tau, \quad u_{\mathrm{p},i} = U_{\mathrm{p},i}/u_\tau \quad d_\mathrm{p} = D_\mathrm{p}/\delta$$

R_i defined using a proper density scale and u_τ,

$$Fr = u_\tau/(g\delta)^{0.5}$$

The particle motion equation in a stratified flow

In the particle motion equation for stratified flows, the term proportional to R_i takes into account that an inertial particle, when moving within a stratified environment, encounters a density variation which can affect its own buoyancy

QUESTION:
When is this term important?
Must we always retain this term in the particle motion equation?

ANSWER:

Consider the ratio between the R_i term (*II*) and the *Froude* term (*I*) in the particle motion equation.

$$\mathrm{II}/\mathrm{I} = (\rho_s/\rho_0)/((\rho_p - \rho_0)/\rho_0)$$

Since $\frac{\rho_s}{\rho_0} << 1$ under the B-approximation, $\frac{II}{I} \approx 1$ if $\rho_p \approx \rho_0$,

That is when the density of the particles is close to the reference one.
This is the case of upwelling of fresh water particles in a salty water one

The Mathematical model:
Lagrangian-Eulerian Approach

-Time advancement of the Eulerian field using DNS/LES

- Only when LES is used: Reconstruction of the SGS velocity field using the approximate deconvolution technique as proposed by Kuerten, Phys. Fluids, 2006

-Interpolation of the Eulerian velocity field onto the particle position

-Solution of the particle motion equation (Armenio & Fiorotto, PoF 2001; Inghilesi et al. 2008)

-Time advancement of the particle position

The Mathematical model: Lagrangian-Eulerian Approach

- Filtered form of the 3D unsteady Navier-Stokes eqs. for incompressible flows + transport equation of passive/active scalars solved using a fractional step technique (Zang et al., JCP, 1994);

- Space discretization: 2nd order accurate, centered finite differences

- dynamic mixed subgrid-scale model (Armenio and Piomelli, FTC, 2000, Armenio and Sarkar JFM 2002)

- Lagrangian particles treated as fluid or inertial particles depending on the problem under investigation

- Interpolation of the Eulerian field onto the particle position using a 2nd order accurate interpolation technique (Marchioli et al., C&F, 2007)

Examples of applications:

Vertical jet of buoyant particles in a stably stratified wind driven Ekman layer
(VJBP)

Inghilesi et al. Int. J. Heat and Fluid Flow, 2008

Scope: Study of upwelling of a fresh water buoyant bubbles in a salt water environment

Relevance: water mixing in coastal regions

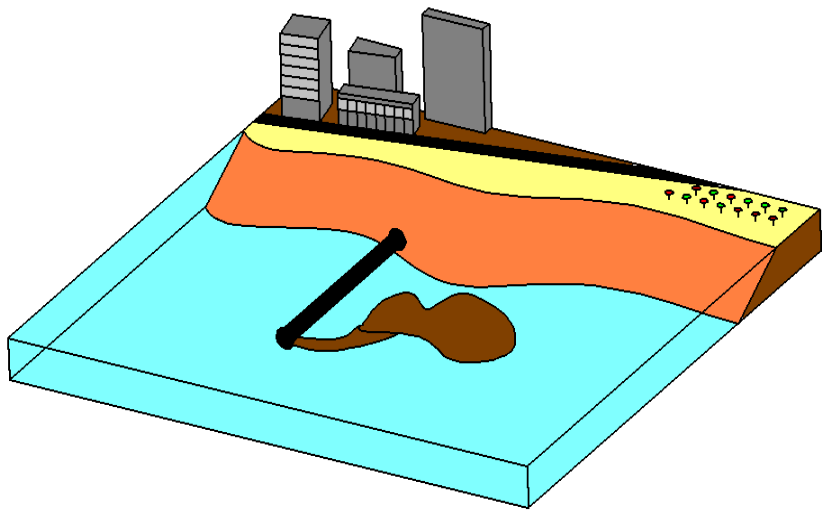
Forcing terms:

-Wind stress

-earth rotation ($\Rightarrow$ Coriolis force)

-Incoming heat flux at the free surface

-Particles: buoyant jet



VJBP: *the numerical setup*

wind stress

Rotation

Near-shore circulation

stratification

$$\tau_w = \mathbf{0.07{-}0.1}\ Pa$$

$$f \approx \mathbf{10^{-4}}\, rad/s$$

$$\mathbf{Re} = \frac{u_\tau \delta}{\nu} = \frac{u_\tau^2}{f\nu} \approx \mathbf{7 \times 10^4}$$

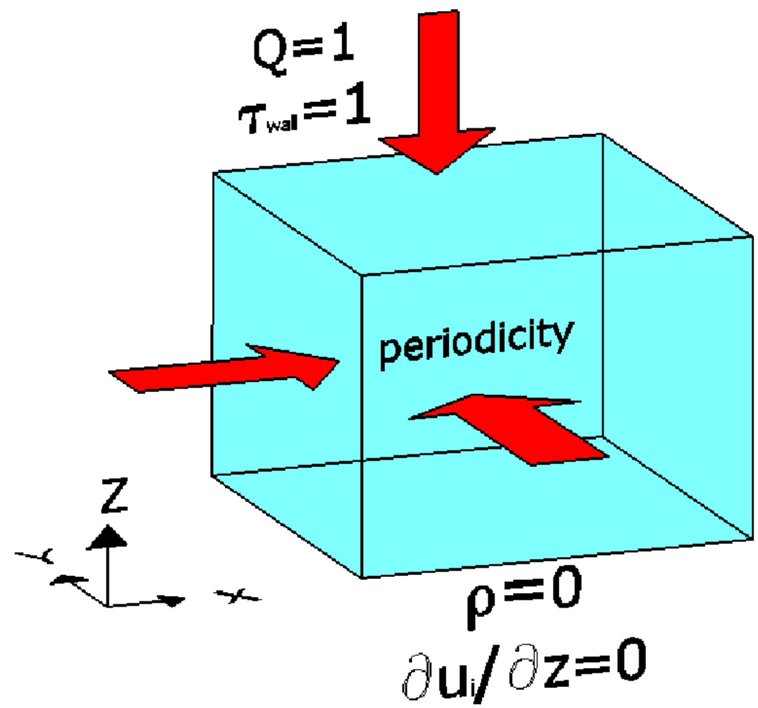
The real-life problem

The archetypal problem:

The wind-driven stratified Ekman layer

$$\mathbf{Re} = \mathbf{10^4}$$

LES with wall functions for the imposed stress and heat flux



VJBP: *the turbulent mean field*

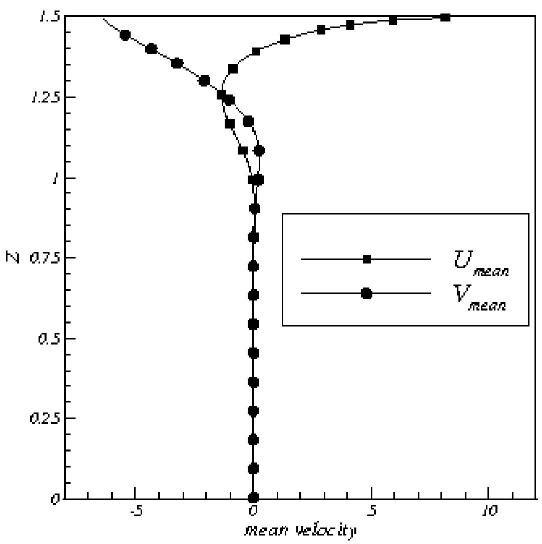
Cases investigated:

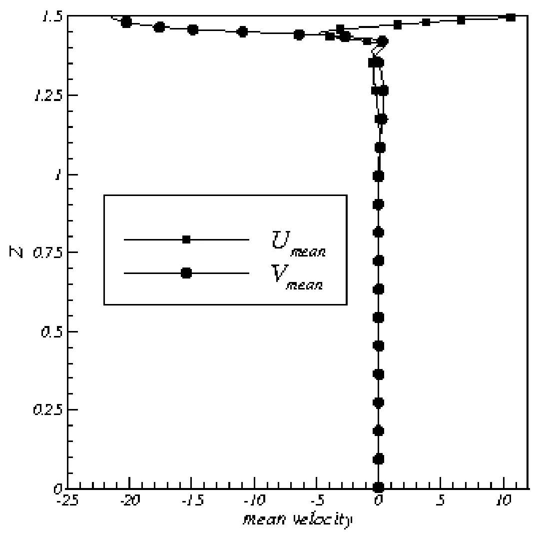
- One value of latitude (Mid-latitude)
- Two levels of stratification
- Particles released continuously in time over a disc of radius *0.05* δ at a depth *0.5* δ

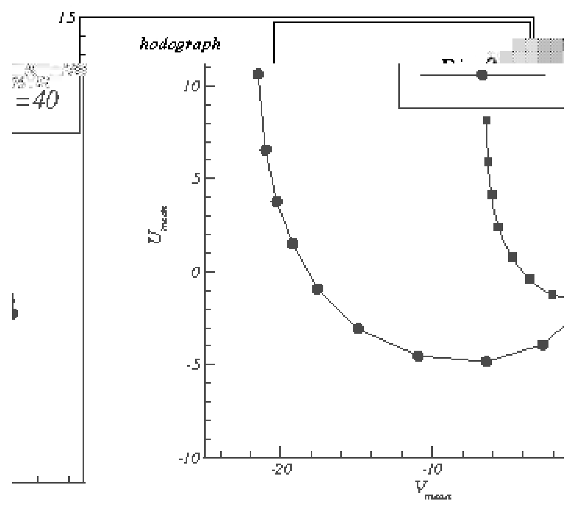
$$\theta = 45^o$$

$$Ri = 0 \;\; , Ri = 40$$

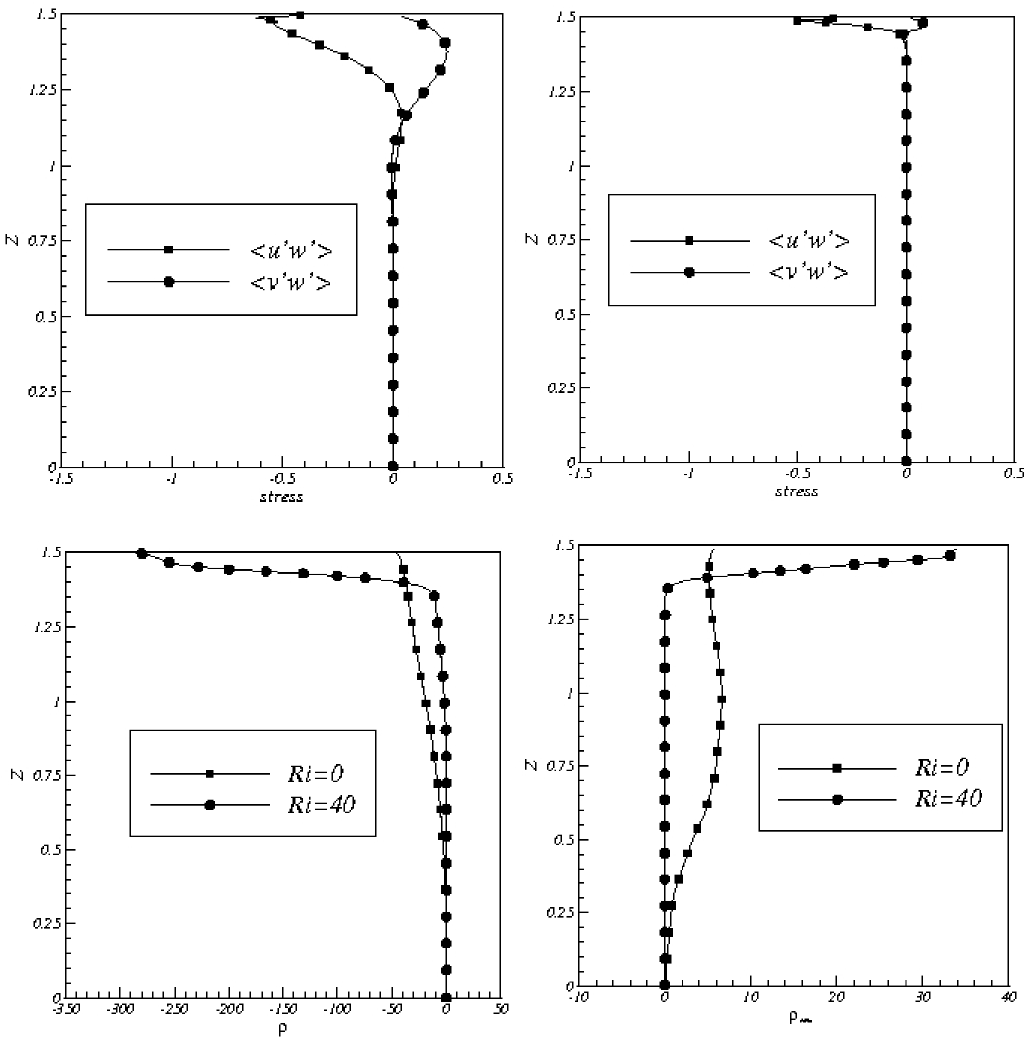
$$\frac{\rho_p}{\rho_0} = 0.976, \frac{r}{\delta} = 10^{-3}$$







VJBP: *the turbulent field*



Stratification:

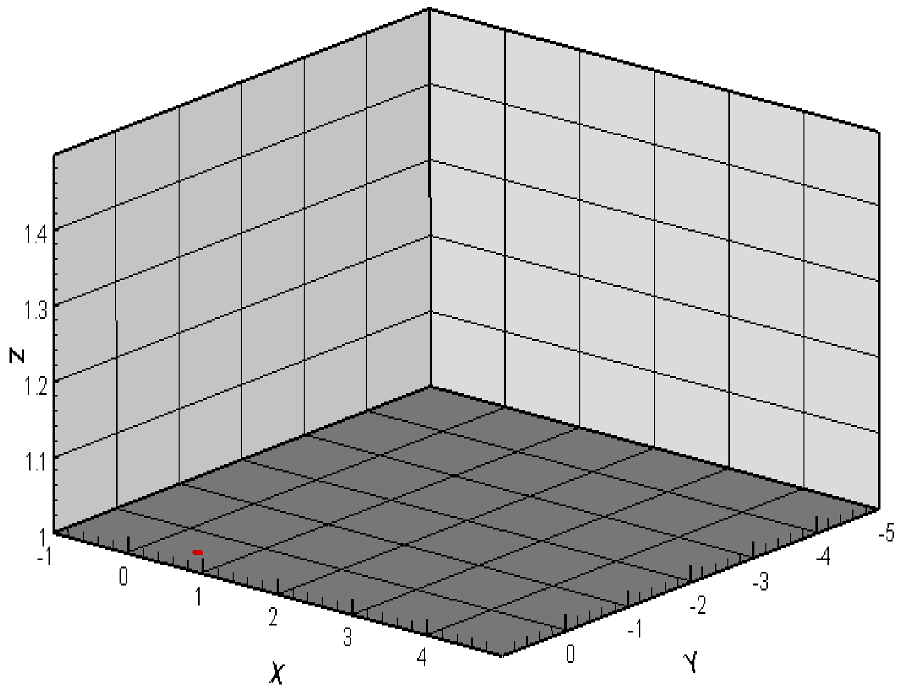
-reduces the turbulent depth

-inhibits vertical mixing

-Increases the angle between the wind stress and the free surface velocity

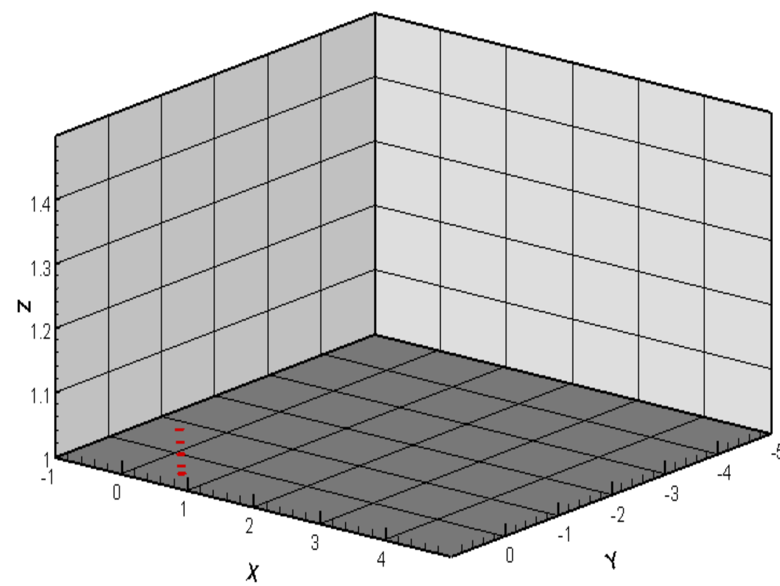
- Reduces the value of fluid density in the free surface region

- Produces a strong picnocline

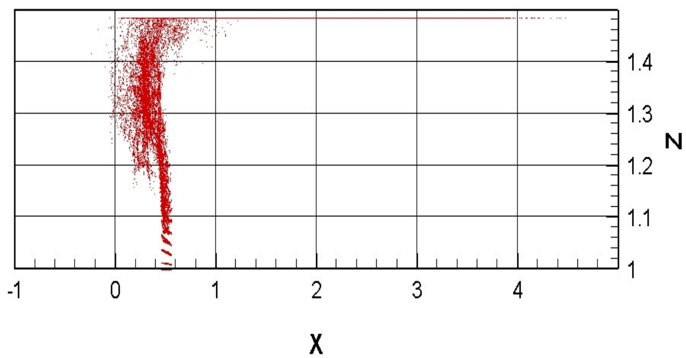


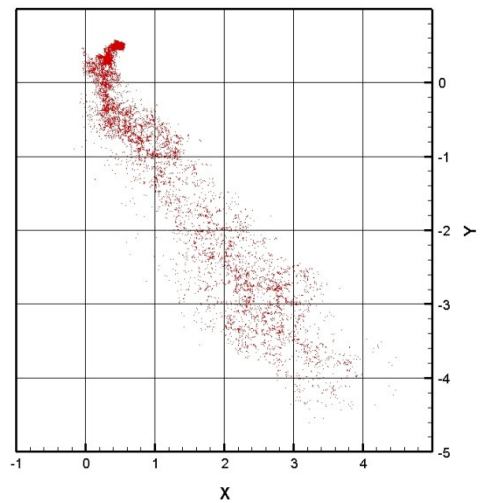
Neutral case

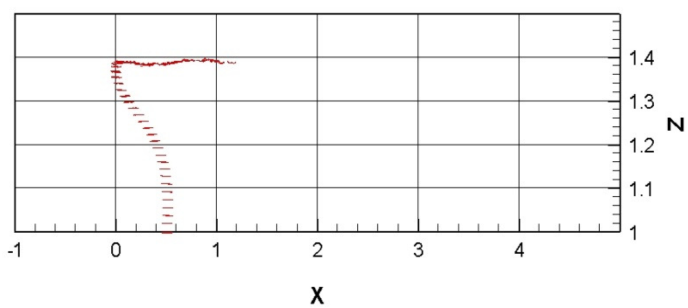
Stratified case

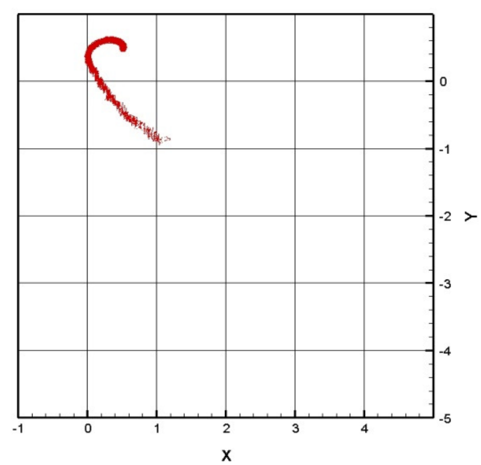


VJBP: *the buoyant plume*









Neutral case

-Loose its structure

-Reaches the free surface

-Travels fast and is rapidly dispersed in the horizontal plane

Stratified case

-Initial cylindrical structure nearly unchanged

-Particles entrapped

- in the low speed region below the thermocline region

Particle dispersion in a stratified bottom Ekman layer (PDSBEL)

Stocca et al., Proc. ISSF 2006

Scandura et al., in preparation

Scope: Study of dispersion of a polluting particulate in the low-atmosphere

Relevance: prediction of PM10 concentration in industrial and urban areas

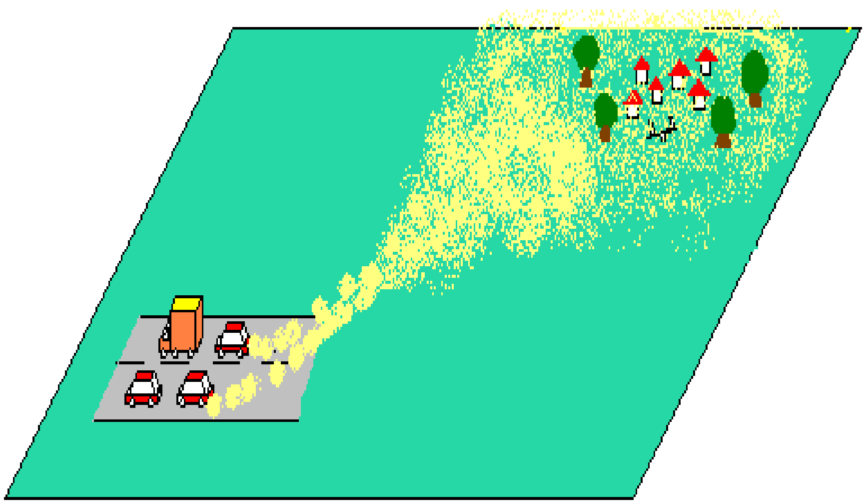
Forcing terms:

-Geostrophic wind

-earth rotation ($\Rightarrow$ Coriolis force)

-Temperature gap between the ground level and the free atmosphere

-Particles: tracers and inertial



PDSBEL: *the numerical setup*

real-life problem

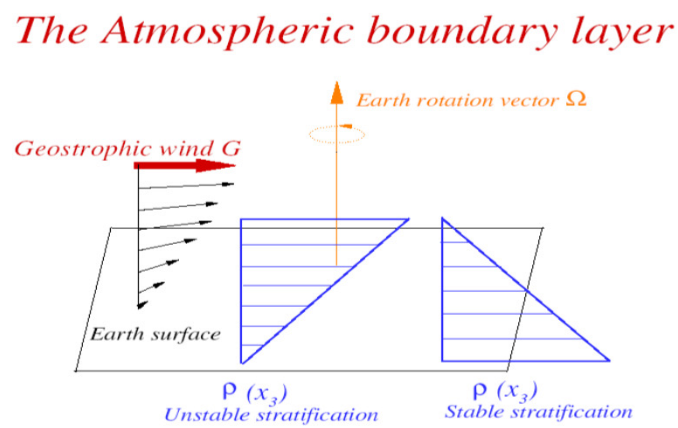
- geostrophic wind
- Rotation
- stratification

$$G \approx 10\ m/s$$

$$f \approx 10^{-4}\ rad/s$$

$$\mathbf{Re} = \frac{G\delta}{\nu} = \frac{\sqrt{2}G}{\sqrt{f\nu}} \approx 5\times10^5$$

$$\Delta T \approx \pm 5 \div 10$$

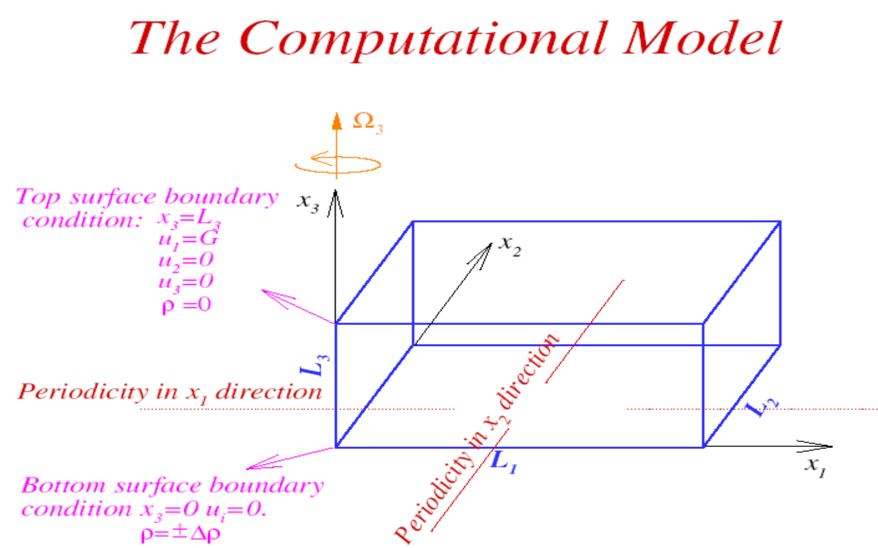


The archetypal problem:

Stratified bottom Ekman layer

$$\mathbf{Re} = 400$$

Wall-Resolving LES



PDSBEL: *the cases studied*

stable

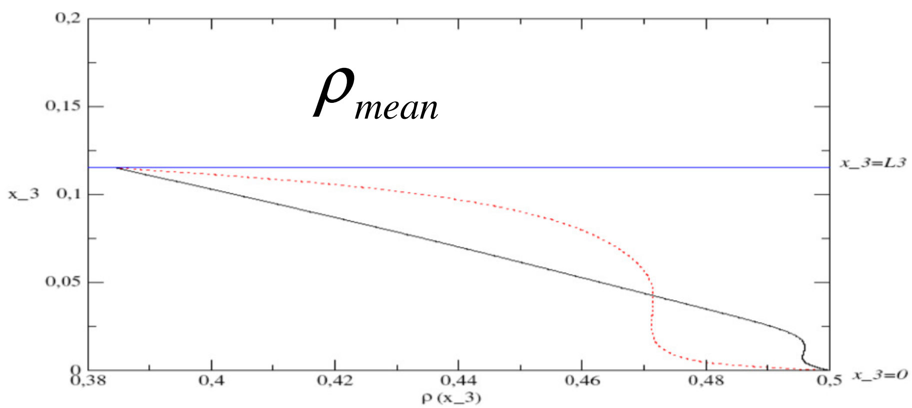
unstable

Passive scalar

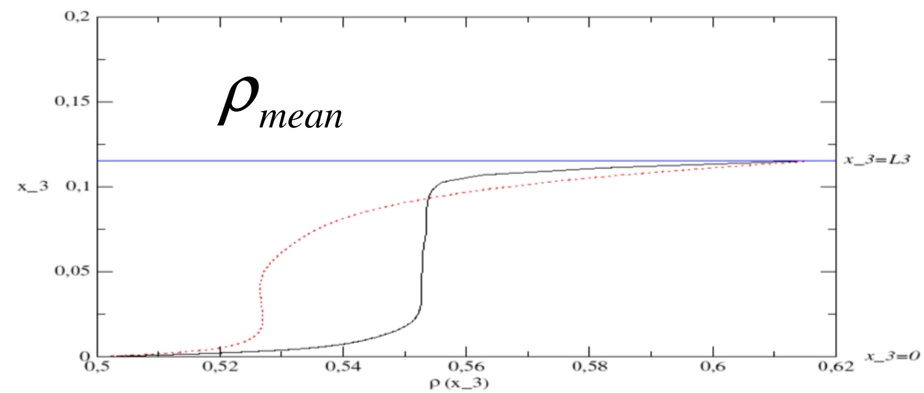
	γ	Ri	Re	Pr
C1	-1	0.001	400	0.7
C2	+1	0.0001	400	0.7
C3	-1	0	400	0.7
C4	+1	0	400	0.7

	Np	dp	ρ part/ρ fluid	St
P1(tracers)	15,000	2.5e-5	1	
P2	15,000	2.5e-5	1217	1.026
P3	15,000	2.5e-5	2435	2.053
P4	15,000	2.5e-5	4869	4.106

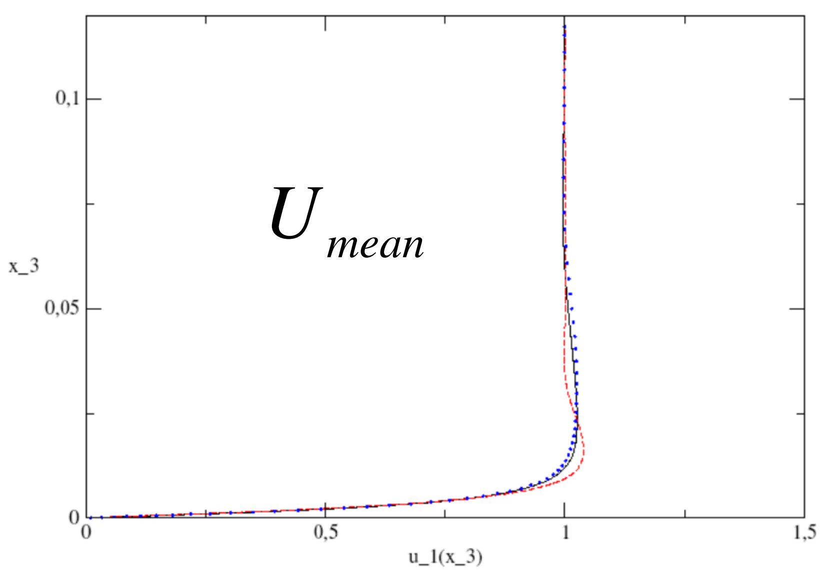
PDSBEL: *the Eulerian Field*

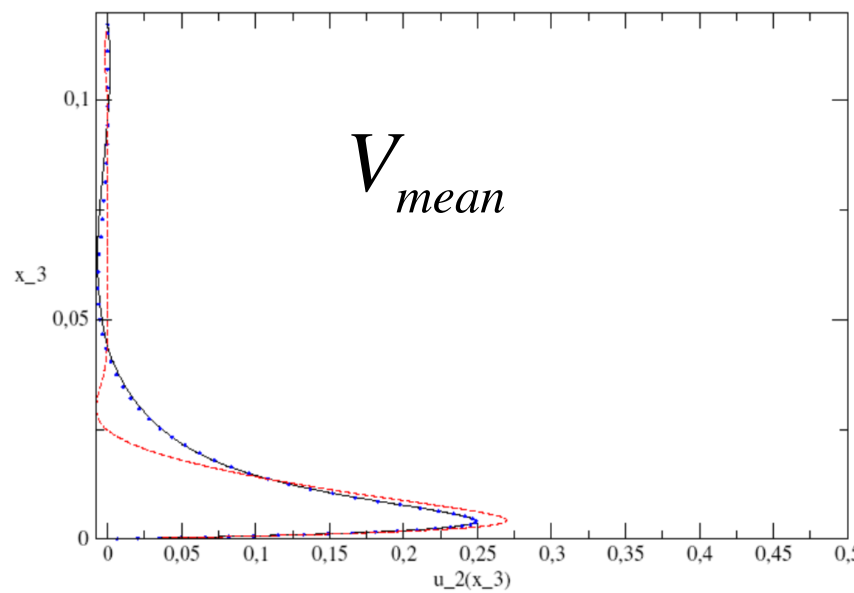


Stable stratification: solid black line C1; red dotted line C3



Unstable stratification: solid black line C2; red dotted line C4

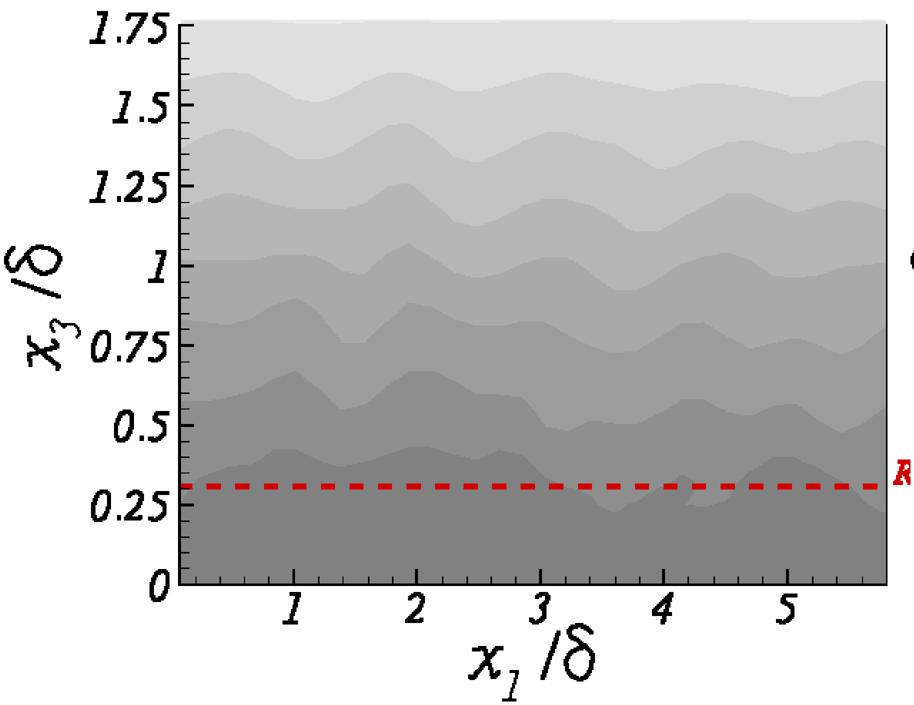




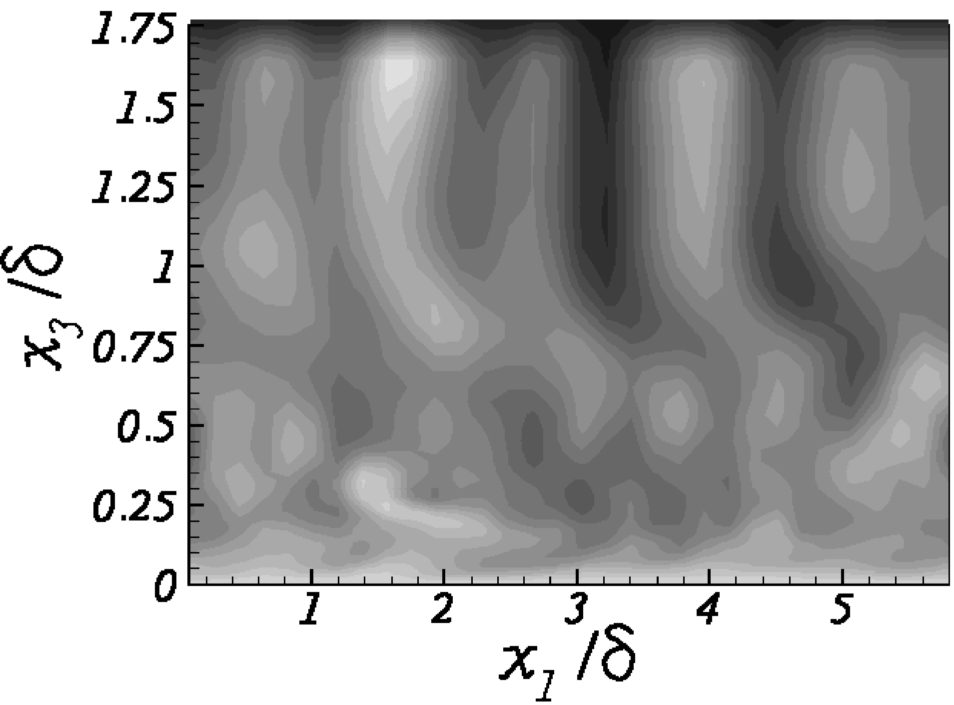
Red dashed lines C1 (stably stratified), blue circles C2 (unstably stratified), black lines cases C3 (passive-scalar case)

PDSBEL: *animation of the density field*

Stable stratification



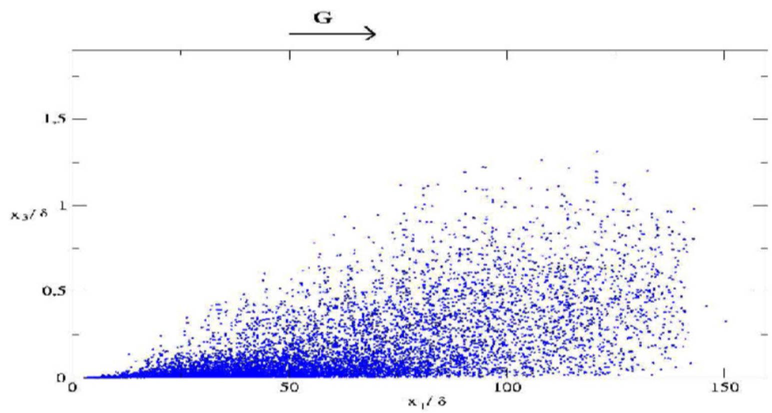
Unstable stratification



PDSBEL: *The particle motion*

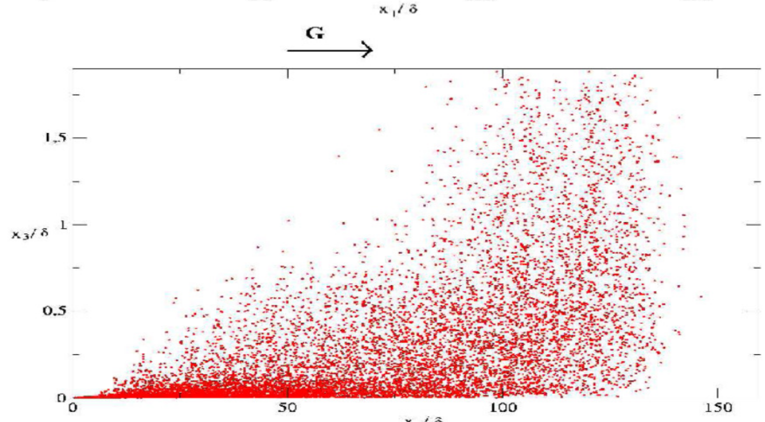
P4: heaviest particles

stable

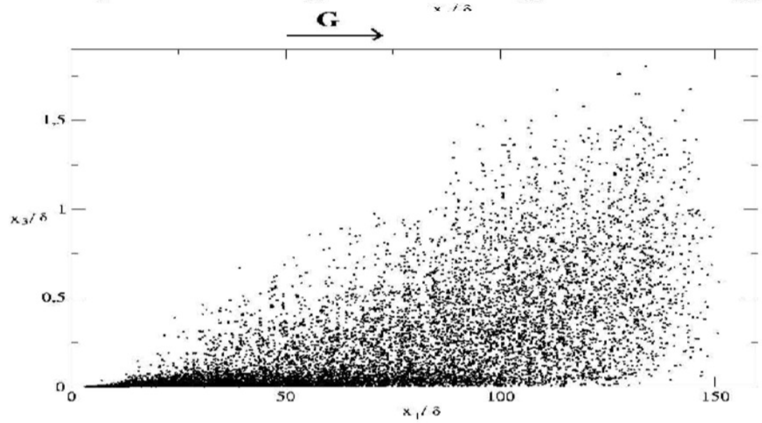


Lateral view of the final particle positions

unstable



neutral



PDSBEL: *The particle motion*

Top view of the final particle positions

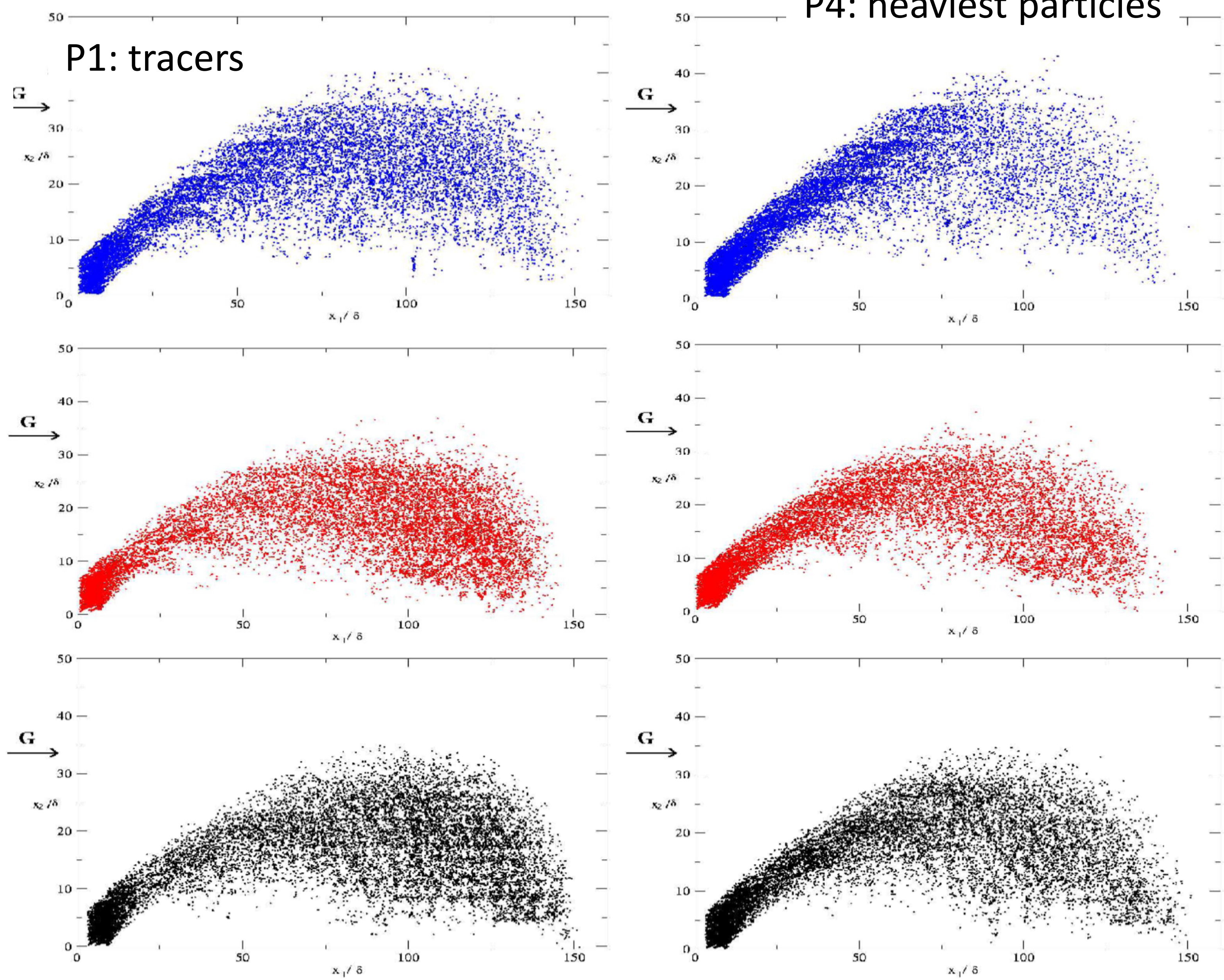
P1: tracers

P4: heaviest particles

stable

unstable

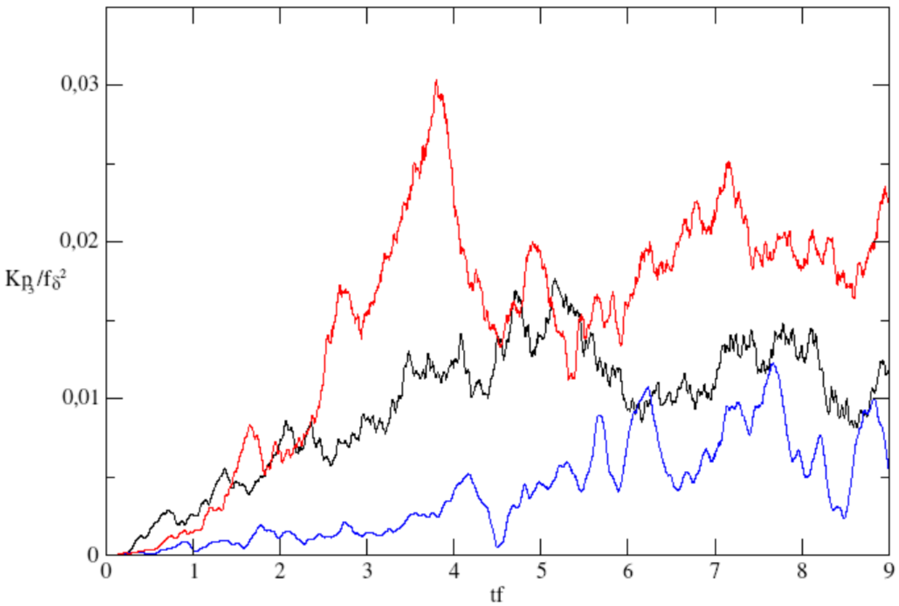
neutral



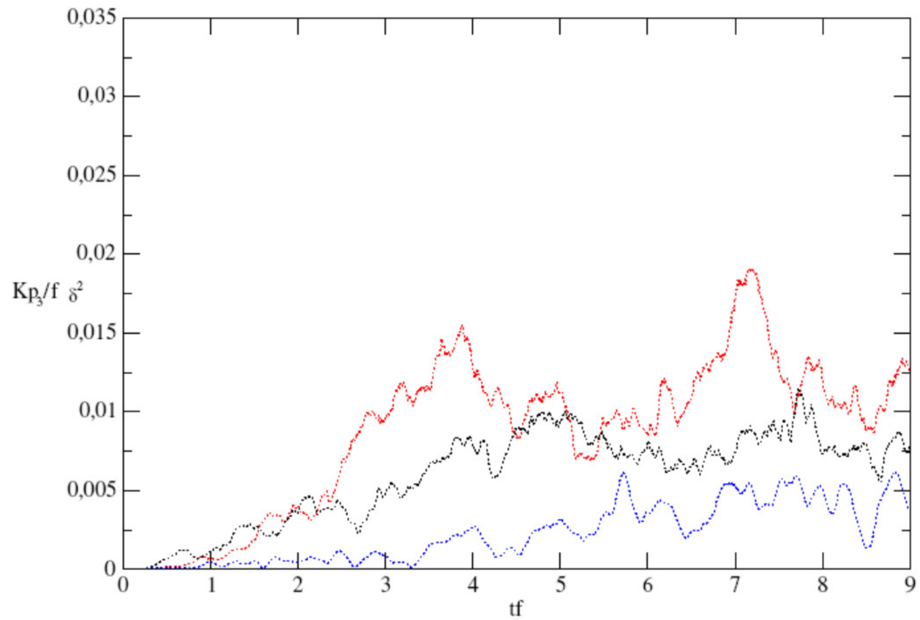
PDSBEL: *The particle diffusivity*

Particle vertical diffusivity: blue lines P-C1 (stable); red lines P-C2 (unstable); black lines P-C3 (neutral)

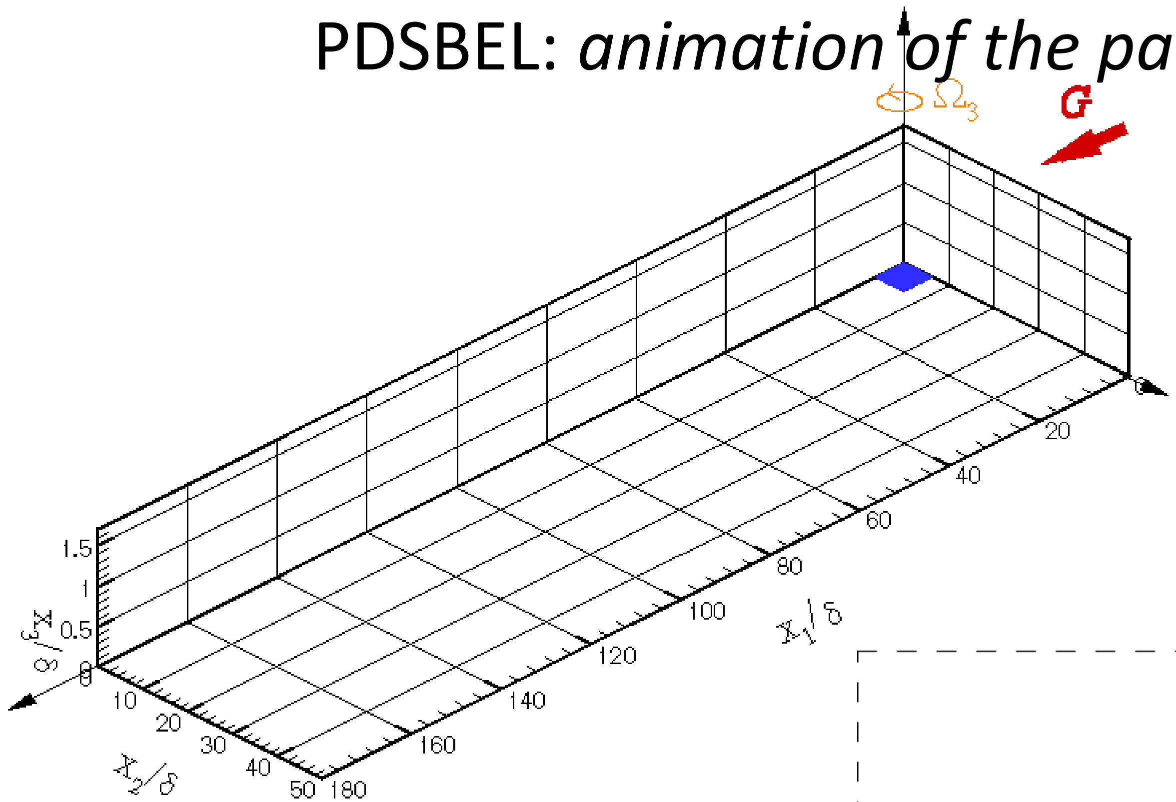
P1: tracers



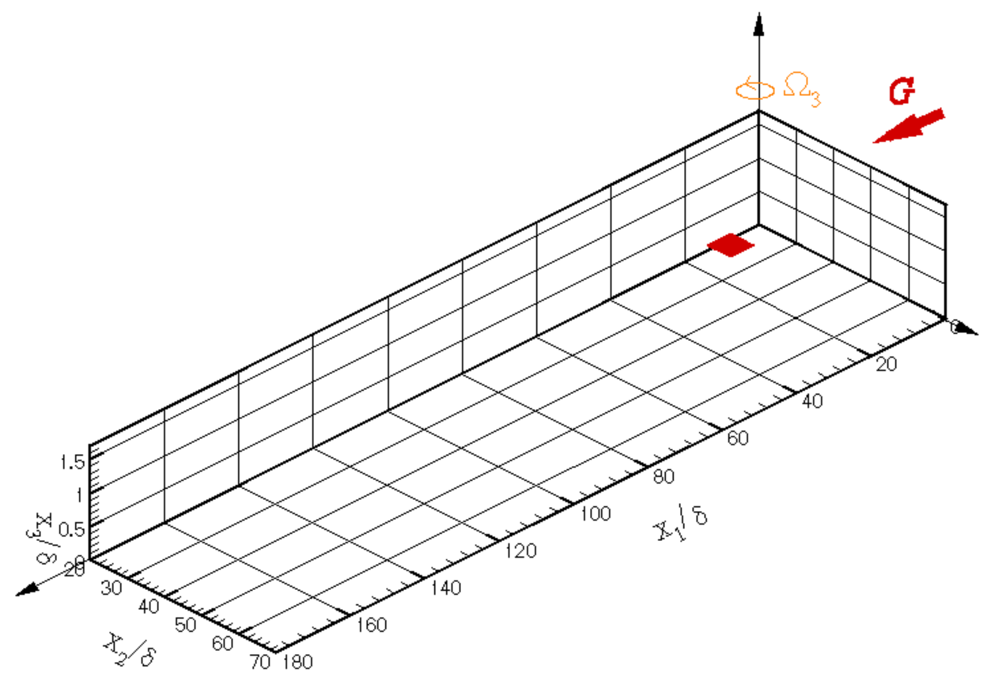
P4: heaviest particles



PDSBEL: *animation of the particle motion*



P4: stable stratification

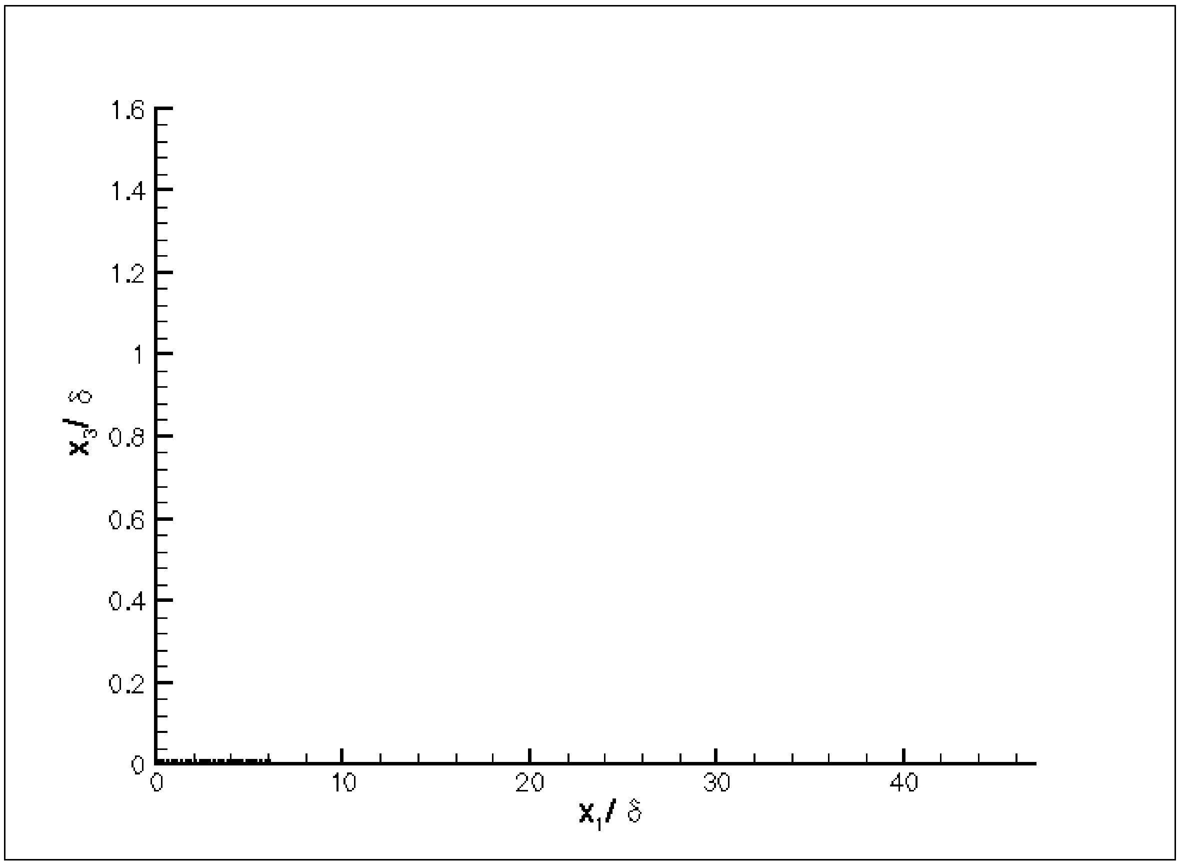


P4: unstable stratification

PDSBEL: *front view animation*

P4: neutral flow

Front view



•Particles are alternatively ejected by the low-speed streaks while travelling along the transversal direction

•Once they reach the upper region with zero transversal velocity they move up and down along vertical columns

Lagrangian particle motion in stratified environment: *remarks*

•Lagrangian particles, either tracers or inertial ones, feel the modifications of the turbulent field due to stratification

•Less vertical transport and modified horizontal transport under stratification

•For particles with density similar to that of the ambient flow, the density variation in the particle motion equation must be considered

Thanks for your kind attention

In the present notes some figures and formulas are taken by the excellent book: Fluid Mechanics, 2^{nd} Edition, Kundu and Cohen

Some figures on homogeneous stratified turbulence are taken by the papers by Sutanu Sarkar and co-authors and from lecture notes of my friend Sutanu